

ANSWERS

Unit 1	Errors in measurement	p.A1 ~ p.A4
Unit 2	Simple factorization & grouping terms	p.A4 ~ p.A5
Unit 3	Algebraic identities & factorization	p.A5 ~ p.A8
Unit 4	Algebraic fractions	p.A9 ~ p.A11
Unit 5	Use of formulae	p.A11 ~ p.A16
Unit 6	Congruent triangles	p.A16 ~ p.A21
Unit 7	Isosceles triangles	p.A21 ~ p.A25
Unit 8	Angles related to polygons	p.A26 ~ p.A30
Unit 9	Rate, ratio & proportion	p.A31 ~ p.A37
Unit 10	Similar triangles	p.A37 ~ p.A44
Unit 11	Graphs of linear equations in two unknowns	p.A44 ~ p.A46
Unit 12	Simultaneous linear equations in two unknowns	p.A46 ~ p.A56
Unit 13	Rational & irrational numbers	p.A56 ~ p.A59
Unit 14	Pythagoras' theorem	p.A59 ~ p.A64
Unit 15	Introduction to trigonometric ratios	p.A64 ~ p.A72
Unit 16	Areas (2): Arcs and sectors	p.A72 ~ p.A78
Unit 17	Volumes(2): Cylinders	p.A78 ~ p.A81
Unit 18	More about statistical charts	p.A81 ~ p.A86

Unit 1 Errors in measurement

1.

	(a)	(b)	(c)	(d)
lower limit	419.5 cm	98.5	256.75 g	6995
upper limit	420.5 cm	99.5	256.85 g	7005
2. (a) The speed = $\frac{31.225 + 31.235}{2} = 31.23 \text{ km/h}$
 (b) 4 sig. fig., or 2 decimal places, or 0.01 km/h
3. The weight lies between 13.95 kg and 14.05 kg.
4. The lower limit = $420\,000 - \frac{100}{2} = \$419\,950$,
 the upper limit = $420\,000 + \frac{100}{2} = \$420\,050$
5. The lower limit = $(38.4 - \frac{0.1}{2}) \div 58 = 0.6612 \text{ minute}$,
 the upper limit = $(38.4 + \frac{0.1}{2}) \div 58 = 0.6629 \text{ minute}$
6. 30 m: lower limit = $30 - \frac{1}{2} = 29.5 \text{ m}$, upper limit = $30 + \frac{1}{2} = 30.5 \text{ m}$
 16 m: lower limit = $16 - \frac{1}{2} = 15.5 \text{ m}$, upper limit = $16 + \frac{1}{2} = 16.5 \text{ m}$
 $\therefore$ lower limit of the perimeter = $2 \times (29.5 + 15.5) = 90 \text{ m}$,
 upper limit of the perimeter = $2 \times (30.5 + 16.5) = 94 \text{ m}$
7. 12 cm: lower limit = $12 - \frac{0.5}{2} = 11.75 \text{ cm}$, upper limit = $12 + \frac{0.5}{2} = 12.25 \text{ cm}$
 8 cm: lower limit = $8 - \frac{0.5}{2} = 7.75 \text{ cm}$, upper limit = $8 + \frac{0.5}{2} = 8.25 \text{ cm}$
 $\therefore$ lower limit of the area = $11.75 \times 7.75 = 91.1 \text{ cm}^2$ (3 sig. fig.),
 upper limit of the area = $12.25 \times 8.25 = 101 \text{ cm}^2$ (3 sig. fig.)
8.

	Max absolute error	Relative error
(a)	0.5 cm^3	$= \frac{0.5}{336} = \frac{1}{672}$
(b)	0.005 kg	$= \frac{0.005}{0.84} = \frac{1}{168}$
(c)	0.05° C	$= \frac{0.05}{19} = \frac{1}{380}$
(d)	$\$ 5$	$= \frac{5}{780} = \frac{1}{156}$
9.

	Max absolute error	% error (3 sig. fig.)
(a)	$= \frac{5}{2} = 2.5 \text{ g}$	$= \frac{2.5}{165} \times 100\% = 1.52\%$
(b)	$= \frac{2}{2} = 1 \text{ mm}$	$= \frac{1}{5.8} \times 100\% = 17.2\%$
(c)	$= \frac{0.01}{2} = 0.005 \text{ L}$	$= \frac{0.005}{22} \times 100\% = 0.0227\%$
(d)	$= \frac{10}{2} = 5 \text{ s}$	$= \frac{5}{960} \times 100\% = 0.521\%$
10. (a) The maximum absolute error = $\frac{10}{2} = 5 \text{ g}$, relative error = $\frac{5}{540} = \frac{1}{108}$,
 $\% \text{ error} = \frac{1}{108} \times 100\% = 0.926\%$ (3 sig. fig.)
 (b) The maximum absolute error = $\frac{5}{2} = 2.5 \text{ cm}^2$, relative error = $\frac{2.5}{240} = \frac{1}{96}$,

$$\% \text{ error} = \frac{1}{96} \times 100\% = 1.04\% \quad (3 \text{ sig. fig.})$$

11. The maximum absolute error = $66^\circ \times \frac{1}{12} = 5.5^\circ$,
 $\therefore$ the true value lies between $(66 \pm 5.5^\circ)$, that is between 60.5° and 71.5° .
12. (a) The maximum error = $49 \times 5\% = 2.45 \text{ kg}$
 (b) The range of actual weight = $(49 \pm 2.45) \text{ kg}$,
 that is between 46.55 kg and 51.45 kg .
13. (a) The maximum absolute error = 0.5 m/s ,
 $\therefore$ the $\% \text{ error} = \frac{0.5}{26} \times 100\% = 1.92\% \quad (3 \text{ sig. fig.})$
 (b) The maximum error in $1 \text{ s} = 0.5 \text{ m}$,
 $\therefore$ the max error in $20 \text{ minutes} = 0.5 \times 20 \times 60 = 600 \text{ m}$
14. The height of the boy = $\frac{154.5 + 165.5}{2} = 165 \text{ cm}$,
 the maximum absolute error = $165.5 - 165 = 0.5 \text{ cm}$,
 $\therefore$ the $\% \text{ error} = \frac{0.5}{165} \times 100\% = 0.303\% \quad (3 \text{ sig. fig.})$
15. (a) The maximum $\% \text{ error} = \frac{0.2}{13} \times 100\% = 1.54\% \quad (3 \text{ sig. fig.})$
 (b) The maximum $\% \text{ error} = \frac{15}{185} \times 100\% = 8.11\% \quad (3 \text{ sig. fig.})$
16. Lower limit of the speed = 8.5 m/s , upper limit of the speed = 9.5 m/s .
 $\therefore$ The least possible distance travelled in 1 day
 $= 8.5 \times (24 \times 60 \times 60) = 734\,400 \text{ m} = 734.4 \text{ km}$
 The greatest possible distance travelled in 1 day
 $= 9.5 \times (24 \times 60 \times 60) = 820\,800 \text{ m} = 820.8 \text{ km}$
17. 8.3 m lies between $(8.3 \pm 0.05) \text{ m}$; 17.4 m lies between $(17.4 \pm 0.05) \text{ m}$.
 $\therefore$ The shortest length of the remaining rope = $17.35 - 8.35 = 9 \text{ m}$
 The longest length of the remaining rope = $17.45 - 8.25 = 9.2 \text{ m}$
18. The area lies between $(64 \pm 0.5) \text{ cm}^2$; the base lies between $(12 \pm 0.5) \text{ cm}$.
 $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}, \therefore \text{height} = \frac{2 \times \text{area}}{\text{base}}$
 $\therefore$ Lower limit of the height = $\frac{2 \times 63.5}{12.5} = 10.16 \text{ cm}$
 Upper limit of the height = $\frac{2 \times 64.5}{11.5} = 11.22 \text{ cm}$
19. The shortest time = $\frac{45 \times 24}{80 + 4} \text{ min} + 45 \times (9 - 1) \text{ s} = 18 \text{ min } 51 \text{ s} \quad (\text{to the nearest s})$
 The longest time = $\frac{45 \times 24}{80 - 4} \text{ min} + 45 \times (9 + 1) \text{ s} = 21 \text{ min } 43 \text{ s} \quad (\text{to the nearest s})$
20. (a) The lower limit = $85 \times (1 - 4\%) = 81.6 \text{ g}$.
 $\therefore 82 \text{ g}$ is greater than the lower limit, $\therefore$ it will be accepted.
 (b) The upper limit of the weight a packet of noodles = $85 \times (1 + 4\%) = 88.4 \text{ g}$.
 $\therefore$ The no. of packets = $\frac{2 \text{ kg}}{88.4 \text{ g}} = \frac{2\,000}{88.4} \approx 22.6$

Ans. The maximum no. of packets is 22.

21. The max absolute error = $\frac{0.5}{2} = 0.25$ cm.

Let the length of the measured object be x cm when the % error is 1%.

$$\therefore \frac{0.25}{x} \times 100\% = 1\%, \quad x = 0.25 \times 100 = 25$$

If the object is shorter than x , the error will be greater than 1%.

Ans. He can measure objects with length between 25 cm and 1 m.

22. (a) 3 min 20 s = $3 \times 60 + 20 = 200$ s; the max error = $0.5x$

$$\therefore \frac{0.5x}{200} \times 100\% = 1.25\%, \quad x = \frac{1.25 \times 200}{100 \times 0.5} = 5$$

(b) The max. error = $\frac{x}{2} = \frac{5}{2} = 2.5$ s, $\therefore$ the range = 3 min 20 s $\pm$ 2.5 s.

$\therefore$ It lies between 3 min 17.5 s and 3 min 22.5 s.

23. (a) 6.5 cm

(b) Maximum absolute error = $\frac{0.5}{2} = 0.25$ cm, relative error = $\frac{0.25}{6.5} = \frac{1}{26}$

(c) Relative error = $\frac{\text{maximum absolute error}}{\text{measured value}} = \frac{0.25}{\text{measured value}}$

If the other crayon is shorter, the measured value is smaller.

The denominator of the relative error is smaller while the numerator remains unchanged, thus the relative error becomes larger.

24. (a) (i) $(8+x)(3+x) = 24 + 11x + x^2$

(ii) $(8-x)(3-x) = 24 - 11x + x^2$

(b) Upper limit of area of triangle = $\frac{(8+k)(3+k)}{2} = \frac{24 + 11k + k^2}{2}$

Lower limit of area of triangle = $\frac{(8-k)(3-k)}{2} = \frac{24 - 11k + k^2}{2}$

Difference = $\frac{24 + 11k + k^2}{2} - \frac{24 - 11k + k^2}{2} = 11k$

25. (a) 0.5 cm

(b) Pentagon = trapezium + rectangle, the required lower limit

$$= \frac{(27.5 + 19.5)(16.5 - 13.5)}{2} + (27.5)(13.5) = 441.75 \text{ cm}^2$$

Alternative method:

Pentagon = rectangle - triangle

$$\text{The required lower limit} = 27.5 \times 16.5 - \frac{1}{2}(27.5 - 19.5)(16.5 - 13.5) = 441.75 \text{ cm}^2$$

(c) Upper limit of the area of the paper

$$= \frac{(28.5 + 20.5)(17.5 - 14.5)}{2} + (28.5)(14.5) = 486.75 \text{ cm}^2 > 483 \text{ cm}^2 \quad \therefore \text{It is possible.}$$

26. (a) The least possible weight = $\left(500 - \frac{1}{2}\right) \text{ g} = 499.5 \text{ g}$

(b) (i) The upper limit of the weight of the bag of flour

$$= \left(22.7 + \frac{0.1}{2}\right) \text{ kg} = 22.75 \text{ kg} = 22\,750 \text{ g.}$$

Actual weight of the bag of flour $< 22\,750 \text{ g}$. $\therefore$ It is impossible.

- (ii) The greatest number of bags $= \frac{22750}{499.5} = 45.545... < 46$. $\therefore$ It is impossible.

Unit 2 Simple factorization & grouping terms

- $= 6(3y^2 + 1)$
 - $= 7x(x - 7)$
 - $= 3x(3y - 4t + 5ty)$
 - $= 4x(4x^2 - 2x + 1)$
 - $= 12bc(3a^2 + 2ac + 5)$
 - $= 15r^3s^2(-3r - 4s + 5s^2)$
- $= (x + y)(1 - k)$
 - $= 10(a^2 + b^2)(3p + 5q)$
 - $= (m + n)^3 [8(m + n) - 1] = (m + n)^3 (8m + 8n - 1)$
 - $= (a + b)(c + d)(1 + c + d)$
- $= (y^2 - y) + (xy - x) = y(y - 1) + x(y - 1) = (y - 1)(x + y)$
 - $= (na + ny) + (a + y) = n(a + y) + (a + y) = (a + y)(n + 1)$
 - $= (3k^3 + 21k^2) - (k + 7) = 3k^2(k + 7) - (k + 7) = (k + 7)(3k^2 - 1)$
 - $= (mn - 2nr) - (my - 2ry) = n(m - 2r) - y(m - 2r) = (m - 2r)(n - y)$
 - $= (2ab^2c + 6a^2b) - (5bc^2 + 15ac) = 2ab(bc + 3a) - 5c(bc + 3a) = (bc + 3a)(2ab - 5c)$
 - $= (e + f)^2 - (e + f) = (e + f)(e + f - 1)$
- $= r(x - y) + s(x - y) = (x - y)(r + s)$
 - $= 8x^2(4a - 3b) - 4x(4a - 3b) = 4x(4a - 3b)(2x - 1)$
- $= (8ay + 4a^2) + (6xy + 3ax) = 4a(2y + a) + 3x(2y + a) = (2y + a)(4a + 3x)$
 - $= (rs - 3r) - (5s - 15) = r(s - 3) - 5(s - 3) = (s - 3)(r - 5)$
 - $= (p^2 - p) - (pq - q) = p(p - 1) - q(p - 1) = (p - 1)(p - q)$
 - $= (c + b) - (ac^2 + abc) = (c + b) - ac(c + b) = (c + b)(1 - ac)$
 - $= (pq + r) + (r^2pq + rp^2q^2) = (pq + r) + rpq(r + pq) = (r + pq)(1 + rpq)$
 - $= (xz - 2x^2z) - (3yz^2 - 6xyz^2) = xz(1 - 2x) - 3yz^2(1 - 2x) = z(1 - 2x)(x - 3yz)$
- $= (6x^2 + 9) + (2ax^3 + 3ax) + (2x^2y + 3y)$
 $= 3(2x^2 + 3) + ax(2x^2 + 3) + y(2x^2 + 3) = (2x^2 + 3)(3 + ax + y)$
 - $= (ah^2 + bhn - hp) - (ahn + bn^2 - np) = h(ah + bn - p) - n(ah + bn - p)$
 $= (ah + bn - p)(h - n)$
 - $= (x^3 - x^2y + x^2) - (3x - 3y + 3) = x^2(x - y + 1) - 3(x - y + 1)$
 $= (x - y + 1)(x^2 - 3)$
 - $= (pt^2 - p^2t + 3rp^2t) + (6rt - 6rp + 18r^2p) = pt(t - p + 3rp) + 6r(t - p + 3rp)$
 $= (t - p + 3rp)(pt + 6r)$
- $-x + 1 = 1 - x$; $\therefore x - 1$ is different.
 - $-a + b = -(a - b)$; $\therefore a + b$ is different.
 - $(x - y)(a - b) = [-(y - x)][-(b - a)] = (y - x)(b - a)$; $\therefore (x + y)(a + b)$ is different.
 - $(m - n)^2 = [-(n - m)]^2 = (n - m)^2$; $\therefore -(n - m)^2$ is different.
 - $(x - y)^3 = [-(y - x)]^3 = (-1)^3(y - x)^3 = -(y - x)^3$; $\therefore (y - x)^3$ is different.
- $= 2(a + b)(p - q) - 4(p - q)(x + y) = 2(p - q)(a + b - 2x - 2y)$
 - $= 18(p - q)^3 - 12(p - q)^2 = 6(p - q)^2(3p - 3q - 2)$
 - $= (y + 3)^2(5y + 1)$
 - $= (y - x)^2(m - n)^3 + (y - x)^3(m - n)^4 = (y - x)^2(m - n)^3 [1 + (y - x)(m - n)]$
- $= (8p + 8q) + (p + q)^2 = 8(p + q) + (p + q)^2 = (p + q)(8 + p + q)$
 - Cannot be factorized.
 - Cannot be factorized.
 - Cannot be factorized.

- (e) $= 4rs^2 - 4t^2 + st - 16rst = (4rs^2 - 16rst) + (st - 4t^2)$
 $= 4rs(s - 4t) + t(s - 4t) = (s - 4t)(4rs + t)$
- (f) Cannot be factorized.
- (g) $= (3n + 4mn) - (6p + 8mp) + (18m + 24m^2)$
 $= n(3 + 4m) - 2p(3 + 4m) + 6m(3 + 4m) = (3 + 4m)(n - 2p + 6m)$
10. $= 4[-(a+b)]^2 - (a+b) = 4(a+b)^2 - (a+b) = (a+b)[4(a+b) - 1] = (a+b)(4a + 4b - 1)$
11. $= (x-y)^3 + 9(x-y)^2 - (x-y) = (x-y)[(x-y)^2 + 9(x-y) - 1]$
 $= (x-y)(x^2 - 2xy + y^2 + 9x - 9y - 1)$
12. $= \ell(s^2t + \ell s^2t + s + \ell s - st^2 - t) = \ell[(s^2t + s) + (\ell s^2t + \ell s) - (t + st^2)]$
 $= \ell[s(st+1) + \ell s(st+1) - t(st+1)] = \ell(st+1)(s + \ell s - t)$
13. $= (1 + x^2 + x^4 + x^6) + x(1 + x^2 + x^4 + x^6) = (1 + x^2 + x^4 + x^6)(1 + x)$
 $= [(1 + x^2) + x^4(1 + x^2)](1 + x) = (1 + x^4)(1 + x^2)(1 + x)$
Ans. Its 3 factors are $(1 + x)$, $(1 + x^2)$, and $(1 + x^4)$.
14. $= 1 + x + 2x + 2x^2 - x^2 - x^3 = (1 + x) + 2x(1 + x) - x^2(1 + x) = (1 + x)(1 + 2x - x^2)$
15. The area of David's rectangle $= 15(2y^3 + y^2 + 2y + 1) = 15[y^2(2y + 1) + (2y + 1)]$
 $= 15(2y + 1)(y^2 + 1) \text{ cm}^2$
 The width of Donna's rectangle $= (30y + 15) = 15(2y + 1) \text{ cm}$
 $\therefore$ the two rectangles have the same area,
 $\therefore$ the length of Donna's rectangle $= \frac{15(2y + 1)(y^2 + 1)}{15(2y + 1)} = (y^2 + 1) \text{ cm}$
16. (a) When $n = 1$, $S = 1^2 + 1 + 41 = 43$; When $n = 2$, $S = 2^2 + 2 + 41 = 47$;
 When $n = 3$, $S = 3^2 + 3 + 41 = 53$; When $n = 4$, $S = 4^2 + 4 + 41 = 61$;
 When $n = 5$, $S = 5^2 + 5 + 41 = 71$
- (b) $S = n^2 + n + 41 = n(n+1) + 41$
 When $n = 41$, $S = 41(42) + 41 = 41(42 + 1)$ which is a composite number.
 When $n = 40$, $S = 40(41) + 41 = 41(40 + 1)$ which is also a composite number.
Ans. The two values of n are 40 and 41.

Unit 3 Algebraic identities & factorization

1. $x^2 + 6x + p \equiv x^2 + 2qx + q^2$, $\therefore 2q = 6$ and $p = q^2$, $q = 3$ and $p = 3^2 = 9$
2. $x^2 + 4x + 4 - x^2 + x + 2 \equiv Ax + (B - A)$, $5x + 6 \equiv Ax + (B - A)$,
 $\therefore A = 5$ and $B - A = 6$; $\therefore B - 5 = 6$, $B = 11$. *Ans. $A = 5$ and $B = 11$.*
3. $x^2 - 6x + 9 + Ax - 3A \equiv x^2 + B$, $x^2 + (A - 6)x + (9 - 3A) \equiv x^2 + B$, $\therefore A - 6 = 0$
 and $9 - 3A = B$. $A = 6$ and $9 - 3(6) = B$, $B = -9$. *Ans. $A = 6$ and $B = -9$.*
4. $3Ax^2 + (3B + A)x + B \equiv 6x^2 + Cx - 2$, $\therefore 3A = 6$, $3B + A = C$ and $B = -2$,
 $A = 2$ and $3(-2) + 2 = C$, $C = -4$. *Ans. $A = 2$, $B = -2$ and $C = -4$.*
5. L.H.S. $= [(x + 3y) + (3y - x)][(x + 3y) - (3y - x)] = (6y)(2x) = 12xy = \text{R.H.S.}$
 $\therefore$ It is an identity.
6. (a) L.H.S. $= 3x + 18 + 2x - 16 = 5x + 2$, R.H.S. $= 7x + 14 - 5x - 9 = 2x + 5 \neq \text{L.H.S.}$
 $\therefore$ It is not an identity.
- (b) L.H.S. $= x^2 - 3xy$, R.H.S. $= x^2 - 3xy - 4y^2 + 4y^2 = x^2 - 3xy = \text{L.H.S.}$
 $\therefore$ It is an identity.
- (c) L.H.S. $= 8x - 3y - 2y - 3z - 3x + 2z = 5x - 5y - z$,
 R.H.S. $= 5x - 5y - 5z + 4z = 5x - 5y - z = \text{L.H.S.}$, $\therefore$ It is an identity.

- (d) L.H.S. = $y^2 + 6y + 9 - 10 = y^2 + 6y - 1$, R.H.S. = $y^2 - 1 \neq$ L.H.S.
 $\therefore$ It is not an identity.
- (e) R.H.S. = $2x^2 - (2x^2 - 3x - 9) = 3x + 9 = 3(x + 3) =$ L.H.S., $\therefore$ It is an identity.
7. L.H.S. = $2[36x^2 + 12x + 1 - (36x^2 + 9x - 10)] = 2(3x + 11) = 6x + 22$,
 R.H.S. = $10x - 18 + 55x + 40 - 60x + x = 6x + 22 =$ L.H.S.
 $\therefore$ It is an identity.
8. (a) = $25x^2 + 40xy + 16y^2$ (b) = $49m^2 - 4n^2$ (c) = $9x^2 - 48x + 64$
 (d) = $1 - 16x^2$ (e) = $36a^2b^2 - 25$
 (f) = $(4t - 9)^2 = 16t^2 - 72t + 81$ (g) = $16p^4 - 56p^2 + 49$
9. (a) = $(10 + 2x)^2 = 100 + 40x + 4x^2$ (b) = $4x^2 - 2(2x)(\frac{1}{x}) + \frac{1}{x^2} = 4x^2 - 4 + \frac{1}{x^2}$
 (c) = $2(36x^2 + 60xy + 25y^2) = 72x^2 + 120xy + 50y^2$
 (d) = $(\frac{2}{m})^2 + 2(\frac{2}{m})(\frac{3}{n}) + (\frac{3}{n})^2 = \frac{4}{m^2} + \frac{12}{mn} + \frac{9}{n^2}$
 (e) = $(7a + \frac{3}{a})(7a - \frac{3}{a}) = 49a^2 - \frac{9}{a^2}$
 (f) = $3(16 - y^4) = 48 - 3y^4$ (g) = $(-6x)^2 - (13y)^2 = 36x^2 - 169y^2$
10. (a) = $(3 - 5y)^2$ (b) = $(2n - 7)^2$ (c) = $(x + 8)^2$
 (d) = $(3y + 1)(3y - 1)$ (e) = $(5m + 2n)^2$ (f) = $(7xy + 2)^2$
 (g) = $(9a - 10b)(9a + 10b)$ (h) = $121b^2 - 16a^2 = (4a + 11b)(11b - 4a)$
11. (a) = $3(x^2 - 25) = 3(x - 5)(x + 5)$ (b) = $5(y^2 - 6y + 9) = 5(y - 3)^2$
 (c) = $(ab + 6pq)^2$ (d) = $1 - (4y^2)^2 = (1 + 4y^2)(1 - 4y^2) = (1 + 4y^2)(1 + 2y)(1 - 2y)$
 (e) = $3(x^4 - 9y^4) = 3(x^2 - 3y^2)(x^2 + 3y^2)$ (f) = $9(16 - 9k^2) = 9(4 - 3k)(4 + 3k)$
 (g) = $4(4m^2 - 9) = 4(2m - 3)(2m + 3)$
 (h) = $[3y - (5y + x)][3y + (5y + x)] = -(2y + x)(8y + x)$
 (i) = $p(p^2 - 10pq + 25q^2) = p(p - 5q)^2$
 (j) = $(x^4 - 1)(x^4 + 1) = (x^2 - 1)(x^2 + 1)(x^4 + 1) = (x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$
 (k) = $[(x - 2y) + 7]^2 = (x - 2y + 7)^2$
 (l) = $[2(a - b)]^2 - [5(a + b)]^2 = [(2a - 2b) - (5a + 5b)][(2a - 2b) + (5a + 5b)]$
 $= -(3a + 7b)(7a + 3b)$
12. (a) not possible (not : $4x^2 + 4x + 1$) (b) not possible (not : $y^2 + 14y + 49$)
 (c) $(1 - 3a)^2$ (d) not possible (not : $n^2 - 12n + 36$)
 (e) not possible (not : $x^2 - 9y^2$) (f) = $5(4k^2 - 1) = 5(2k + 1)(2k - 1)$
 (g) not possible (not : $16y^2 + 8y + 1$) (h) = $m^2 + 6m + 9 = (m + 3)^2$
 (i) = $(\frac{a}{3})^2 - 2(\frac{a}{3})(\frac{b}{2}) + (\frac{b}{2})^2 = (\frac{a}{3} - \frac{b}{2})^2$ (j) = $(x)^2 + 2(x)(\frac{1}{x}) + (\frac{1}{x})^2 = (x + \frac{1}{x})^2$
13. (a) = $[(a + b) + c]^2 = (a + b)^2 + 2c(a + b) + c^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
 (b) = $[(x + 3) - 4y][(x + 3) + 4y] = (x + 3)^2 - (4y)^2 = x^2 + 6x + 9 - 16y^2$
 (c) = $[(2x - y) + 6]^2 = (2x - y)^2 + 12(2x - y) + 36$
 $= 4x^2 + y^2 - 4xy + 24x - 12y + 36$
 (d) = $[p + (3q - r)][p - (3q - r)] = p^2 - (3q - r)^2$
 $= p^2 - (9q^2 - 6qr + r^2) = p^2 - 9q^2 + 6qr - r^2$
14. = $(\frac{m-1}{m} + \frac{m+1}{m})(\frac{m-1}{m} - \frac{m+1}{m}) = (\frac{2m}{m})(\frac{-2}{m}) = \frac{-4}{m}$
15. = $(10001 + 9999)(10001 - 9999) = 20000 \times 2 = 40000$
16. = $(x^2 + 2x + 1) + (y + xy) = (x + 1)^2 + y(x + 1) = (x + 1)(x + y + 1)$

17. $= (m^2 + 2mn + n^2) + (m^2n + mn^2) = (m+n)^2 + mn(m+n) = (m+n)(m+n+mn)$
18. $= y^2 - 4 + \frac{4}{y^2} = y^2 - 2(y)(\frac{2}{y}) + (\frac{2}{y})^2 = (y - \frac{2}{y})^2$
19. $= mn + 4 - 2m - 2n = (mn - 2m) - (2n - 4) = m(n-2) - 2(n-2) = (n-2)(m-2)$
20. $= (s^2 - 4rs + 4r^2) - \frac{s^2}{4} = (s-2r)^2 - (\frac{s}{2})^2$
 $= [(s-2r) + \frac{s}{2}][(s-2r) - \frac{s}{2}] = (\frac{3s}{2} - 2r)(\frac{s}{2} - 2r)$
21. $= y(x^2 - y^2 - 2x + 1) = y[(x^2 - 2x + 1) - y^2] = y[(x-1)^2 - y^2] = y(x-1+y)(x-1-y)$
22. $= (2x+1+x^2)(2x+1-x^2) = (x+1)^2(2x+1-x^2)$
23. $= (x^2+9+6x)(x^2+9-6x) = (x+3)^2(x-3)^2$
24. $= x^2 - y^2 - 6x + 6y = (x+y)(x-y) - 6(x-y) = (x-y)(x+y-6)$
25. $= x^2 + y^2 + 2xy + 2xy - 1 - x^2y^2 = (x^2 + 2xy + y^2) - (1 - 2xy + x^2y^2) = (x+y)^2 - (1-xy)^2$
 $= [(x+y) + (1-xy)][(x+y) - (1-xy)] = (x+y+1-xy)(x+y-1+xy)$
26. $(a + \frac{1}{a})^2 = 2^2, \quad a^2 + 2a(\frac{1}{a}) + \frac{1}{a^2} = 4, \quad a^2 + 2 + \frac{1}{a^2} = 4, \quad a^2 + \frac{1}{a^2} = 2$
27. $= h^2 - k^2 - h + k = (h^2 - k^2) - (h - k) = (h-k)(h+k) - (h-k) = (h-k)(h+k-1)$
28. $= [(a^2 + 6a) + 9]^2 = [(a+3)^2]^2 = (a+3)^4$
29. (a) $(p^2 + 8q^2)^2$ (b) $= (p^4 + 16p^2q^2 + 64q^4) - 16p^2q^2$
 $= (p^2 + 8q^2)^2 - (4pq)^2 = (p^2 + 8q^2 - 4pq)(p^2 + 8q^2 + 4pq)$
30. (a) $(a^2 + b^2)^2$ (b) $= (a^4 + 2a^2b^2 + b^4) - a^2b^2 = (a^2 + b^2)^2 - (ab)^2$
 $= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$
31. (a) $= \frac{1}{3}(x^2 + 6x + 9) = \frac{1}{3}(x+3)^2$
 (b) $= \frac{1}{3}(p+q)^2 + 2p + 2q + 3 = \frac{1}{3}(p+q)^2 + 2(p+q) + 3$
 Put $p+q = x$ in part (a), the result $= \frac{1}{3}[(p+q)+3]^2 = \frac{1}{3}(p+q+3)^2$
32. (a) $= [(\frac{3}{x} + \frac{x}{3}) + (\frac{3}{x} - \frac{x}{3})][(\frac{3}{x} + \frac{x}{3}) - (\frac{3}{x} - \frac{x}{3})] = (\frac{6}{x})(\frac{2x}{3}) = 4$
 (b) From (a), $(\frac{3}{x} + \frac{x}{3})^2 - (\frac{3}{x} - \frac{x}{3})^2 = 4$,
 $\therefore (\frac{3}{x} + \frac{x}{3})^2 - (\sqrt{5})^2 = 4, \quad (\frac{3}{x} + \frac{x}{3})^2 = 9, \quad \frac{3}{x} + \frac{x}{3} = 3 \text{ or } -3$
33. (a) R.H.S. $= (x+y)(x^2 - xy + y^2) = x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 = x^3 + y^3 = \text{L.H.S.}$
 $\therefore$ It is an identity.
 (b) R.H.S. $= (x-y)(x^2 + xy + y^2) = x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3 = x^3 - y^3 = \text{L.H.S.}$
 $\therefore$ It is an identity.
 (c) $x^6 - y^6 = (x^3 + y^3)(x^3 - y^3) = (x^3 + y^3)(x-y)(x^2 + xy + y^2)$ [from (b)]
 $= (x+y)(x^2 - xy + y^2)(x-y)(x^2 + xy + y^2)$ [from (a)]
Ans. Its factors are $(x+y)$, $(x-y)$, $(x^2 - xy + y^2)$ and $(x^2 + xy + y^2)$.
34. Dividend $= 16x^2 + kx + 4$, divisor $= 2x + 5$, quotient $= mx + n$, remainder $= -1$.
 $\therefore$ Dividend $\equiv$ divisor $\times$ quotient + remainder
 $\therefore 16x^2 + kx + 4 \equiv (2x+5)(mx+n) - 1, \quad 16x^2 + kx + 4 \equiv 2mx^2 + 2nx + 5mx + 5n - 1$

Comparing the coefficients, we get:

$$16 = 2m, \quad 4 = 5n - 1 \quad \text{and} \quad k = 2n + 5m,$$

$$\therefore m = \frac{16}{2} = 8, \quad n = \frac{4+1}{5} = 1, \quad k = 2(1) + 5(8) = 42.$$

$$\text{Ans. } m = 8, \quad n = 1, \quad k = 42.$$

$$35. (a) \quad x^2 - 33 = 16y^2, \quad x^2 - 16y^2 = 33, \quad (x + 4y)(x - 4y) = 33;$$

$$\text{but } 1 + 4y = x, \quad 1 = x - 4y; \quad \therefore (x + 4y)(1) = 33, \quad x + 4y = 33$$

$$(b) \quad x = 33 - 4y \dots (i), \quad \text{and } x = 1 + 4y \dots (ii).$$

$$\text{From (i) and (ii), } 1 + 4y = 33 - 4y, \quad 8y = 32, \quad y = 4$$

$$\therefore x = 4(4) + 1 = 17 \quad \text{Ans. } x = 17, \quad y = 4.$$

$$36. \quad x^2 - 2x - y^2 + 4y - 3 = (x^2 - 2x + 1) - (y^2 - 4y + 4) = (x - 1)^2 - (y - 2)^2$$

$$= [(x - 1) + (y - 2)][(x - 1) - (y - 2)] = (x + y - 3)(x - y + 1)$$

$$37. \quad = x^4 + 18^2 - 100x^2 = [x^4 - 2(18)x^2 + 18^2] + 2(18)x^2 - 100x^2$$

$$= (x^2 - 18)^2 - 64x^2 = (x^2 - 18)^2 - (8x)^2 = (x^2 - 18 + 8x)(x^2 - 18 - 8x)$$

$$38. (a) (i) \quad (x - A)^2 = x^2 - 2Ax + A^2$$

$$(ii) \quad (4 + 5x)(5x - 4) = (5x + 4)(5x - 4) = (5x)^2 - 4^2 = 25x^2 - 16$$

$$(b) \quad \text{L.H.S.} = x^2 - 2Ax + A^2 + Bx(3 - x)$$

$$= x^2 - 2Ax + A^2 + 3Bx - Bx^2 = (1 - B)x^2 + (3B - 2A)x + A^2$$

$$\text{R.H.S.} = 5C - (4 + 5x)(5x - 4)$$

$$= 5C - (25x^2 - 16) = 5C - 25x^2 + 16 = -25x^2 + (5C + 16)$$

By comparing coefficients,

$$1 - B = -25 \dots (1), \quad 3B - 2A = 0 \dots (2), \quad 5C + 16 = A^2 \dots (3)$$

$$\text{From (1), } B = 26;$$

$$\text{From (2), } 3(26) - 2A = 0, \quad A = 39;$$

$$\text{From (3), } 5C + 16 = 39^2, \quad C = 301$$

$$39. (a) (i) \quad = (4x)^2 - 2(4x)(1) + 1^2 = (4x - 1)^2$$

$$(ii) \quad = (4x - 1)^2 - (5y)^2 = (4x - 1 - 5y)(4x - 1 + 5y)$$

$$(b) \quad = 16x^2 - 8x - 4x - 25y^2 - 5y + 1 + 1$$

$$= (16x^2 - 8x + 1 - 25y^2) - 4x + 1 - 5y$$

$$= (4x - 1 - 5y)(4x - 1 + 5y) - (4x - 1 + 5y) = (4x - 1 + 5y)[(4x - 1 - 5y) - 1]$$

$$= (4x - 1 + 5y)(4x - 2 - 5y)$$

$$40. (a) \quad \text{L.H.S.} = k^2 + x(x + 3k) = k^2 + x^2 + 3kx$$

$$\text{R.H.S.} = (x + k)^2 + kx = x^2 + 2kx + k^2 + kx = k^2 + x^2 + 3kx = \text{L.H.S.}$$

$\therefore$ It is an identity.

$$(b) (i) \quad x = y - 2k,$$

$$\therefore x + k = y - 2k + k = y - k, \quad (x + k)^2 = (y - k)^2$$

$$(ii) \quad \text{L.H.S. of the identity in (a)}$$

$$= k^2 + x(x + 3k)$$

$$= k^2 + (y - 2k)[(y - 2k) + 3k]$$

$$= k^2 + (y - 2k)(y + k)$$

$$\text{R.H.S. of the identity in (a)}$$

$$= (x + k)^2 + kx$$

$$= (y - k)^2 + k(y - 2k) \quad [\text{From (b(i))}]$$

$$\therefore \text{From (a), } k^2 + (y - 2k)(y + k) \equiv (y - k)^2 + k(y - 2k).$$

$\therefore$ The claim is agreed.

$$41. (a) \quad \text{Area of P} = (9a^2 + 42a + 49) = [(3a)^2 + 2(3a)(7) + 7^2] = (3a + 7)^2$$

$\therefore$ The length of each side of P is $(3a + 7)$.

$$(b) \quad \text{Area of Q} = [(9a^2 + 42a + 49) - (42a + 74)]$$

$$= 9a^2 - 25$$

$$= (3a - 5)(3a + 5)$$

$$(c) \quad \therefore \text{Area of Q} = (\text{width})(\text{length}), \quad \therefore \text{width of Q} = 3a + 5.$$

Unit 4 Algebraic fractions

1. (a) $\frac{3}{5y^4}$ (b) $\frac{3x^4}{2}$ (c) $\frac{3b^2}{7}$ (d) $\frac{3(x+2y)}{x-2y}$
 (e) $\frac{5x^2(z-4)}{z}$ (f) $\frac{4}{3(2a+b)}$
2. (a) $= \frac{9x^2(6x-1)}{9x} = x(6x-1)$ (b) $= \frac{2(3m-n)}{10x(3m-n)} = \frac{1}{5x}$
 (c) $= \frac{a-b}{(a-b)(a+b)} = \frac{1}{a+b}$
 (d) $= \frac{4(x^2-2x+1)}{x(x-a)-(x-a)} = \frac{4(x-1)^2}{(x-a)(x-1)} = \frac{4(x-1)}{x-a}$
 (e) $= \frac{3x(x^2-a)+(x^2-a)}{7x(3x+1)} = \frac{(3x+1)(x^2-a)}{7x(3x+1)} = \frac{x^2-a}{7x}$
3. (a) $= \frac{2(3-2a)}{-3(3-2a)} = -\frac{2}{3}$ (b) $= \frac{8(1-k)}{-k^2(1-k)} = -\frac{8}{k^2}$
 (c) $= \frac{-y(y^2+5)}{y^2+5} = -y$ (d) $= \frac{9(x-y)^2}{12(x-y)} = \frac{3}{4}(x-y)$
 (e) $= \frac{-3x^2(5y-2x)}{7(5y-2x)} = -\frac{3x^2}{7}$
4. (a) $\frac{5}{6}$ (b) $= \frac{12m}{7} \times \frac{21}{4m} = 9$ (c) $= \frac{x^4}{3(x-2)} \times \frac{3}{x^3} = \frac{x}{x-2}$
 (d) $= 9ab^3 \times \frac{6}{a^2b^2} = \frac{54b}{a}$ (e) $= \frac{1}{5}ab^2 \times \frac{b^2}{5a} \times a^2 = \frac{a^2b^4}{25}$
 (f) $= n^2x^3 \times \frac{3}{2nx^2} \times \frac{3}{2n} = \frac{9x}{4}$ (g) $= \frac{3(b+2)}{6(b-2)} \times \frac{4(b-2)}{5(2+b)} = \frac{2}{5}$
 (h) $= \frac{m(1-a)}{b(x-1)} \div \frac{c(1-a)}{n(1-x)} = \frac{m(1-a)}{b(x-1)} \times \frac{n(1-x)}{c(1-a)} = -\frac{mn}{bc}$
5. (a) $= \frac{4(b+1)}{a-3} \times \frac{18(a-3)}{4(a-1)} \times \frac{1}{6(b+1)} = \frac{3}{a-1}$
 (b) $= \frac{(x-y)(x+y)}{x^2+y^2} \times \frac{1}{(x+y)^2} \times [(x^2)^2 - (y^2)^2]$
 $= \frac{(x-y)}{(x+y)(x^2+y^2)} \times (x^2+y^2)(x^2-y^2) = \frac{(x-y)(x+y)(x-y)}{x+y} = (x-y)^2$
 (c) $= \frac{b(a+b)}{a(2a-b)} \times \frac{5(2a-b)}{b^2(3a+1)} \times \frac{1}{a+b} = \frac{5}{ab(3a+1)}$
 (d) $= \frac{x(y+1)-(y+1)}{x^2(x-1)-(x-1)} \times \frac{-12(x-1)}{8(y+1)} = \frac{(x-1)(y+1)}{(x^2-1)(x-1)} \times \frac{-3(x-1)}{2(y+1)} = \frac{-3(x-1)}{2(x-1)(x+1)} = \frac{-3}{2(x+1)}$
6. (a) $= \frac{10y-9y}{12} = \frac{y}{12}$ (b) $= \frac{12r-(2r+3)}{9} = \frac{10r-3}{9}$
 (c) $= \frac{5(a-3)+2(a+2)}{30} = \frac{5a-15+2a+4}{30} = \frac{7a-11}{30}$

- (d) $= \frac{y-7+5}{5} = \frac{y-2}{5}$ (e) $= \frac{4x-(x+2)}{4} = \frac{4x-x-2}{4} = \frac{3x-2}{4}$
7. (a) $\frac{1+5x}{xy}$ (b) $= \frac{3-(4+b)}{3a} = \frac{3-4-b}{3a} = -\frac{b+1}{3a}$
- (c) $= \frac{3(2x+a)-2(a-3x)-(x+a)}{6x} = \frac{6x+3a-2a+6x-x-a}{6x} = \frac{11x}{6x} = \frac{11}{6}$
- (d) $\frac{15m-2}{5m}$ (e) $= \frac{8x-(1-6x)}{4x} = \frac{8x-1+6x}{4x} = \frac{14x-1}{4x}$
8. (a) $\frac{12-y}{3(y+5)}$ (b) $= \frac{14-12}{21(x-y)} = \frac{2}{21(x-y)}$
- (c) $= \frac{b}{a-b} + \frac{a}{a-b} = \frac{b+a}{a-b}$ (d) $= \frac{8-x}{6-x} - \frac{2}{6-x} = \frac{8-x-2}{6-x} = \frac{6-x}{6-x} = 1$
- (e) $= \frac{10y+9y-6y}{12(a-b)} = \frac{13y}{12(a-b)}$
9. (a) $= \frac{x(x-1)-2x^2}{x-1} = \frac{x^2-x-2x^2}{x-1} = \frac{x(1+x)}{1-x}$ (b) $= \frac{m+n+m-n}{m+n} = \frac{2m}{m+n}$
- (c) $= \frac{3y(x+y)}{3xy} - \frac{x(x-y)}{3xy} = \frac{3xy+3y^2-x^2+xy}{3xy} = \frac{4xy+3y^2-x^2}{3xy}$
- (d) $= \frac{2(k+2)-3(k-3)}{(k-3)(k+2)} = \frac{2k+4-3k+9}{(k-3)(k+2)} = \frac{13-k}{(k-3)(k+2)}$
- (e) $= \frac{y(2y-x)-x(2x-y)}{(2x-y)(2y-x)} = \frac{2y^2-xy-2x^2+xy}{(2x-y)(2y-x)} = \frac{2(y+x)(y-x)}{(2x-y)(2y-x)}$
10. (a) $= \frac{x-4+3x}{x(x-1)(x-4)} = \frac{4(x-1)}{x(x-1)(x-4)} = \frac{4}{x(x-4)}$
- (b) $= \frac{y-2(y+3)}{(y+3)^2} = \frac{y-2y-6}{(y+3)^2} = -\frac{y+6}{(y+3)^2}$
- (c) $= \frac{4(1-x)-2(x-2)}{(3x-4)(x-2)(1-x)} = \frac{4-4x-2x+4}{(3x-4)(x-2)(1-x)} = \frac{8-6x}{(3x-4)(x-2)(1-x)}$
- $= \frac{-2(3x-4)}{(3x-4)(x-2)(1-x)} = \frac{2}{(x-2)(x-1)}$
- (d) $= \frac{5(a+2)-4(a+1)}{(a+1)(a+6)(a+2)} = \frac{5a+10-4a-4}{(a+1)(a+6)(a+2)}$
- $= \frac{a+6}{(a+1)(a+6)(a+2)} = \frac{1}{(a+1)(a+2)}$
- (e) $= \frac{2(1+k)+2(1-k)-2}{(1+k)(1-k)} = \frac{2+2k+2-2k-2}{(1+k)(1-k)} = \frac{2}{(1+k)(1-k)}$
- (f) $= \frac{4(x-2)+3(x+2)-7x}{(x+2)(x-2)} = \frac{4x-8+3x+6-7x}{(x+2)(x-2)} = \frac{-2}{(x+2)(x-2)}$
11. (a) $= \frac{5}{2(a-4b)} + \frac{a-b}{b(a-4b)} = \frac{5b+2(a-b)}{2b(a-4b)} = \frac{5b+2a-2b}{2b(a-4b)} = \frac{3b+2a}{2b(a-4b)}$
- (b) $= \frac{2}{(x+3)(x-4)} - \frac{1-x}{(4-x)(4+x)} = \frac{2(x+4)+(1-x)(x+3)}{(x+3)(x-4)(x+4)}$
- $= \frac{2x+8+3-2x-x^2}{(x+3)(x-4)(x+4)} = \frac{11-x^2}{(x+3)(x-4)(x+4)}$

$$(c) = \frac{2}{x(x+2)} + \frac{4}{(x+2)(x+6)} = \frac{2(x+6)+4x}{x(x+2)(x+6)} = \frac{2x+12+4x}{x(x+2)(x+6)}$$

$$= \frac{6(x+2)}{x(x+2)(x+6)} = \frac{6}{x(x+6)}$$

$$(d) = \frac{a+b}{b(b-a)} - \frac{a+b}{a(b-a)} = \frac{a+b}{b-a} \left(\frac{1}{b} - \frac{1}{a} \right) = \frac{a+b}{b-a} \left(\frac{a-b}{ab} \right) = -\frac{a+b}{ab}$$

$$12. \frac{x^2(x-y)+y^2(x-y)}{(x^2)^2-(y^2)^2} = \frac{(x^2+y^2)(x-y)}{(x^2+y^2)(x^2-y^2)} = \frac{x-y}{(x+y)(x-y)} = \frac{1}{x+y}$$

$$13. \frac{2a-b}{(3a+b)(x-y)} + \frac{3-x}{4x(x-y)} = \frac{4x(2a-b)}{4x(3a+b)(x-y)} + \frac{(3a+b)(3-x)}{4x(x-y)(3a+b)}$$

$$= \frac{8ax-4bx+9a-3ax+3b-bx}{4x(3a+b)(x-y)} = \frac{5ax-5bx+9a+3b}{4x(3a+b)(x-y)}$$

$$14. \frac{m-3n}{(3m-5n)^2} - \frac{m+n}{(3m-5n)(4+n)} = \frac{(4+n)(m-3n)}{(4+n)(3m-5n)^2} - \frac{(3m-5n)(m+n)}{(3m-5n)^2(4+n)}$$

$$= \frac{4m-12n+mn-3n^2-(3m^2-2mn-5n^2)}{(4+n)(3m-5n)^2} = \frac{4m-12n+3mn+2n^2-3m^2}{(4+n)(3m-5n)^2}$$

$$15. 8ab^2+12+8b+16ab+4b^2+24a = 4(2ab^2+3+2b+4ab+b^2+6a)$$

$$= 4[(3+6a)+(2b+4ab)+(b^2+2ab^2)] = 4(1+2a)(3+2b+b^2)$$

$$\therefore \text{The given expression} = \frac{5(3+2b+b^2)}{4(1+2a)(3+2b+b^2)} - \frac{7a-4}{3(2a-1)(2a+1)}$$

$$= \frac{5}{4(1+2a)} - \frac{7a-4}{3(2a-1)(2a+1)} = \frac{5 \cdot 3(2a-1) - 4(7a-4)}{12(1+2a)(2a-1)}$$

$$= \frac{30a-15-28a+16}{12(1+2a)(2a-1)} = \frac{2a+1}{12(2a+1)(2a-1)} = \frac{1}{12(2a-1)}$$

$$16. = \left(\frac{p^2-q^2}{pq} \right) \div \left(\frac{q-p}{pq} \right) = \frac{(p+q)(p-q)}{pq} \times \frac{pq}{q-p} = -(p+q)$$

$$17. = \frac{2ab+a^2+b^2}{ab} \div \frac{a^2-b^2}{ab} = \frac{(a+b)^2}{ab} \cdot \frac{ab}{(a+b)(a-b)} = \frac{a+b}{a-b}$$

$$18. = 1 \div \left(1 + \frac{(2y) \cdot y}{\left(\frac{1}{y}\right) \cdot y} \right) = 1 \div \left(1 + \frac{2y^2}{1-y^2} \right) = 1 \div \left(\frac{1-y^2+2y^2}{1-y^2} \right) = 1 \div \frac{1+y^2}{1-y^2} = \frac{1-y^2}{1+y^2}$$

$$19. (a) \text{ The other adjacent side} = \text{area} \times 2 \div \text{side} = 2(4x^2 - 20x + 25) \div (2x - 5)$$

$$= 2(2x - 5)^2 \div (2x - 5) = 2(2x - 5) \text{ cm}$$

$$(b) 2(2x - 5) > (2x - 5), \therefore 2(2x - 5) = 18, \quad 2x - 5 = 9, \quad \therefore x = 7$$

$$\therefore \text{Its area} = 4(7)^2 - 20(7) + 25 = 81 \text{ cm}^2$$

Unit 5 Use of formulae

$$1. (a) A = lw \quad (c) T = \frac{D}{S} \quad (d) C = \frac{a}{12} \times m + b \times n, \therefore C = \frac{am}{12} + bn$$

$$2. (a) a = \frac{180(12-2)}{12} = \frac{1800}{12} = 150 \quad (b) 99 = \frac{1}{2}(11)(4+b), \quad 18 = 4+b, \quad \therefore b = 14$$

$$(c) D = (-6)^2 - 4(3)(-2) = 36 + 24 = 60 \quad (d) \frac{1}{6} = \frac{1}{4} - \frac{1}{v}, \quad \frac{1}{v} = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}, \quad \therefore v = 12$$

- (e) $7600 = 4000(1 + 0.15t)$, $1.9 = 1 + 0.15t$, $0.15t = 0.9$, $\therefore t = 6$
3. (a) $s = (10)(3) + \frac{1}{2}(8)(3)^2 = 30 + 36 = 66$
- (b) $84 = (6)(4) + \frac{1}{2}a(4)^2$, $60 = 8a$, $\therefore a = 7.5$
4. (a) $p - b = 2ax$, $\therefore x = \frac{p-b}{2a}$
- (b) $ax + bx = c$, $x(a+b) = c$, $\therefore x = \frac{c}{a+b}$
- (c) $\frac{x}{a} = c - b$, $\therefore x = a(c-b)$
- (d) $m - x = n^2$, $\therefore x = m - n^2$
- (e) $2rx = q - k$, $\therefore x = \frac{q-k}{2r}$
- (f) $2x = 5b - e$, $\therefore x = \frac{5b-e}{2}$
- (g) $8m = h - 4x$, $4x = h - 8m$, $\therefore x = \frac{h-8m}{4}$
- (h) $3y + 3 = x + u$, $\therefore x = 3y + 3 - u$
- (i) $\frac{f}{x} = g - 1$, $\therefore x = \frac{f}{g-1}$
- (j) $ax + b = ct$, $ax = ct - b$, $x = \frac{ct-b}{a}$
5. (a) $m + n = x - ax$, $m + n = x(1-a)$, $\therefore x = \frac{m+n}{1-a}$
- (b) $ax + 2a = bx - 3b$, $2a + 3b = bx - ax$, $2a + 3b = x(b-a)$, $\therefore x = \frac{2a+3b}{b-a}$
- (c) $nx + mx = mns$, $x(n+m) = mns$, $\therefore x = \frac{mns}{n+m}$
- (d) $xy + a = b$, $xy = b - a$, $\therefore x = \frac{b-a}{y}$
- (e) $sh + sx = kr - rx$, $sx + rx = kr - sh$, $x(s+r) = kr - sh$, $\therefore x = \frac{kr-sh}{s+r}$
- (f) $x^2 - cx = x^2 - ax + bx - ab$, $ab = cx - ax + bx$, $ab = x(c-a+b)$,
 $\therefore x = \frac{ab}{c-a+b}$
- (g) $mqx - 3pq = 4pnx$, $mqx - 4pnx = 3pq$, $x(mq - 4pn) = 3pq$,
 $\therefore x = \frac{3pq}{mq-4pn}$
- (h) $\frac{6nx+3m}{2k} = -x$, $6nx+3m = -2kx$, $6nx+2kx = -3m$, $x(6n+2k) = -3m$,
 $\therefore x = \frac{-3m}{6n+2k}$
6. (a) $\frac{s}{180} = n - 2$, $\therefore n = \frac{s}{180} + 2$
- (b) $2A = bh$, $\therefore b = \frac{2A}{h}$
- (c) $y - c = mx$, $\therefore m = \frac{y-c}{x}$
- (d) $A - 2\pi r^2 = 2\pi rh$, $\therefore h = \frac{A-2\pi r^2}{2\pi r}$
- (e) $3v = \pi r^2 h$, $\therefore h = \frac{3v}{\pi r^2}$
- (f) $x - a = (n-1)d$, $\frac{x-a}{d} = n-1$, $\therefore n = \frac{x-a}{d} + 1$

7. (a) $b^2 = c^2 - a^2$, $\therefore b = \sqrt{c^2 - a^2}$ (b) $\frac{2T}{n} = a + b$, $\therefore a = \frac{2T}{n} - b$
- (c) $s - ut = \frac{1}{2}at^2$, $\therefore a = \frac{2(s - ut)}{t^2}$ (d) $\frac{v + u}{uv} = \frac{1}{f}$, $\therefore f = \frac{uv}{u + v}$
- (e) $\frac{1}{u} - \frac{1}{f} = \frac{1}{v}$, $\frac{f - u}{uf} = \frac{1}{v}$, $\therefore v = \frac{uf}{f - u}$
- (f) $bx + ay = ab$, $ay = ab - bx$, $ay = b(a - x)$, $\therefore b = \frac{ay}{a - x}$
- (g) $s(R - 1) = a(R^n - 1)$, $\therefore a = \frac{s(R - 1)}{R^n - 1}$
- (h) $aH = bH - bh$, $bh = bH - aH$, $bh = H(b - a)$, $\therefore H = \frac{bh}{b - a}$
- (i) $kx - ky = mn + p - y$, $y - ky = mn + p - kx$, $y(1 - k) = mn + p - kx$,
 $\therefore y = \frac{mn + p - kx}{1 - k}$
- (j) $40as - 40bs = abc$, $40as = abc + 40bs$, $40as = b(ac + 40s)$,
 $\therefore b = \frac{40as}{ac + 40s}$
- (k) $2s = 2an + n(n - 1)d$, $2s - 2an = n(n - 1)d$, $\therefore d = \frac{2s - 2an}{n(n - 1)}$
8. (a) $C = \frac{5}{9}(F - 32)$, $\frac{9C}{5} = F - 32$, $\therefore F = \frac{9C}{5} + 32$
- (b) Water boils at 100°C . When $C = 100$, $F = \frac{9(100)}{5} + 32 = 212$.
Ans. Water boils at 100°C and 212°F .
- (c) Water freezes at 0°C . When $C = 0$, $F = \frac{9(0)}{5} + 32 = 32$.
Ans. Water freezes at 0°C and 32°F .
- (d) When $F = 99.5$, $C = \frac{5}{9}(99.5 - 32) = 37.5$.
Ans. The normal body temperature is 37.5°C .
- (e) When $C = 40.5$, $F = \frac{9(40.5)}{5} + 32 = 104.9$.
Ans. His body temperature is 104.9°F .
9. When $n = 1$, $S = 9$. When $n = 2$, $S = 13 = 9 + 4(1)$.
 When $n = 3$, $S = 17 = 9 + 4(2)$. $\therefore S = 9 + 4(n - 1) = 4n + 5$
10. When $w = 0$, $l = 15$. When $w = 10$, $l = 18 = 15 + \frac{3}{10}(10)$.
 When $w = 20$, $l = 21 = 15 + \frac{3}{10}(20)$. When $w = 30$, $l = 24 = 15 + \frac{3}{10}(30)$.
 $\therefore l = 15 + \frac{3}{10}w$
11. $B = \frac{A + C}{2}$, $\therefore C = 2B - A$
12. (a) $\therefore (n - 1)$ cards should be sent by each of the n students. $\therefore T = n(n - 1)$.
 (b) When $n = 36$, $T = 36(36 - 1) = 1260$. *Ans. The number of cards sent is 1260.*

13. (a) $k = an + b$ (b) $k = an + b$, $k - b = an$, $\therefore n = \frac{k-b}{a}$
- (c) $k = c(n-1) + d$, $k = c(\frac{k-b}{a} - 1) + d$, $k = \frac{kc-bc}{a} - c + d$,
 $ak = kc - bc - ac + ad$, $ak - kc = ad - bc - ac$,
 $k(a-c) = ad - bc - ac$, $\therefore k = \frac{ad-bc-ac}{a-c}$
14. (a) $1 + 2 + 3 + \dots + 1000 = \frac{1000(1000+1)}{2} = 500500$
- (b) $2 + 4 + 6 + \dots + 1000 = 2(1 + 2 + 3 + \dots + 500) = 2 \times \frac{500(500+1)}{2} = 250500$
- (c) $1 + 3 + 5 + \dots + 999$
 $= (1 + 2 + 3 + \dots + 1000) - (2 + 4 + 6 + \dots + 1000) = 500500 - 250500 = 250000$
- (d) $500 + 501 + 502 + \dots + 1000$
 $= (1 + 2 + 3 + \dots + 1000) - (1 + 2 + 3 + \dots + 499)$
 $= 500500 - \frac{499(499+1)}{2} = 500500 - 124750 = 375750$
- (e) $\text{Sum} = 3 + 6 + 9 + \dots + 999 = 3(1 + 2 + 3 + \dots + 333) = 3 \times \frac{333(333+1)}{2} = 166833$
15. (a) When $t = 0$, $s = -5(0-3)^2 + 80 = -45 + 80 = 35$
Ans. The height of the building is 35 m.
- (b) $\therefore (t-3)^2 \geq 0$, $\therefore -5(t-3)^2 \leq 0$, $-5(t-3)^2 + 80 \leq 80$, $s \leq 80$
Ans. The maximum height the stone can reach is 80 m.
- (c) The height is maximum when $(t-3)^2 = 0$, i.e. $t = 3$
Ans. The stone takes 3 s to reach the maximum height.
16. (a) $\frac{1}{a}$ of the hall will be painted. (b) $\frac{1}{a} + \frac{1}{b} = \frac{1}{x}$
- (c) When $a = 24$, $x = 8$, $\frac{1}{24} + \frac{1}{b} = \frac{1}{8}$, $\frac{1}{b} = \frac{1}{12}$, $b = 12$,
Ans. It takes 12 hours for Simon to complete the work alone.
17. (a) By Formula A, when $P = 81$, $N = 10\sqrt{81} = 10(9) = 90$
 By Formula B, when $P = 81$, $N = 0.85(81) + 15 = 83.85$
- (b) By Formula A, when $N = 50$, $50 = 10\sqrt{P}$, $5 = \sqrt{P}$, $P = 5^2 = 25$
 By Formula B, when $N = 50$, $50 = 0.85P + 15$, $35 = 0.85P$, $P = 41.2$
Ans. Jane's original score is 25, and Susan's original score is 41.2.
- (c) From the result of (b), original scores of 25 and 41.2 will become 50 using Formula A and B respectively. Therefore, more students will pass the examination if Formula A is adopted.
18. (a) $\frac{24}{24-h} = \frac{8}{x}$, $3x = 24 - h$, $\therefore h = 24 - 3x$
- (b) $V = \frac{(x+8)h}{2} \times 18 = 9(x+8)(24-3x) = 9(192 - 3x^2)$, $\therefore V = 1728 - 27x^2$
- (c) When $x = 6$, $V = 1728 - 27(6)^2 = 1728 - 972 = 756$.
Ans. The volume of water is 756 cm³.

- (d) Let H cm be the height of the container, $\frac{2}{3}H = 24 - 3(4) = 12$, $H = 18$.

When the container is full of water, $h = 18$ and $x = a$,

$$\therefore 18 = 24 - 3a, \quad 3a = 6, \quad a = 2.$$

- (e) When $x = 2$, $V = 1728 - 27(2)^2 = 1728 - 108 = 1620$

Ans. The capacity of the container is 1620 cm^3 .

19. (a) Height of the larger pyramid $= (h + y)$, base area $= b^2$

Height of the smaller pyramid $= h$, base area $= a^2$

$$\therefore \text{volume of the frustum} = \frac{1}{3}b^2(h + y) - \frac{1}{3}a^2h = \frac{1}{3}(b^2h - a^2h + b^2y)$$

$$(b) \quad \frac{a}{b} = \frac{h}{h + y}, \quad \therefore bh = ah + ay, \quad bh - ah = ay, \quad \therefore h = \frac{ay}{b - a}$$

- (c) Combining (a) and (b), volume of the frustum

$$= \frac{1}{3}[(b^2 - a^2)h + b^2y] = \frac{1}{3}[(b + a)(b - a)\frac{ay}{b - a} + b^2y] = \frac{1}{3}[(b + a)ay + b^2y]$$

$$= \frac{1}{3}(aby + a^2y + b^2y) = \frac{1}{3}y(a^2 + ab + b^2)$$

20. (a) $3 + 12s - 4r = 7 - 12s$, $-4r = 4 - 24s$, $r = -1 + 6s$

$$(b) \quad (i) \quad r = -1 + 6\left(\frac{4t}{3}\right), \quad r = -1 + 8t, \quad t = \frac{r + 1}{8}$$

$$(ii) \quad \text{If } r = 15, \quad t = \frac{15 + 1}{8} = 2$$

21. (a) (i) $\text{RHS} = \frac{(a+b)(a-b)}{ab} = \frac{a^2 - b^2}{ab} = \frac{a^2}{ab} - \frac{b^2}{ab} = \frac{a}{b} - \frac{b}{a} = \text{LHS}$

$$(ii) \quad \frac{\frac{a}{b} - \frac{b}{a}}{\frac{a}{a-b} - \frac{b}{a-b}} = \left(\frac{\frac{a}{b} - \frac{b}{a}}{\frac{a-b}{ab}}\right) \times \frac{1}{a-b} = \frac{(a+b)(a-b)}{ab} \times \frac{1}{a-b} = \frac{a+b}{ab}$$

$$(b) \quad (i) \quad c = \frac{\frac{a}{b} - \frac{b}{a}}{\frac{a}{a-b} - \frac{b}{a-b}}, \quad \text{from the result of (a)(ii),} \quad c = \frac{a+b}{ab},$$

$$\therefore abc = a + b, \quad abc - a = b, \quad a(bc - 1) = b, \quad a = \frac{b}{bc - 1}$$

$$(ii) \quad \text{If } b = 4 \text{ and } c = -\frac{3}{2}, \quad a = \frac{4}{4\left(-\frac{3}{2}\right) - 1} = -\frac{4}{7}$$

22. (a) When $r = 3$ and $h = \frac{5}{10} = 0.5$, $A = 2\pi(3)^2 + 2\pi(3)(0.5) = 21\pi$

Ans. Total surface area $= 21\pi \text{ cm}^2$.

$$(b) \quad (i) \quad A = 2\pi r^2 + 2\pi rh, \quad 2\pi rh = A - 2\pi r^2, \quad h = \frac{A - 2\pi r^2}{2\pi r}$$

$$(ii) \quad h = \frac{396 - 2\left(\frac{22}{7}\right)(7)^2}{2\left(\frac{22}{7}\right)(7)} = \frac{88}{44} = 2$$

- (c) When
- $h = 1$
- ,
- $A = 2\pi r^2 + 2\pi r = 2\pi r(r + 1)$
- .

Let A' be the new area.

$$A' = 2\pi(r - 1)[(r - 1) + 1] = 2\pi r(r - 1),$$

$$A - A' = 2\pi r(r + 1) - 2\pi r(r - 1) = 2\pi r[(r + 1) - (r - 1)] = 2\pi r(2) = 4\pi r$$

Ans. The required decrease is $4\pi r \text{ cm}^2$.

23. (a) $C = P + Q = 5k + \frac{440}{n} + 4k + \frac{520}{n} = 9k + \frac{960}{n}$

(b) When $n = 80$, $C = 129$. $129 = 9k + \frac{960}{80}$, $k = 13$

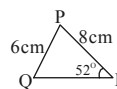
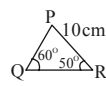
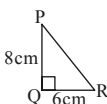
(c) When $n = 48$, $C = 9(13) + \frac{960}{48} = 137$.

$$\text{Profit percentage} = \frac{10550 - (137)(48)}{(137)(48)} \times 100\% = 60.4\% \quad (3 \text{ sig. fig.})$$

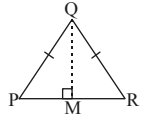
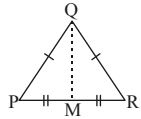
(d) When $C = 120$, $9(13) + \frac{960}{n} = 120$, $n = 320$.

*Ans. At least 320 suits.***Unit 6 Congruent triangles**

1. (a) $\triangle ABC \cong \triangle PRQ$ (b) $\triangle XYZ \cong \triangle LMN$
2. (a) $a = 55^\circ$, $b = 11$ (b) $x = 32^\circ$, $y = 5$
(c) $x = 90^\circ$, $y = 20$ (d) $a = 180^\circ - 35^\circ - 28^\circ = 117^\circ$, $b = 8.5$
3. (a) $z = NM = QO$ (corr. sides, $\cong \Delta$ s), $QO = QP = 5 \text{ cm}$ (corr. sides, $\cong \Delta$ s),
 $\therefore z = 5 \text{ cm}$. $\angle QOP = 53^\circ$ (base $\angle$ s, isos. Δ);
 $\angle Q = 180^\circ - 53^\circ \times 2 = 74^\circ$ ($\angle$ sum of isos. Δ)
 $x = \angle NMO = \angle QOP = 53^\circ$ (corr. $\angle$ s, $\cong \Delta$ s); $y = \angle Q = 74^\circ$ (corr. $\angle$ s, $\cong \Delta$ s)
- (b) $AB = CD$ (corr. sides, $\cong \Delta$ s), $\therefore 2a - 1 = \frac{3}{2}a + 4$, $\frac{1}{2}a = 5$, $a = 10$
 $BC = DA$ (corr. sides, $\cong \Delta$ s), $\therefore 4b + 1 = 15$, $4b = 14$, $b = 3.5$
- (c) $BE = BD$ (corr. sides, $\cong \Delta$ s), $\therefore x = 12$
 $AB = CB$ (corr. sides, $\cong \Delta$ s), $\therefore y + 12 = 9 + x = 9 + 12 = 21$, $y = 21 - 12 = 9$
4. (a) 14cm (b) 17cm (c) 8cm
5. (a) 53° (b) $\angle X = 180^\circ - 53^\circ - 66^\circ = 61^\circ$ (c) 61°
6. (a) $\triangle PQR \cong \triangle LMN$ (SAS) (b) $\triangle ABC \cong \triangle YXZ$ (RHS)
(c) $\triangle PQR \cong \triangle FED$ (AAS) (d) $\triangle MNL \cong \triangle YXZ$ (ASA)
7. (a) yes, $\triangle ABC \cong \triangle ZXY$ (SAS) (b) yes, $\triangle LMN \cong \triangle ZXY$ (SSS)
(c) no (d) yes, $\triangle XYZ \cong \triangle LMN$ (AAS)
(e) yes, $\triangle ABC \cong \triangle EFD$ (RHS) (f) no
8. (a) $\triangle ABC \cong \triangle ZYX$ (RHS) (b) $\triangle PQR \cong \triangle XYZ$ (or: $\triangle YXZ$) (SAS)
(c) $\triangle PQR \cong \triangle ZYX$ (AAS)
9. $\triangle DAB \cong \triangle DEC$ (RHS)

10. $\angle CDE = \angle FED$ ($\because \angle CDF + \angle FDE = \angle FEC + \angle CED$),
 $\angle CED = \angle FDE$, $DE = ED$, $\therefore \triangle CED \cong \triangle FDE$ (ASA)
11. (a) In $\triangle ADB$ and $\triangle BCA$, $AB = BA$ (common), $AD = BC$ and $BD = AC$ (given),
 $\therefore \triangle ADB \cong \triangle BCA$ (SSS)
- (b) $\angle DAB = \angle CBA$ (corr. $\angle$ s, $\cong \Delta$ s)
12. (a) In $\triangle DAB$ and $\triangle DEC$, $AB = EC$ and $AD = ED$ (given), $\angle BAD = \angle CED = 90^\circ$ (given).
 $\therefore \triangle DAB \cong \triangle DEC$ (RHS)
- (b) $\angle ECD = \angle ABD$ (corr. $\angle$ s, $\cong \Delta$ s)
 $\angle ABD = 180^\circ - 90^\circ - 68^\circ = 22^\circ$ ($\angle$ sum of Δ), $\therefore \angle ECD = 22^\circ$
13. (a) In $\triangle PQN$ and $\triangle PRM$,
 $PN = PM$ (given), $\angle PNQ = \angle PMR = 90^\circ$ (given),
 $\angle QPN = \angle RPM$ (common), $\therefore \triangle QPN \cong \triangle RPM$ (ASA).
- (b) $QN = RM$ (corr. sides, $\cong \Delta$ s)
14. $\angle EGF = 180^\circ - 60^\circ - 70^\circ = 50^\circ$ ($\angle$ sum of Δ)
 In $\triangle EFG$ and $\triangle HGF$, $FG = GF$ (common), $\angle EFG = \angle HGF$ (given),
 $\angle EGF = \angle HFG$ (proved), $\therefore \triangle EFG \cong \triangle HGF$ (ASA)
15. (a) In $\triangle CEF$ and $\triangle CDH$, $\angle C = \angle C$ (common), $CF = CH$ (given),
 $\angle CFE = 90^\circ = \angle CHD$ (given), $\therefore \triangle CEF \cong \triangle CDH$ (ASA).
- (b) $CD = CE$ (corr. sides, $\cong \Delta$ s)
16. (a)  No, the two triangles will be congruent ($\because$ SAS)
- (b)  or  No, the two triangles will be congruent ($\because$ AAS)
- (c)  No, the two triangles will be congruent ($\because$ ASA)
- (d)  No, the two triangles will be congruent ($\because$ AAS)
- (e)   No, the two triangles will be congruent ($\because$ SAS)
- (f)  No, the two triangles will be congruent ($\because$ RHS)
- (g)  No, the two triangles will be congruent ($\because$ SAS)
17. $\angle RPQ = \angle SPQ$ and $\angle PQR = \angle PQS$ (given), $PQ = PQ$ (common),
 $\therefore \triangle RPQ \cong \triangle SPQ$ (ASA), $\therefore RP = SP$ (corr. sides, $\cong \Delta$ s)
18. (a) $\triangle QNP$. In $\triangle RMP$ and $\triangle QNP$, $PM = PN$ (given), $PR = 2PN = PQ$ (given),
 $\angle RPM = \angle QPN$ (common), $\therefore \triangle RMP \cong \triangle QNP$ (SAS)
- (b) $\angle PNQ = \angle PMR$ (corr. $\angle$ s, $\cong \Delta$ s)
19. Join AC. In $\triangle ABC$ and $\triangle ADC$, $AB = AD$ and $BC = DC$ (given),
 $AC = AC$ (common), $\therefore \triangle ABC \cong \triangle ADC$ (SSS), $\therefore \angle B = \angle D$ (corr. $\angle$ s, $\cong \Delta$ s)
20. $\angle CED + \angle FEC = \angle FDE + \angle CDF$ (given), i.e. $\angle FED = \angle CDE$
 In $\triangle EDC$ and $\triangle DEF$, $\angle CED = \angle FDE$ (given), $\angle FED = \angle CDE$ (proved),
 $ED = DE$ (common), $\therefore \triangle EDC \cong \triangle DEF$ (ASA), $\therefore EC = DF$ (corr. sides, $\cong \Delta$ s)

21. (a) In $\triangle PMQ$ and $\triangle RMQ$,
 $PM = RM$ (given), $PQ = RQ$ (given), $QM = QM$ (common)
 $\therefore \triangle PMQ \cong \triangle RMQ$ (SSS),
 $\therefore \angle PMQ = \angle RMQ$ (corr. $\angle$ s, $\cong \Delta$ s),
 $\therefore \angle PMQ + \angle RMQ = 180^\circ$ (adj. $\angle$ s on st. line),
 $\therefore \angle PMQ = \angle RMQ = 90^\circ$, i.e. $QM \perp PR$
- (b) In $\triangle PMQ$ and $\triangle RMQ$,
 $QM \perp PR$ (given), $PQ = RQ$ (given), $QM = QM$ (common)
 $\therefore \triangle PMQ \cong \triangle RMQ$ (RHS), $\therefore PM = RM$ (corr. sides, $\cong \Delta$ s)
22. Join QS. In $\triangle QPS$ and $\triangle SRQ$,
 $PS = RQ$ and $PQ = RS$ (given), $QS = SQ$ (common).
 $\therefore \triangle QPS \cong \triangle SRQ$ (SSS), $\therefore \angle QSP = \angle SQR$ (corr. $\angle$ s, $\cong \Delta$ s)
23. $\angle EGF = 180^\circ - 73^\circ - 59^\circ = 48^\circ$
 In $\triangle FGE$ and $\triangle GFH$, $FG = GF$ (common),
 $\angle EFG = \angle HGF = 59^\circ$ (given), $\angle EGF = \angle HFG = 48^\circ$ (proved),
 $\therefore \triangle FGE \cong \triangle GFH$ (ASA), $\therefore EF = HG$ (corr. sides, $\cong \Delta$ s)
24. In $\triangle QLM$ and $\triangle QNM$, $QL = QN$ (given), $QM = QM$ (common),
 $\angle QLM = \angle QNM = 90^\circ$ (given), $\therefore \triangle QLM \cong \triangle QNM$ (RHS),
 $\therefore LM = NM$ (corr. sides, $\cong \Delta$ s).
 In $\triangle LPM$ and $\triangle NRM$, $LM = NM$ (proved), $\angle MLP = \angle MNR = 90^\circ$ (given),
 $PM = RM$ (given), $\therefore \triangle LPM \cong \triangle NRM$ (RHS)
25. (a) In $\triangle ABC$ and $\triangle EDC$, $\angle A = \angle E$ (given), $\angle C = \angle C$ (common),
 $BC = DC$ (given), $\therefore \triangle ABC \cong \triangle EDC$ (AAS),
 $\therefore AB = ED$ (corr. sides, $\cong \Delta$ s)
- (b) $\therefore \triangle ABC \cong \triangle EDC$ (proved), $\therefore AC = EC$ (corr. sides, $\cong \Delta$ s),
 but $BC = DC$ (given),
 $\therefore AC - DC = EC - BC$, i.e. $AD = EB$
26. In $\triangle AEB$ and $\triangle DEC$, $AE = ED$ and $BE = EC$ (given),
 $\angle AEB = \angle DEC$ (vert. opp. $\angle$ s), $\therefore \triangle AEB \cong \triangle DEC$ (S.A.S.),
 $\therefore \angle ABE = \angle DCE$ (corr. $\angle$ s, $\cong \Delta$ s), $\therefore AB \parallel CD$ (alt. $\angle$ s equal)
27. (a) $\angle CDE + \angle CDF = 90^\circ$, $\angle CDE + \angle ADE = 90^\circ$, $\therefore \angle CDF = \angle ADE$
 (b) $AD = CD$, $\angle EAD = 90^\circ = \angle FCD$, $\angle ADE = \angle CDF$ [from (a)],
 $\therefore \triangle AED \cong \triangle CDF$ (ASA)
28. $\angle BAD = 60^\circ = \angle CAE$, $BA = CA$, $AD = AE$, $\therefore \triangle BAD \cong \triangle CAE$ (SAS)
29. In $\triangle ABE$ and $\triangle CBD$, $AB = CB$ and $BE = BD$ (equilateral Δ s),
 $\angle ABE = \angle CBD = 60^\circ$ (equilateral Δ s), $\therefore \triangle ABE \cong \triangle CBD$ (SAS),
 $\therefore \angle BAE = \angle BCD$ (corr. $\angle$ s, $\cong \Delta$ s)
30. In $\triangle ABD$ and $\triangle CBE$, $AB = CB$ and $BD = BE$ (equilateral Δ s),
 $\angle ABD = \angle ABC - \angle DBF = 60^\circ - \angle DBF$ (equilateral Δ),
 $\angle CBE = \angle DBE - \angle DBF = 60^\circ - \angle DBF$ (equilateral Δ), $\therefore \angle ABD = \angle CBE$,
 $\therefore \triangle ABD \cong \triangle CBE$ (SAS), $\therefore AD = CE$ (corr. sides, $\cong \Delta$ s)
31. (a) In $\triangle ABF$ and $\triangle CBE$, $AB = CB$ and $BF = BE$ (equilateral Δ s),
 $\angle ABF = \angle ABC - \angle FBC = 60^\circ - \angle FBC$ (equilateral Δ),



- $\angle CBE = \angle FBE - \angle FBC = 60^\circ - \angle FBC$ (equilateral Δ), $\therefore \angle ABF = \angle CBE$,
 $\therefore \triangle ABF \cong \triangle CBE$ (SAS), $\therefore AF = CE$ (corr. sides, $\cong \Delta$ s)
- (b) $\triangle BCE$. In $\triangle FDE$ and $\triangle BCE$, $DE = CE$ and $FE = BE$ (equilateral Δ s),
 $\angle DEF = \angle DEC - \angle FEC = 60^\circ - \angle FEC$ (equilateral Δ),
 $\angle CEB = \angle FEB - \angle FEC = 60^\circ - \angle FEC$ (equilateral Δ),
 $\therefore \angle DEF = \angle CEB$, $\therefore \triangle FDE \cong \triangle BCE$ (SAS).
 $\therefore \triangle FDE \cong \triangle BCE$ (proved), $\therefore FD = BC$ (corr. sides, $\cong \Delta$ s),
 but $AB = BC$ (corr. sides, $\cong \Delta$ s), $\therefore FD = AB$
32. (a) In $\triangle ABD$ and $\triangle ACE$, $AB = AC$ and $AD = AE$ (equilateral Δ s),
 $\angle BAD = \angle CAE = 60^\circ$ (equilateral Δ s), $\therefore \triangle ABD \cong \triangle ACE$ (SAS),
 $\therefore \angle ABD = \angle ACE$ (corr. $\angle$ s, $\cong \Delta$ s)
- (b) In $\triangle CDF$ and $\triangle BDA$, $\angle ACE = \angle ABD$ (proved),
 $\angle CDF = \angle BDA$ (vert. opp. $\angle$ s), $\therefore \angle DFC = \angle DAB$ (the 3rd $\angle$ of Δ).
 $\angle DAB = 60^\circ$ (equilateral Δ), but $\angle DFC = x + y$ (ext. $\angle$ of Δ),
 $\therefore x + y = 60^\circ$
33. (a) In $\triangle ACE$ and $\triangle DBF$, $AE = DF$ and $CE = BF$ (given),
 $AC = AB + BC$ and $DB = DC + BC$, but $AB = DC$ (given), $\therefore AC = DB$,
 $\therefore \triangle ACE \cong \triangle DBF$ (SSS), $\therefore \angle A = \angle D$ (corr. $\angle$ s, $\cong \Delta$ s),
 $\therefore AE \parallel DF$ (alt. $\angle$ s eq.)
- (b) In $\triangle BAE$ and $\triangle CDF$, $\angle A = \angle D$ (proved), $AE = DF$ and
 $AB = DC$ (given), $\therefore \triangle BAE \cong \triangle CDF$ (SAS),
 $\therefore EB = FC$ (corr. sides, $\cong \Delta$ s)
34. (a) In $\triangle BAE$ and $\triangle CDE$, $BA = CD$ (given), $BE = CE$ (given),
 $AE = DE = \frac{1}{2}AD$ (given),
 $\therefore \triangle BAE \cong \triangle CDE$ (SSS), $\therefore \angle BAD = \angle CDA$ (corr. $\angle$ s, $\cong \Delta$ s)
- (b) In $\triangle BAD$ and $\triangle CDA$, $AB = DC$ (given), $AD = DA$ (common),
 $\angle BAD = \angle CDA$ (proved), $\therefore \triangle BAD \cong \triangle CDA$ (SAS),
 $\therefore BD = CA$ (corr. sides, $\cong \Delta$ s)
35. (a) $\angle BAE = \angle BAC - \angle CAE$, $\angle ACD = \angle FDC - \angle CAE$ (ext. $\angle$ of Δ),
 but $\angle BAC = \angle FDC$ (given), $\therefore \angle BAE = \angle ACD$
- (b) In $\triangle ABE$ and $\triangle CAD$, $AB = CA$ (given), $\angle BAE = \angle ACD$ (proved),
 $\angle AEB = 180^\circ - \angle BEF$ and $\angle CDA = 180^\circ - \angle FDC$ (adj. $\angle$ s on st. line),
 but $\angle BEF = \angle FDC$ (given), $\therefore \angle AEB = \angle CDA$,
 $\therefore \triangle ABE \cong \triangle CAD$ (AAS), $\therefore BE = AD$ (corr. sides, $\cong \Delta$ s)
36. In $\triangle BCD$ and $\triangle ACE$, $BC = AC$ and $CD = CE$ (equilateral Δ s),
 $\angle ACB = \angle ECD = 60^\circ$ (equilateral Δ s), $\angle ACD = 60^\circ - 50^\circ = 10^\circ$,
 $\therefore \angle ECA = 60^\circ - 10^\circ = 50^\circ = \angle DCB$, $\therefore \triangle BCD \cong \triangle ACE$ (SAS),
 $\therefore \theta = \angle CBD$ (corr. $\angle$ s, $\cong \Delta$ s), $\therefore \theta = 180^\circ - 100^\circ - 50^\circ = 30^\circ$ ($\angle$ sum of Δ)
37. (a) In $\triangle ABP$ and $\triangle CBS$, $PB = SB$ (given), $\angle APB = \angle CSB = 90^\circ$ (square),

$\angle ABP = \angle CBS$ (vert. opp. $\angle$ s), $\therefore \triangle ABP \cong \triangle CBS$ (ASA),

$\therefore AB = CB$ (corr. sides, $\cong$ Δs)

- (b) In $\triangle ABR$ and $\triangle CBR$, $BR = BR$ (common), $\angle ABR = \angle CBR$ (given),
 $AB = CB$ (proved), $\therefore \triangle ABR \cong \triangle CBR$ (SAS), $\therefore AR = CR$ (corr. sides, $\cong$ Δs).

$\therefore \triangle ABR \cong \triangle CBR$ (proved),

$\therefore$ the $\perp$ distance from B to AR = the $\perp$ distance from B to RC

$$= BS = \frac{1}{2} \times 9 = 4.5 \text{ cm}$$

38. In $\triangle QBP$ and $\triangle RCQ$, $a_1 = a_2$ (given), $QB = RC = 6 \text{ cm}$ (given),

$\angle BQP + a_3 = a_2 + \angle CRQ$ (ext. $\angle$ of Δ), $\therefore \angle BQP = \angle CRQ$ ($a_3 = a_2$),

$\therefore \triangle QBP \cong \triangle RCQ$ (ASA), $\therefore BP = CQ = 3 \text{ cm}$ (corr. sides, $\cong$ Δs)

39. (a) $PQ = 2PA = 2AQ$ and $PR = 2PB = 2BR$ (mid-points)

$\therefore PQ = PR$, $\therefore 2PA = 2AQ = 2PB = 2BR$, $PA = AQ = PB = BR$

In $\triangle BPQ$ and $\triangle APR$, $PQ = PR$ (given), $PB = PA$ (proved),

$\angle BPQ = \angle APR$ (common $\angle$), $\therefore \triangle BPQ \cong \triangle APR$ (SAS), $\therefore \angle Q = \angle R$ (corr. $\angle$ s,

$\cong$ Δs)

- (b) In $\triangle ACQ$ and $\triangle BCR$, $\angle ACQ = \angle BCR$ (vert. opp. $\angle$ s), $AQ = BR$ (proved),

$\angle Q = \angle R$ (proved), $\therefore \triangle ACQ \cong \triangle BCR$ (ASA)

40. (a) $\angle ABC = 60^\circ$ (equil. Δ), $\angle EBF = 60^\circ - 18^\circ = 42^\circ$,

$\angle BED = 180^\circ - 90^\circ - 42^\circ = 48^\circ$ ($\angle$ sum of Δ)

- (b) In $\triangle BEF$ and $\triangle BDF$, $\angle BDF = 48^\circ$ (given), $\angle BEF = 48^\circ$ (from (a)),

$\therefore \angle BEF = \angle BDF$, $\angle BFE = \angle BFD = 90^\circ$ (given), $BF = BF$ (common side),

$\therefore \triangle BEF \cong \triangle BDF$ (AAS)

- (c) (i) In $\triangle AEF$ and $\triangle ADF$, $EF = DF$ (corr. sides, $\cong$ Δs), $\angle AFE = \angle AFD = 90^\circ$ (given),

$AF = AF$ (common side), $\therefore \triangle AEF \cong \triangle ADF$ (SAS)

(ii) $\angle EAF = 60^\circ$ (property of equil. Δ), $\angle DAF = \angle EAF$ (corr. $\angle$ s, $\cong$ Δs) $= 60^\circ$

$\angle EBF = 42^\circ \neq \angle DAF$, $\therefore AD$ is not parallel to EB (alt. $\angle$ s not equal)

41. (a) (i) $\angle BAD = \angle EAC$ (given), $\angle BAD - \angle CAD = \angle EAC - \angle CAD$, $\angle BAC = \angle EAD$,

$BA = EA$ (given), $\angle B = \angle E$ (given), $\therefore \triangle ABC \cong \triangle AED$ (ASA)

(ii) $AC = AD$ (corr. sides, $\cong$ Δs), $\therefore \angle ADC = \angle ACD$ (base $\angle$ s, isos. Δ) $= 80^\circ$,

$\angle CAD = 180^\circ - 2(80^\circ) = 20^\circ$ ($\angle$ sum of Δ)

- (b) Let $\angle BAC = 3k$, $\angle CBA = 4k$. $\angle EAD = \angle BAC$ (corr. $\angle$ s, $\cong$ Δs) $= 3k$

$\angle ACB = \angle CAE$ (alt. $\angle$ s, $BC \parallel AE$)

$$= 20^\circ + 3k,$$

$\angle BAC + \angle CBA + \angle ACB = 180^\circ$ ($\angle$ sum of Δ)

$$3k + 4k + (20^\circ + 3k) = 180^\circ, k = 16^\circ$$

$\angle ADE = \angle ACB$ (corr. $\angle$ s, $\cong$ Δs)

$$= 20^\circ + 3(16^\circ) = 68^\circ$$

42. (a) $AS = BQ$ (given), $PS = PQ$ (given), $\angle ASP = \angle BQP = 90^\circ$ (given),

$\therefore \triangle SPA \cong \triangle QPB$ (SAS)

- (b) (i) $\angle BPC : \angle CPS : \angle SPA = 3 : 1 : 2$.

Let $\angle BPC = 3k$, $\angle CPS = k$ and $\angle SPA = 2k$.

$\angle QPB = \angle SPA$ (corr. $\angle$ s, $\cong$ Δs) $= 2k$

$\angle QPB + \angle BPC + \angle CPS = \angle QPS = 90^\circ$, $2k + 3k + k = 90^\circ$, $k = 15^\circ$,

$\therefore \angle SPA = 2(15^\circ) = 30^\circ$

- (ii) $\angle APC = \angle APS + \angle SPC = 2k + k = 3k = \angle BPC$,

PC = PC (common), PA = PB (corr. sides, $\cong \Delta$ s), $\therefore \Delta PAC \cong \Delta PBC$ (SAS)

$$(iii) \angle CPS : \angle SPA = 1 : 2, \therefore \angle CPS = \frac{1}{2}(30^\circ) = 15^\circ$$

$$\angle PCS = 180^\circ - \angle SPC - \angle PSC = 180^\circ - 15^\circ - 90^\circ = 75^\circ$$

$$\angle PCB = \angle PCS \text{ (corr. } \angle\text{s, } \cong \Delta\text{s)} = 75^\circ$$

$$\begin{aligned} \angle RCB &= 180^\circ - \angle PCS - \angle PCB \text{ (adj. } \angle\text{s on a st. line)} \\ &= 180^\circ - 75^\circ - 75^\circ = 30^\circ \end{aligned}$$

Unit 7 Isosceles triangles

1. (a) $y = \frac{180^\circ - 90^\circ}{2} = 45^\circ$
 (b) $2(2y - 8)^\circ + 3y^\circ = 180^\circ, \quad 7y^\circ = 196^\circ, \quad \therefore y = 28$
 (c) $(180^\circ - 2y^\circ) \times 2 + y^\circ = 180^\circ, \quad 360^\circ - 4y^\circ + y^\circ = 180^\circ, \quad \therefore y = 60$
 (d) $(y^\circ + y^\circ) \times 2 + 4y^\circ = 180^\circ, \quad 8y^\circ = 180^\circ, \quad \therefore y = 22.5$
 (e) $2(y + 5)^\circ = (3y - 18)^\circ, \quad 2y + 10 = 3y - 18, \quad \therefore y = 28$
 (f) $180^\circ - 2(2y^\circ) = 5y^\circ, \quad 9y^\circ = 180^\circ, \quad \therefore y = 20$
2. (a) The third angle $= 180^\circ - 32^\circ - 74^\circ = 74^\circ$ ($\angle$ sum of Δ),
 $\therefore 4y - 21 = y + 12$ (sides opp., eq. $\angle$ s), $3y = 33, \quad \therefore y = 11$
 (b) $\angle R = \angle S = y$ (base $\angle$ s, isos. Δ); $y + y = 72^\circ$ (ext. $\angle$ of Δ), $2y = 72^\circ, \quad \therefore y = 36^\circ$
 (c) $\angle EGF = 180^\circ - 120^\circ = 60^\circ$ (adj. $\angle$ s on st. line);
 $m + 60^\circ + 60^\circ = 180^\circ$ ($\angle$ sum of isos. Δ), $\therefore m = 60^\circ$
 (d) $\angle A = \angle ABD = 35^\circ$ (base $\angle$ s, isos. Δ); $\angle DBC = n$ (base $\angle$ s, isos. Δ);
 $n + n + 35^\circ + 35^\circ = 180^\circ$ ($\angle$ sum of Δ), $2n = 110^\circ, \quad \therefore n = 55^\circ$
 (e) $\angle CDE = 76^\circ$ (alt. $\angle$ s, $AB \parallel CD$); $\angle ABE = 115^\circ - 76^\circ = 39^\circ$ (ext. $\angle$ of Δ);
 $\angle C = \angle ABE = 39^\circ$ (alt. $\angle$ s, $AB \parallel CD$), $\therefore x = \angle C = 39^\circ$ (base $\angle$ s, isos. Δ)
 (f) $\angle LMN = 180^\circ - 90^\circ - 63^\circ = 27^\circ$ ($\angle$ sum of Δ);
 $\angle KLN = 27^\circ$ (alt. $\angle$ s, $LK \parallel NM$); $\angle KML = \angle KLN = 27^\circ$ (base $\angle$ s, isos. Δ);
 $b + 27^\circ + 27^\circ = 180^\circ$ ($\angle$ sum of Δ), $\therefore b = 126^\circ$
 (g) $\angle BCA = \angle A = 50^\circ$ (base $\angle$ s, isos. Δ); $\angle CBD = x$ (base $\angle$ s, isos. Δ);
 $x + x = 50^\circ$ (ext. $\angle$ of Δ), $2x = 50^\circ, \quad \therefore x = 25^\circ$
 (h) $\angle CEB = 180^\circ - 25^\circ - 85^\circ = 70^\circ$ ($\angle$ s, isos. Δ);
 $y + 70^\circ = 85^\circ$ (alt. $\angle$ s, $AB \parallel DF$), $\therefore y = 15^\circ$
 (i) $\angle S = \angle SQR$ (base $\angle$ s, isos. Δ); $2\angle SQR + 46^\circ = 180^\circ$ ($\angle$ sum of Δ),
 $\angle SQR = 67^\circ$; $\angle QRP = \angle SQR = 67^\circ$ (alt. $\angle$ s, $SQ \parallel RP$);
 $\therefore PR = PQ$ (given), $\therefore \angle QRP = \angle PQR = 67^\circ$;
 $a + 67^\circ + 67^\circ = 180^\circ$ ($\angle$ sum of Δ), $\therefore a = 46^\circ$
3. $\angle C = \angle ABC$ (base $\angle$ s, isos. Δ);
 $\therefore 2\angle C + 38^\circ = 180^\circ$ ($\angle$ sum of Δ),
 $\angle BDC = \angle C = 71^\circ$ (base $\angle$ s, isos. Δ); $\angle C = 71^\circ$
 $\therefore \angle ABD = 71^\circ - 38^\circ = 33^\circ$ (ext. $\angle$ of Δ)
4. $\angle R = \angle PQR$ (base $\angle$ s, isos. Δ);
 $\therefore 2\angle R + y = 180^\circ$ ($\angle$ sum of Δ), $R = 90^\circ - \frac{y}{2}$

$$x + \angle R = 90^\circ \text{ (ext. } \angle \text{ of } \Delta), \quad x = 90^\circ - \left(90^\circ - \frac{y}{2}\right) = \frac{y}{2}, \quad \therefore y = 2x$$

5. $\angle CBD = a$ (base $\angle$ s, isos. Δ); $\angle A = \angle ACB$ (base $\angle$ s, isos. Δ);

$$\angle A + \angle ACB = a \text{ (ext. } \angle \text{ of } \Delta), \quad 2\angle A = a, \quad \angle A = \frac{a}{2}$$

$$b = \angle D + \angle A = a + \frac{a}{2} = \frac{3a}{2} \text{ (ext. } \angle \text{ of } \Delta)$$

6. (a) In ΔACD , $\angle D = x$ (base $\angle$ s, isos. Δ).

$$\text{In } \Delta BCD, \quad \angle D = y \text{ (base } \angle \text{s, isos. } \Delta); \quad \therefore x = y.$$

- (b) $\angle ABC = \angle D + y = 2x$ (ext. $\angle$ of Δ)

$$\angle ACB = \angle ABC = 2x \text{ (base } \angle \text{s, isos. } \Delta)$$

$$x + 2x + 2x = 180^\circ \text{ (} \angle \text{ sum of } \Delta), \quad 5x = 180^\circ, \quad \therefore x = 36^\circ$$

7. In ΔABC , $\angle BAC = \theta$ (base $\angle$ s, isos. Δ), $\therefore \angle B = 180^\circ - 2\theta$ ($\angle$ sum of Δ).

$$\text{In } \Delta ADC, \quad \angle ADC = \theta \text{ (base } \angle \text{s, isos. } \Delta), \quad \therefore \angle DAB = \angle B = 180^\circ - 2\theta.$$

$$\angle B + \angle DAB = \angle ADC \text{ (ext. } \angle \text{ of } \Delta), \quad 2(180^\circ - 2\theta) = \theta, \quad 5\theta = 360^\circ, \quad \therefore \theta = 72^\circ$$

8. $\angle ABC = 18^\circ$ (base $\angle$ s, isos. Δ)

$$\angle BCD = 18^\circ + 18^\circ = 36^\circ \text{ (ext. } \angle \text{ of } \Delta), \quad \therefore \angle BDC = 36^\circ \text{ (base } \angle \text{s, isos. } \Delta)$$

$$\angle EBD = 36^\circ + 18^\circ = 54^\circ \text{ (ext. } \angle \text{ of } \Delta).$$

$$\therefore BD = DE \text{ (given), } \therefore \angle BED = 54^\circ \text{ (base } \angle \text{s, isos. } \Delta);$$

$$\angle EDF = 54^\circ + 18^\circ = 72^\circ \text{ (ext. } \angle \text{ of } \Delta)$$

$$\therefore DE = EF \text{ (given), } \therefore \angle EFD = 72^\circ \text{ (base } \angle \text{s, isos. } \Delta),$$

$$\therefore \theta = 72^\circ + 18^\circ = 90^\circ \text{ (ext. } \angle \text{ of } \Delta)$$

9. $\angle DBC = 20^\circ + 28^\circ = 48^\circ$ (ext. $\angle$ of Δ);

$$\angle C = 180^\circ - 28^\circ - 20^\circ - 84^\circ = 48^\circ \text{ (} \angle \text{ sum of } \Delta).$$

$$\therefore \angle DBC = \angle C = 48^\circ, \quad \therefore DB = DC \text{ (sides opp., eq. } \angle \text{s)}, \quad \therefore \Delta BCD \text{ is isosceles.}$$

10. Join BD.

$$\angle CBD = \angle CDB \text{ (base } \angle \text{s, isos. } \Delta), \quad \text{but } \angle ABC = \angle ADC \text{ (given),}$$

$$\therefore \angle ABC - \angle CBD = \angle ADC - \angle CDB,$$

$$\text{i.e. } \angle ABD = \angle ADB, \quad \therefore AB = AD \text{ (sides opp., eq. } \angle \text{s)}$$

11. $\angle XYZ = \angle XZY$ (base $\angle$ s, isos. Δ),

$$\text{but } \angle XYA = \angle AYZ \text{ and } \angle XZA = \angle AZY \text{ (given)}$$

$$\therefore 2\angle AYZ = 2\angle AZY, \quad \angle AYZ = \angle AZY, \quad \therefore AY = AZ \text{ (sides opp., eq. } \angle \text{s)}$$

$$\therefore \Delta AYZ \text{ is also isosceles.}$$

12. In ΔABD and ΔFEC , $AB = FE$ (given), $\angle B = \angle E$ (given),

$$BD = BC + CD = DE + CD = EC \text{ (} \because BC = DE), \quad \therefore \Delta ABD \cong \Delta FEC \text{ (SAS),}$$

$$\therefore \angle GCD = \angle GDC \text{ (corr. } \angle \text{s, } \cong \Delta \text{s)}, \quad \therefore GC = GD \text{ (sides opp., eq. } \angle \text{s)},$$

$$\therefore \Delta CDG \text{ is isosceles.}$$

13. (a) In ΔABC and ΔCDA , $AC = CA$ (common),

$$\angle ACB = a_2 = a_1 = \angle CAD \text{ (given),}$$

$$\angle CAB = a_1 + b_2 = a_2 + b_1 = \angle ACD \text{ (given), } \therefore \Delta ABC \cong \Delta CDA \text{ (ASA)}$$

- (b) $\therefore \Delta ABC \cong \Delta CDA$ (proved), $\therefore BC = DA$ (corr. sides, $\cong \Delta$ s);

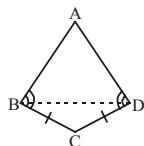
$$\therefore a_1 = a_2 \text{ (given), } \therefore EA = EC \text{ (sides opp., eq. } \angle \text{s);}$$

$$BE = BC - EC = DA - EA = DE, \quad \therefore \Delta BDE \text{ is isosceles.}$$

14. $\angle BPQ = \angle BQP$ (base $\angle$ s, isos. Δ); $\angle BPQ = x + z$ (ext. $\angle$ of Δ),

$$\therefore \angle BQP = \angle BQP = x + z \text{ (base } \angle \text{s, isos. } \Delta); \quad \angle CQR = y - z \text{ (ext. } \angle \text{ of } \Delta);$$

$$\therefore \angle BQP = \angle CQR \text{ (vert. opp. } \angle \text{s)},$$



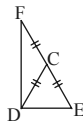
$$\therefore x + z = y - z, \quad 2z = y - x, \quad \therefore z = \frac{1}{2}(y - x)$$

15. $\because QR = RS$ (given), $\therefore \angle RSQ = 40^\circ$ (base $\angle$ s, isos. Δ);
 $\angle SRQ = 180^\circ - 40^\circ - 40^\circ = 100^\circ$ ($\angle$ sum of Δ). $\angle PQR = 60^\circ$ (equi. Δ),
 $\angle PRS = \angle PRQ + \angle SRQ = 60^\circ + 100^\circ = 160^\circ$.

$$\because PR = RS$$
 (given), $\therefore \angle PSR = \theta$ (base $\angle$ s, isos. Δ)

$$2\theta + 160^\circ = 180^\circ \text{ (}\angle \text{ sum of } \Delta\text{)}, \quad 2\theta = 20^\circ, \quad \therefore \theta = 10^\circ$$

16. $\because CD = CE$ (given), $\therefore \angle CDE = \angle E$ (base $\angle$ s, isos. Δ);
 $\because CD = CF$ (given), $\therefore \angle CDF = \angle F$ (base $\angle$ s, isos. Δ);
 $\angle CDE + \angle E + \angle CDF + \angle F = 180^\circ$ ($\angle$ sum of Δ),
 $2\angle CDE + 2\angle CDF = 180^\circ, \quad \angle CDE + \angle CDF = 90^\circ,$
 $\therefore \angle FDE = 90^\circ$



17. $\angle A = \angle C$ (base $\angle$ s, isos. Δ); $\angle BQC = \theta + \angle A$ (ext. $\angle$ of Δ);
 $\angle BQC + \angle C = 3\theta$ (ext. $\angle$ of Δ); $\therefore \theta + \angle A + \angle A = 3\theta, \quad 2\angle A = 2\theta, \quad \angle A = \theta,$
 $\therefore \Delta AQB$ is isosceles (sides opp., eq. $\angle$ s).

18. (a) In ΔABC , $\angle C = 2a$ (base $\angle$ s, isos. Δ),
 $\therefore 2a + 2a + 100^\circ = 180^\circ$ ($\angle$ sum of Δ), $4a = 80^\circ, \quad a = 20^\circ.$
 $2a + a + 2b = 180^\circ$ (ext. $\angle$ of Δ), $100^\circ + a = 2b, \quad 2b = 120^\circ, \quad b = 60^\circ.$
 $\therefore \angle DEA = 2a + b = 2(20^\circ) + 60^\circ = 100^\circ$ (ext. $\angle$ of Δ)

- (b) In ΔABD and ΔAED , $AD = AD$ (common), $\angle BAD = \angle DAE$ (given),
 $\angle DEA = \angle B = 100^\circ$ (proved), $\therefore \Delta ABD \cong \Delta AED$ (AAS),
 $\therefore DB = DE$ (corr. sides, $\cong$ As)

19. (a) $\because QP = QR$ (given), $\therefore \angle P = \angle QRP$ (base $\angle$ s, isos. Δ);
 $\because AP = AB$ (given), $\therefore \angle P = \angle ABP$ (base $\angle$ s, isos. Δ),
 $\therefore AB \parallel QD$ (corr. $\angle$ s eq.)

- (b) In ΔABC and ΔDRC , $AB = DR$ (common), $\angle ABC = \angle DRC$ and
 $\angle BAC = \angle RDC$ (alt. $\angle$ s, $AB \parallel QD$), $\therefore \Delta ABC \cong \Delta DRC$ (ASA),
 $\therefore BC = RC$ (corr. sides, $\cong$ As), i.e. C is the mid-point of BR.

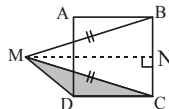
20. $\because \Delta ABC \cong \Delta EDC$ (given), $\therefore x = 68^\circ$ (corr. $\angle$ s, $\cong$ As);
 $\because BC = DC$ (corr. sides, $\cong$ As), $\therefore \angle BDC = x = 68^\circ$ (base $\angle$ s, isos. Δ),
 $y + 68^\circ + 68^\circ = 180^\circ$ ($\angle$ sum of Δ), $\therefore y = 44^\circ$

21. Draw $MN \perp BC$.

In ΔBMN and ΔCMN , $MN = MN$ (common),
 $MB = MC$ (given), $\angle MNB = \angle MNC = 90^\circ$ (construction),
 $\therefore \Delta BMN \cong \Delta CMN$ (R.H.S.)

$$\therefore BN = CN = \frac{1}{2}BC \text{ (corr. sides, } \cong \text{As),}$$

$$\therefore \text{Area of } \Delta DMC = \frac{1}{2} \times DC \times CN = \frac{1}{2}(\sqrt{18})\left(\frac{1}{2}\sqrt{18}\right) = \frac{18}{4} = 4.5 \text{ cm}^2$$



22. (a) $\because \angle ACB = a$ (base $\angle$ s, isos. Δ), $\therefore \angle CBD = 2a$ (ext. $\angle$ of Δ),
 $\therefore \angle FDE = 2a$ (corr. $\angle$ s, $BC \parallel DF$).

In ΔDEF , $\angle DFE = b$ (base $\angle$ s, isos. Δ),

$$\therefore b + b + 2a = 180^\circ \text{ (}\angle \text{ sum of } \Delta\text{)}, \quad 2a + 2b = 180^\circ, \quad \therefore a + b = 90^\circ$$

- (b) $a + b + \angle AGE = 180^\circ$ ($\angle$ sum of Δ), $90^\circ + \angle AGE = 180^\circ, \quad \therefore \angle AGE = 90^\circ$

23. Let $\angle PHQ = \angle PQH = a$ (base $\angle$ s, isos. Δ); $\therefore \angle PQR = a - \theta$
 Let $\angle RQK = \angle RQK = b$ (base $\angle$ s, isos. Δ).

$$\angle PQR + \angle RQP = 100^\circ, \text{ i.e. } (a - \theta) + b = 100^\circ, \quad a + b = 100^\circ + \theta$$

$$\text{In } \triangle HKQ, \quad a + b + \theta = 180^\circ (\angle \text{ sum of } \triangle), \quad \therefore a + b = 180^\circ - \theta.$$

$$\therefore 100^\circ + \theta = 180^\circ - \theta, \quad 2\theta = 80^\circ, \quad \therefore \theta = 40^\circ$$

24. (a) $\angle PLM = a$ (base $\angle$ s, isos. $\triangle$), $\therefore \angle P = 180^\circ - 2a$ ($\angle$ sum of $\triangle$);
 $\angle RNM = b$ (base $\angle$ s, isos. $\triangle$), $\therefore \angle R = 180^\circ - 2b$ ($\angle$ sum of $\triangle$);
 $\angle P + \angle R + \theta = 180^\circ$ ($\angle$ sum of $\triangle$),
 $(180^\circ - 2a) + (180^\circ - 2b) + \theta = 180^\circ, \quad 360^\circ - 2a - 2b + \theta = 180^\circ, \quad \therefore \theta = 2a + 2b - 180^\circ$
- (b) $\theta = 2a + 2b - 180^\circ, \quad 2a + 2b = \theta + 180^\circ, \quad \therefore a + b = \frac{\theta}{2} + 90^\circ$
 $a + b + \angle LMN = 180^\circ$ (adj. $\angle$ s on st. line),
 $\frac{\theta}{2} + 90^\circ + \angle LMN = 180^\circ, \quad \therefore \angle LMN = 90^\circ - \frac{\theta}{2}$
25. (a) In $\triangle ABC$, $\angle A = \angle ACB$ (base $\angle$ s, isos. $\triangle$),
 $\therefore 2\angle A + 20^\circ = 180^\circ$ ($\angle$ sum of $\triangle$), $\angle A = 80^\circ$ and $\angle ACB = 80^\circ$.
 In $\triangle ACD$, $\angle A = \angle ADC$ (base $\angle$ s, isos. $\triangle$),
 $2\angle A + \angle ACD = 180^\circ$ ($\angle$ sum of $\triangle$),
 $2(80^\circ) + \angle ACD = 180^\circ, \quad \angle ACD = 20^\circ, \quad \therefore \angle DCE = 80^\circ - 20^\circ = 60^\circ$.
 In $\triangle DCE$, $\angle DEC = \angle DCE$ (base $\angle$ s, isos. $\triangle$),
 $\angle CDE + 2(60^\circ) = 180^\circ$ ($\angle$ sum of $\triangle$), $\therefore \angle CDE = 60^\circ$
- (b) $\angle EDB = 60^\circ - 20^\circ = 40^\circ$ (ext. $\angle$ of $\triangle$); $\therefore \angle EFD = 40^\circ$ (base $\angle$ s, isos. $\triangle$);
 $\angle FEB = 40^\circ - 20^\circ = 20^\circ$ (ext. $\angle$ of $\triangle$); $\therefore \angle FEB = \angle B = 20^\circ$,
 $\therefore BF = EF$ (sides opp., eq. $\angle$ s), $\therefore AC = EF = BF = 6$ cm
26. (a) In $\triangle ABC$ and $\triangle CDE$, $BC = DE$ and $AB = CD$ (given),
 $\angle ABC = \angle CDE = 180^\circ - 94^\circ = 86^\circ$ (adj. $\angle$ s on st. line),
 $\therefore \triangle ABC \cong \triangle CDE$ (SAS), $\therefore AC = CE$ (corr. sides, $\cong \triangle$ s),
 $\therefore \triangle ACE$ is isosceles.
- (b) $\therefore \triangle ABC \cong \triangle CDE$ (proved), $\therefore \angle BAC = y$ (corr. $\angle$ s, $\cong \triangle$ s),
 $x + y = 94^\circ$ (ext. $\angle$ of $\triangle$); but $35^\circ + x + \theta + y = 180^\circ$ (adj. $\angle$ s on st. line),
 $\therefore \theta + 94^\circ + 35^\circ = 180^\circ, \quad \therefore \theta = 51^\circ$
- (c) In $\triangle ACE$, $AC = CE$ (proved), $\therefore \angle CAE = \angle CEA$ (base $\angle$ s, isos. $\triangle$),
 $\therefore 2\angle CEA + 51^\circ = 180^\circ$ ($\angle$ sum of $\triangle$), $\angle CEA = 64.5^\circ$,
 $\therefore \angle CED = 180^\circ - 52^\circ - 64.5^\circ = 63.5^\circ$ (adj. $\angle$ s on st. line);
 But $x = \angle CED$ (corr. $\angle$ s, $\cong \triangle$ s),
 $\therefore x = 63.5^\circ$ and $y = 94^\circ - x = 94^\circ - 63.5^\circ = 30.5^\circ$
27. (a) $AC = \sqrt{AB^2 + BC^2} = \sqrt{(4+3)^2 + 24^2} = \sqrt{625} = 25$ cm;
 Area of $\triangle ABC = \frac{7 \times 24}{2} = \frac{BD \times 25}{2}, \quad \therefore BD = 6.72$ cm
- (b) Let $a = \angle BCE = \angle DCF$. $\angle BEC = 180^\circ - 90^\circ - a = 90^\circ - a$ ($\angle$ sum of $\triangle$);
 $\angle DFC = 180^\circ - 90^\circ - a = 90^\circ - a$ ($\angle$ sum of $\triangle$);
 But $\angle BFE = \angle DFC$ (vert. opp. $\angle$ s), $\therefore \angle BFE = 90^\circ - a = \angle BEC$,
 $\therefore \triangle BEF$ is isosceles (sides opp., eq. $\angle$ s)
- (c) $BF = BE = 4$ cm (proved), $\therefore FD = BD - BF = 6.72 - 4 = 3.72$ cm
28. (a) Let $\angle AQR = x$ and $\angle APB = y$.
 $\therefore BP = BA$ (given), $\therefore \angle PAB = y$ (base $\angle$ s, isos. $\triangle$)
 $\therefore RQ = RA$ (given), $\therefore \angle RAQ = x$ (base $\angle$ s, isos. $\triangle$)

$$\angle BAR = 180^\circ - x - y \text{ (adj. } \angle \text{s on st. line),}$$

$$\angle PQR = 180^\circ - x - y \text{ (} \angle \text{ sum of } \Delta \text{), } \therefore \angle BAR = \angle PQR$$

- (b) In $\triangle ABR$ and $\triangle RCQ$, $\angle BAR = \angle CRQ$ (proved),
 $\angle ABR = \angle RCQ$ (corr. $\angle$ s, $QC \parallel AB$), $RA = RQ$ (given),
 $\therefore \triangle ABR \cong \triangle RCQ$ (A.A.S.), $\therefore AB = CR$ (corr. sides, $\cong \Delta$ s)

29. $\angle BCE + \angle ECD = \angle A + \angle CBA$ (ext. $\angle$ of Δ),
 $\therefore 90^\circ + \angle ECD = 90^\circ + \angle CBA$, i.e. $\angle ECD = \angle CBA$.
 In $\triangle ECD$ and $\triangle CBA$, $\angle ECD = \angle CBA$ (proved),
 $CD = AB$ (given), $\angle EDC = \angle CAB = 90^\circ$ (given), $\therefore \triangle ECD \cong \triangle CBA$ (AAS),
 $\therefore EC = CB$ (corr. sides, $\cong \Delta$ s), i.e. $\triangle BCE$ is isosceles

30. (a) $\angle QRS + \angle PQR = 180^\circ$ (int. $\angle$ s, $PQ \parallel SR$)
 $\angle QRS + 123^\circ = 180^\circ$, $\angle QRS = 57^\circ$
 $\angle QSR = \angle QRS = 57^\circ$ (base $\angle$ s, isos. Δ),
 $\angle RQS = 180^\circ - \angle QSR - \angle QRS$ ($\angle$ sum of Δ)
 $= 180^\circ - 57^\circ - 57^\circ = 66^\circ$
 (b) $\angle PQS = \angle QSR = 57^\circ$ (alt. $\angle$ s, $PQ \parallel SR$)
 $\angle PSQ = 180^\circ - \angle PQS - \angle QPS$ ($\angle$ sum of Δ)
 $= 180^\circ - 57^\circ - 66^\circ = 57^\circ$
 $\therefore \angle PQS = \angle PSQ$, $\therefore PQ = PS$ (sides opp., eq. $\angle$ s)

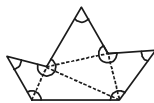
31. (a) $\angle ACB = \angle ABC$ (base $\angle$ s, isos. Δ)
 $= \frac{180^\circ - 20^\circ}{2} = 80^\circ$ ($\angle$ sum of Δ)
 $\angle BDC = \angle BCD = 80^\circ$ (base $\angle$ s, isos. Δ),
 $\angle CBD = 180^\circ - \angle BDC - \angle BCD$ ($\angle$ sum of Δ)
 $= 180^\circ - 80^\circ - 80^\circ = 20^\circ$
 $\angle DBE = \angle ABC - \angle CBD = 80^\circ - 20^\circ = 60^\circ$
 $\angle BED = \angle DBE = 60^\circ$ (base $\angle$ s, isos. Δ)
 $\angle BDE = 180^\circ - \angle DBE - \angle BED$ ($\angle$ sum of Δ)
 $= 180^\circ - 60^\circ - 60^\circ = 60^\circ = \angle DBE$
 $\therefore BE = DE$ (sides opp., eq. $\angle$ s), but $DE = BD$ (given),
 $\therefore \triangle BDE$ is an equilateral triangle.
 (b) $\angle EDF = 180^\circ - \angle BDC - \angle BDE$ (adj. $\angle$ s on st. line)
 $= 180^\circ - 80^\circ - 60^\circ = 40^\circ$
 $\angle DFE = \angle EDF$ (base $\angle$ s, isos. Δ) $= 40^\circ$
 $20^\circ + \angle AEF = \angle DFE$ (ext. $\angle$ of Δ), $\angle AEF = 40^\circ - 20^\circ = 20^\circ$
 $\angle AEF = \angle EAF$, $\therefore AF = EF$ (sides opp., eq. $\angle$ s),
 i.e. $\triangle AEF$ is an isosceles triangle.
 (c) $\angle BEF = 180^\circ - \angle AEF$ (adj. $\angle$ s on st. line)
 $= 180^\circ - 20^\circ = 160^\circ$
 $BE = DE = EF$
 $\therefore \angle BFE = \angle EBF$ (base $\angle$ s, isos. Δ)
 $= \frac{180^\circ - 160^\circ}{2}$ ($\angle$ sum of Δ) $= 10^\circ$,
 $\angle BFC = \angle DFE - \angle BFE = 40^\circ - 10^\circ = 30^\circ$

Unit 8 Angles related to polygons

1. (a) $\triangle GFE, \triangle GCE, \triangle DCE, \triangle DAE$ (b) $\triangle BCF, \triangle DCE$
(c) No. (d) Yes, it is an ext. $\angle$ of $\triangle DCG$ and $\triangle DCE$. (e) No.
2. (a) $y = 180^\circ - 135^\circ = 45^\circ$ (adj. $\angle$ s on st. line)
 $x + y = 128^\circ$ (ext. $\angle$ of $\triangle$), $\therefore x = 128^\circ - 45^\circ = 83^\circ$
(b) $\angle C = 120^\circ - 35^\circ = 85^\circ$ (ext. $\angle$ of $\triangle$). $x = \angle C = 85^\circ$ (alt. $\angle$ s, $AB \parallel CD$)
(c) $\angle STR = 180^\circ - 40^\circ - 80^\circ = 60^\circ$ ($\angle$ sum of $\triangle$)
 $y = \angle STR = 60^\circ$ (vert. opp. $\angle$ s).
 $x + y = 110^\circ$ (ext. $\angle$ of $\triangle$), $\therefore x = 110^\circ - 60^\circ = 50^\circ$
(d) $\angle CEF = 180^\circ - 108^\circ = 72^\circ$ (adj. $\angle$ s on st. line),
 $\therefore \angle BCA = 72^\circ + 35^\circ = 107^\circ$ (ext. $\angle$ of $\triangle$),
 $\therefore \theta = 54^\circ + 107^\circ = 161^\circ$ (ext. $\angle$ of $\triangle$)
(e) $\angle BAD = y$ (base $\angle$ s, isos. $\triangle$), $\angle BDC = 32^\circ$ (base $\angle$ s, isos. $\triangle$)
 $\angle BAD + y = \angle BDC$ (ext. $\angle$ of $\triangle$), $\therefore y + y = 32^\circ$, $y = 16^\circ$
(f) reflex $\angle EFG = 360^\circ - \theta$ ($\angle$ s at a pt.)
 $28^\circ + 56^\circ + 46^\circ + (360^\circ - \theta) = 360^\circ$ ($\angle$ sum of quad.), $\therefore \theta = 130^\circ$
3. (a) The angle sum = $180^\circ(4-2) = 360^\circ$ (b) The angle sum = $180^\circ(5-2) = 540^\circ$
(c) The angle sum = $180^\circ(6-2) = 720^\circ$ (d) The angle sum = $180^\circ(8-2) = 1080^\circ$
(e) The angle sum = $180^\circ(15-2) = 2340^\circ$ (f) The angle sum = $180^\circ(18-2) = 2880^\circ$
4. (a) $x + 2x + 118^\circ + (x + 23^\circ) + 151^\circ = 180^\circ \times 3$ ($\angle$ sum of polygon), $4x = 248^\circ$, $x = 62^\circ$
(b) $y + (y - 65^\circ) + 119^\circ + 108^\circ + 90^\circ = 180^\circ \times 3$ ($\angle$ sum of polygon), $2y = 288^\circ$, $y = 144^\circ$
(c) $x + 50^\circ + (180^\circ - 88^\circ) + 100^\circ = 360^\circ$ (sum of ext. $\angle$ s), $x = 118^\circ$
(d) $43^\circ + a + 130^\circ + 39^\circ + (360^\circ - 172^\circ) = 180^\circ \times 3$ ($\angle$ sum of polygon), $a = 140^\circ$
5. (a) Let the no. of sides be n . $180(n-2) = 2520$, $n = \frac{2520}{180} + 2 = 16$
(b) The no. of side = $\frac{3060}{180} + 2 = 19$ (c) The no. of side = $\frac{4860}{180} + 2 = 29$
6. Let x be the no. of sides. $180(x-2) = 16 \times 90$, $x-2 = 8$, $x = 10$
Ans. The number of sides is 10.
7. Let n be the no. of sides. $180(n-2) = 2 \times 180(14-2)$, $n-2 = 24$, $n = 26$
Ans. The number of sides is 26.
8. (a) A polygon is a closed area made up of straight lines.
A regular polygon is a polygon with all sides equal and all angles equal.
(b) Equilateral triangle, square, regular octagon.
9. (a) Each exterior angle = $180^\circ - 140^\circ = 40^\circ$, $\therefore$ no. of sides = $\frac{360}{40} = 9$
(b) No. of sides = $\frac{360}{180-156} = \frac{360}{24} = 15$ (c) No. of sides = $\frac{360}{180-172} = \frac{360}{8} = 45$
10. $37 + 88 + 96 + 101 + 144 + x + (\frac{2}{3}x - 11) = 180 \times (7-2)$, $\frac{5}{3}x = 445$, $x = 267$
11. Let the size of the remaining angle be x . $\therefore$ Each of the equal angles = $x + 42^\circ$.
 $\therefore 5(x + 42^\circ) + x = 180^\circ(6-2)$, $6x = 510^\circ$, $x = 85^\circ$.
Ans. The remaining angle is 85° .
12. The sum of interior angles = 360° ,

- $\therefore$ the largest angle $= 360^\circ \times \frac{6}{2+3+5+6} = 360^\circ \times \frac{6}{16} = 135^\circ$
13. (a) 360° (b) 360°
14. (a) no. of sides $= \frac{360}{45} = 8$ (b) no. of sides $= \frac{360}{30} = 12$
- (c) no. of sides $= \frac{360}{15} = 24$
15. Let the no. of sides be n . $180(n-2) = 360$, $(n-2) = 2$, $n = 4$. *Ans. It has 4 sides.*
16. Let the no. of sides be n . $180(n-2) = 4 \times 360$, $(n-2) = 8$, $n = 10$
- $\therefore$ The size of each interior angle $= \frac{4 \times 360^\circ}{10} = 144^\circ$
17. Let each exterior angle be x . $\therefore$ Each interior angle $= 6x + 12^\circ$
- $\therefore x + (6x + 12^\circ) = 180^\circ$, $7x = 168^\circ$, $x = 24^\circ$. $\therefore$ The no. of sides $= \frac{360}{24} = 15$
18. (a) In $\triangle ACI$, $\angle JIE = b + e$ (ext. $\angle$ of $\triangle$). In $\triangle BDJ$, $\angle IJE = a + d$ (ext. $\angle$ of $\triangle$).
In $\triangle JIE$, $\angle JIE + \angle IJE + c = 180^\circ$ ($\angle$ sum of $\triangle$), $\therefore b + e + a + d + c = 180^\circ$
- (b) In $\triangle ACE$, $a + c + e = 180^\circ$ ($\angle$ sum of $\triangle$). In $\triangle BDF$, $b + d + f = 180^\circ$ ($\angle$ sum of $\triangle$).
 $\therefore a + b + c + d + e + f = 180^\circ + 180^\circ = 360^\circ$
- (c) $\angle AIG = e + f$ (ext. $\angle$ of $\triangle$); $\angle EGH = a + b$ (ext. $\angle$ of $\triangle$);
 $\angle CHI = d + c$ (ext. $\angle$ of $\triangle$).
 $\angle AIG + \angle EGH + \angle CHI = 360^\circ$ (sum of ext. $\angle$ s), $\therefore a + b + c + d + e + f = 360^\circ$
- (d) $a + b + c + d + e + f + g + h + i = 180^\circ(10 - 2) = 1440^\circ$ ($\angle$ sum of polygon)
- (e) The sum of interior angles of the hexagon $= 180^\circ(6 - 2) = 720^\circ$
 $a + b + c + d + e + f = 360^\circ \times 6 - 720^\circ = 1440^\circ$
19. The smallest exterior angle $= 360^\circ \times \frac{1}{1+2+3+4+5} = 360^\circ \times \frac{1}{15} = 24^\circ$
- $\therefore$ The largest interior angle $= 180^\circ - 24^\circ = 156^\circ$
20. $\angle AFE = a + b$ (ext. $\angle$ of $\triangle$), $\angle AFE + c = d$ (ext. $\angle$ of $\triangle$)
 $\therefore (a + b) + c = d$, $c = d - a - b$
21. Let the original no. of sides be n .
 $\therefore$ The no. of sides of the new polygon is $2n$.
 $\frac{180^\circ(n-1)}{n} : \frac{180^\circ(n-2)}{n} = 4:3$,
 $\therefore \frac{n-1}{n-2} = \frac{4}{3}$, $3n - 3 = 4n - 8$, $n = 5$, $\therefore 2n = 10$.
22. Angles at a point $= 360^\circ$; to tessellate, the interior angle must be a factor of 360° .
Each interior angle of a regular hexagon $= \frac{180^\circ \times (6-2)}{6} = 120^\circ$ which is a factor of 360,
 $\therefore$ regular hexagons can tessellate.
Each interior angle of a regular octagon $= \frac{180^\circ(8-2)}{8} = 135^\circ$ which is not a factor of 360,
 $\therefore$ regular octagons cannot tessellate.
23. $b = 60^\circ$ (alt. $\angle$ s, $\parallel$ lines), $2a + 60^\circ = 2b$ (ext. $\angle$ of $\triangle$),
 $\therefore 2a + 60^\circ = 60^\circ \times 2$, $a = 30^\circ$
24. Let $a = \angle QPS = \angle SPR$, $b = \angle QRS = \angle SRP$
 $\therefore 2a + 54^\circ + 2b = 180^\circ$ ($\angle$ sum of $\triangle$), $a + b = 63^\circ$

- $\theta + a + b = 180^\circ$ ($\angle$ sum of Δ), $\therefore \theta + 63^\circ = 180^\circ$, $\theta = 117^\circ$
25. Let $x = \angle ABE = \angle EBC$, $y = \angle ACE = \angle ECD$
 $\angle ACD = 80^\circ + 2x$ (ext. $\angle$ of Δ), $\therefore 2y = 80^\circ + 2x$, $y = 40^\circ + x$, $y - x = 40^\circ$
 $\angle EBC + \angle BEC = \angle ECD$ (ext. $\angle$ of Δ), $\therefore x + \angle BEC = y$, $\angle BEC = y - x = 40^\circ$
26. $(x - 5^\circ) + 2(x + 10^\circ) + x + (x + 30^\circ) + [180^\circ - (x + 25^\circ)] = 360^\circ$ (sum of ext. $\angle$ s)
 $\therefore 5x - x + 200^\circ = 360^\circ$, $4x = 160^\circ$, $x = 40^\circ$
27. $\angle C = y$ (corr. $\angle$ s, $DE \parallel AC$), $\therefore x + y + 90^\circ = 180^\circ$ ($\angle$ sum of Δ), $x + y = 90^\circ$ (i)
 $x + 2y = 125^\circ$ (ext. $\angle$ of Δ) (ii)
(ii) - (i), $\therefore 2y - y = 125^\circ - 90^\circ$, $y = 35^\circ$; $x = 90^\circ - 35^\circ = 55^\circ$
28. $2x + y + 100^\circ + x + y = 180^\circ \times 3$ ($\angle$ sum of polygon), $\therefore 3x + 2y = 440^\circ$ (i)
 $60^\circ + 100^\circ + x + y = 360^\circ$ ($\angle$ sum of quad.), $x + y = 200^\circ$ (ii)
(i) - (ii) $\times 2$, $\therefore 3x + 2y - 2(x + y) = 440^\circ - 2 \times 200^\circ$, $x = 40^\circ$
 $y = 200^\circ - x = 200^\circ - 40^\circ = 160^\circ$
29. $\angle FAE = \angle AFE = 60^\circ$ (equil. Δ). $\angle BAE = \frac{180^\circ \times (5-2)}{5} = 108^\circ$ ($\angle$ of regular polygon)
 $\therefore \angle BAF = 108^\circ - 60^\circ = 48^\circ$. $\therefore AB = AE$, $AE = AF$, $\therefore AB = AF$
In ΔABF , $\angle AFB = \frac{180^\circ - 48^\circ}{2} = 66^\circ$ ($\angle$ sum of isos. Δ)
Similarly, $\angle EFD = 66^\circ$, $\angle AFB + 60^\circ + \angle EFD + x = 360^\circ$ ($\angle$ s at a pt.)
 $\therefore x = 360^\circ - 60^\circ - 66^\circ \times 2 = 168^\circ$
30. Let $\angle A = \theta$. $\angle C = \angle A = \theta$ (base $\angle$ s, isos. Δ), $\angle B = \angle A = \theta$ (base $\angle$ s, isos. Δ)
 $\angle BFC = \angle A + \angle B = \theta + \theta = 2\theta$ (ext. $\angle$ of Δ).
 $\therefore CE = CF$, $\therefore \angle CEF = \angle BFC = 2\theta$ (base $\angle$ s, isos. Δ).
 $\angle C + \angle CEF + \angle BFC = 180^\circ$ ($\angle$ sum of Δ), $\angle \theta + 2\theta + 2\theta = 180^\circ$, $\theta = 36^\circ$, $\therefore \angle A = 36^\circ$
31. (a) $\angle EAB = \angle ABC = \frac{180^\circ \times (5-2)}{5} = 108^\circ$ ($\angle$ of regular polygon)
 $AB = BC$, $\therefore \angle BCA = \frac{180^\circ - 108^\circ}{2} = 36^\circ$ ($\angle$ sum of isos. Δ)
Similarly, $\angle ABE = 36^\circ$, $\therefore \angle FBC = 108^\circ - 36^\circ = 72^\circ$
 $\angle EFC = \angle FBC + \angle BCA$ (ext. $\angle$ of Δ), $\therefore \angle EFC = 72^\circ + 36^\circ = 108^\circ$
(b) $\angle AED = 108^\circ$ (regular pentagon)
 $\angle AEB = \angle ABE = 36^\circ$ (base $\angle$ s, isos. Δ), $\therefore \angle FED = 108^\circ - 36^\circ = 72^\circ$
 $\angle EFC + \angle FED = 108^\circ + 72^\circ = 180^\circ$, $\therefore ED \parallel AC$ (int. $\angle$ s supp.)
32. $\angle QPT + \angle PTS + (180^\circ - 49^\circ) + 103^\circ + (180^\circ - \theta) = 180^\circ \times (5-2)$ ($\angle$ sum of polygon),
but $\angle QPT + \angle PTS = 180^\circ$, (int. $\angle$ s, $QP \parallel ST$),
 $\therefore 180^\circ + 131^\circ + 103^\circ + 180^\circ - \theta = 540^\circ$, $\theta = 54^\circ$
33. (a) A concave polygon has at least one interior angle greater than 180° (reflex angle).
Convex polygons have all interior angles smaller than 180° .
(b) Yes. Any n -sided concave polygons can be divided into $(n-2)$ triangles. For example, the given 7-sided concave polygon can be divided into 5 triangles.
(c) An exterior $\angle = 180^\circ$ - its adjacent interior $\angle$,
 $\therefore$ the reflex $\angle$ of a concave polygon has no adjacent exterior $\angle$,
 $\therefore$ we can't find the sum of exterior angles of a concave polygon.



34. Let the equal angles be x .

Produce BC to H.

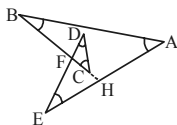
$$\angle BHE = \angle B + \angle A = x + x = 2x \quad (\text{ext. } \angle \text{ of } \Delta)$$

$$\angle EFC = \angle D + \angle DCF = x + x = 2x \quad (\text{ext. } \angle \text{ of } \Delta)$$

$$\angle E + \angle BHE + \angle EFC = 180^\circ \quad (\angle \text{ sum of } \Delta),$$

$$\therefore x + 2x + 2x = 180^\circ, \quad 5x = 180^\circ, \quad x = 36^\circ$$

Ans. The size of each of these equal angles is 36° .



35. In $\triangle ABD$, $a + b + d + \angle MND = 360^\circ$ ($\angle$ sum of quad.)

$$\text{In } \triangle CMN, c + \angle CMN + \angle MNC = 180^\circ \quad (\angle \text{ sum of } \Delta)$$

$$\text{In } \triangle EFG, e + f + g + \angle LMN = 360^\circ \quad (\angle \text{ sum of quad.})$$

$$\therefore a + b + c + d + e + f + g + \angle MND + \angle MNC + \angle CMN + \angle LMN = 360^\circ + 180^\circ + 180^\circ + 360^\circ = 900^\circ,$$

$$\text{but } \angle MND + \angle MNC = 180^\circ \quad \text{and} \quad \angle CMN + \angle LMN = 180^\circ \quad (\text{adj. } \angle \text{ s on st. line}),$$

$$\therefore a + b + c + d + e + f + g + 180^\circ + 180^\circ = 900^\circ$$

$$\text{sum of the marked angles} = 900^\circ - 180^\circ - 180^\circ = 540^\circ$$

36. In $\triangle ACS$, $a + \frac{a}{2} + b = 180^\circ$ ($\angle$ sum of Δ), $b = 180^\circ - \frac{3}{2}a$ (i)

$$\text{In } \triangle BER, \angle SRD = \frac{b}{3} + (a + 4^\circ) \quad (\text{ext. } \angle \text{ of } \Delta)$$

$$\text{In } \triangle SRD, b = \angle SRD + 3a - 14^\circ \quad (\text{ext. } \angle \text{ of } \Delta)$$

$$\therefore b = \left[\frac{b}{3} + (a + 4^\circ) \right] + 3a - 14^\circ, \quad \frac{2}{3}b = 4a - 10^\circ \quad \text{..... (ii)}$$

$$\text{Put (i) into (ii), } \frac{2}{3} \left(180^\circ - \frac{3}{2}a \right) = 4a - 10^\circ,$$

$$120^\circ - a = 4a - 10^\circ, \quad 5a = 130^\circ, \quad a = 26^\circ, \quad \therefore b = 180^\circ - \frac{3}{2}(26^\circ) = 141^\circ$$

37. In $\triangle ABCDEJ$, $a + b + c + d + e + \text{reflex } \angle EJA = 180^\circ \times 4$ ($\angle$ sum of polygon)

$$\text{In } \triangle FGHJ, f + g + h + i + \angle FJI = 180^\circ \times 3 \quad (\angle \text{ sum of polygon})$$

$$\therefore a + b + c + d + e + f + g + h + i + \text{reflex } \angle EJA + \angle FJI$$

$$= 180^\circ \times 4 + 180^\circ \times 3 = 180^\circ \times 7$$

But $\angle FJI = \angle EJA$ (vert. opp. $\angle$ s),

$$\therefore \text{reflex } \angle EJA + \angle FJI = \text{reflex } \angle EJA + \angle EJA = 360^\circ \quad (\angle \text{ s at a pt.})$$

$$\therefore \text{Sum of the marked angles} = 180^\circ \times 7 - 360^\circ = 900^\circ$$

38. (a) $BC = AB$ (regular polygon) $= BI$ (given),

$$\therefore \angle BIC = \angle BCI \quad (\text{base } \angle \text{ s, isos. } \Delta) = 52^\circ, \quad \angle CBI = 180^\circ - 52^\circ - 52^\circ = 76^\circ$$

$$\angle ABC = \frac{(8-2) \times 180^\circ}{8} \quad (\angle \text{ of regular polygon}) = 135^\circ$$

$$\angle ABI = \angle ABC - \angle CBI = 135^\circ - 76^\circ = 59^\circ,$$

$$\angle AIB = \angle BAI \quad (\text{base } \angle \text{ s, isos. } \Delta)$$

$$= \frac{180^\circ - 59^\circ}{2} \quad (\angle \text{ sum of } \Delta) = 60.5^\circ$$

- (b) Join AC. $\therefore AB = BC$ (given),

$$\therefore \angle BAC = \angle ACB \quad (\text{base } \angle \text{ s, isos. } \Delta)$$

$$= \frac{180^\circ - 135^\circ}{2} \quad (\angle \text{ sum of } \Delta) = 22.5^\circ$$

$$\angle CAI = \angle BAI - \angle BAC = 60.5^\circ - 22.5^\circ = 38^\circ$$

$$\angle ACI = \angle BCI - \angle ACB = 52^\circ - 22.5^\circ = 29.5^\circ,$$

$\therefore \angle ACI \neq \angle CAI, \therefore AI \neq CI$. Thus, the claim is disagreed.

39. (a) $\angle CDE = \frac{(5-2) \times 180^\circ}{5} = 108^\circ$ ($\angle$ of regular polygon)
 $\angle EDI = \frac{(6-2) \times 180^\circ}{6} = 120^\circ$
 $\angle JDK = \angle CDE + \angle EDI - 180^\circ = 108^\circ + 120^\circ - 180^\circ = 48^\circ$
- (b) $\angle BCD = \angle CBK = 108^\circ$,
 $\angle BKD + 48^\circ + 108^\circ + 108^\circ = (4-2) \times 180^\circ$ ($\angle$ sum of polygon),
 $\angle BKD = 96^\circ$, $\angle DJK = 180^\circ - 48^\circ - 96^\circ = 36^\circ$ ($\angle$ sum of Δ)
- (c) When $\angle CDI$ is an interior angle of an n -sided regular polygon, its corresponding exterior angle is $\angle JDK$ which is 48° [from part (a)].
 $n = \frac{360}{48} = 7.5$ which is not an integer. Thus, no.
40. (a) $\angle ABC = \frac{(5-2) \times 180^\circ}{5} = 108^\circ$ ($\angle$ of regular polygon)
- (b) Let n be the number of sides of Y . $\angle CBF = \frac{108^\circ}{2} \times 3 = 162^\circ$
 $(n-2) \times 180^\circ = 162^\circ \times n$, $180n - 360 = 162n$, $n = 20$.
Ans. The number of sides of Y is 20.
- (c) $\angle ABF = 360^\circ - \angle ABC - \angle CBF$ ($\angle$ s at a pt.) $= 360^\circ - 108^\circ - 162^\circ = 90^\circ$.
 $AB = BF$ (regular polygon),
 $\therefore \angle BAF = \angle AFB$ (base $\angle$ s, isos. Δ)
 $= \frac{180^\circ - 90^\circ}{2}$ ($\angle$ sum of Δ) $= 45^\circ$
41. (a) Size of an interior angle of $ABCDE = \frac{(5-2) \times 180^\circ}{5} = 108^\circ$
 $\therefore \angle ABC = \angle BCD = \angle CDE = 108^\circ$
 $\therefore CD = DE$ (regular polygon),
 $\therefore \angle DCE = \angle CED$ (base $\angle$ s, isos. Δ)
 $= \frac{180^\circ - 108^\circ}{2}$ ($\angle$ sum of Δ) $= 36^\circ$
 $\angle BCE = \angle BCD - \angle DCE = 108^\circ - 36^\circ = 72^\circ$
 $\angle ABC + \angle BCE = 108^\circ + 72^\circ = 180^\circ$, $\therefore AB \parallel CE$ (int. $\angle$ s supp.)
- (b) Note that $\angle ADE = \angle DAE = 36^\circ$.
 $\angle CGD = \angle ADE + \angle CED = 36^\circ + 36^\circ = 72^\circ$.
 $\angle CDG = 180^\circ - 72^\circ - 36^\circ$ ($\angle$ sum of Δ)
 $= 72^\circ$
 $= \angle CGD$
 $\therefore CG = CD$ (sides opp., eq. $\angle$ s), and $CD = BC$ (regular polygon)
 $\therefore BC = CG$
- (c) $BC = CG$, $\therefore \angle BGC = \angle CBG$ (base $\angle$ s, isos. Δ),
 $\angle BGC = \frac{180^\circ - 72^\circ}{2}$ ($\angle$ sum of Δ) $= 54^\circ$
 But $\angle CGD = 72^\circ \neq \angle BGC$, $\therefore$ the claim is disagreed.

Unit 9 Rate, ratio & proportion

1. (a) $\frac{1200}{25} = 48$ (b) $\frac{128}{32} = 4$ (c) $\frac{48}{30} = 1.6$
2. $90 \text{ km/day} = \frac{90 \times 1000 \text{ m}}{24 \times 60 \text{ min}} = 62.5 \text{ m/min}$, $\therefore 70 \text{ m/min}$ is the higher rate
3. $\text{Speed} = \frac{15}{6} = 2.5 \text{ m/s}$, $\therefore \text{Time taken} = \frac{100}{2.5} = 40 \text{ s}$
4. (a) $\text{Time taken} = \frac{100}{80} = 1.25 \text{ h}$
 (b) $\text{Distance travelled} = 80 \times \frac{3}{4} = 60 \text{ km}$
5. (a) $= \frac{96}{24} : \frac{360}{24} = 4 : 15$
 (b) $= 9.8 \times 10 : 4.2 \times 10 = \frac{98}{14} : \frac{42}{14} = 7 : 3$
 (c) $= \frac{3}{10} : \frac{32}{15} = \frac{3}{10} \times 30 : \frac{32}{15} \times 30 = 9 : 64$
 (d) $= (3 \times 60 \times 60) \text{ seconds} : 200 \text{ seconds} = \frac{10800}{200} : \frac{200}{200} = 54 : 1$
 (e) $= (2.5 \times 100) \text{ ¢} : 15 \text{ ¢} = \frac{250}{5} : \frac{15}{5} = 50 : 3$
 (f) $= 240 \text{ g} : (1.2 \times 1000) \text{ g} = \frac{240}{240} : \frac{1200}{240} = 1 : 5$
 (g) $= (3 \times 1000) \text{ m} : 800 \text{ m} = \frac{3000}{200} : \frac{800}{200} = 15 : 4$
 (h) $= (0.4 \times 10000) \text{ cm}^2 : 500 \text{ cm}^2 = \frac{4000}{500} : \frac{500}{500} = 8 : 1$
6. (a) $\frac{x}{3} = \frac{7}{2}$, $2x = 21$, $\therefore x = 10\frac{1}{2}$
 (b) $\frac{6}{y} = \frac{5}{4}$, $24 = 5y$, $\therefore y = 4\frac{4}{5}$
7. (a) $\frac{800 \text{ g}}{1000 \text{ g}} = \frac{\$ a}{\$ 120}$, $\frac{4}{5} = \frac{a}{120}$, $480 = 5a$, $\therefore a = 96$
 (b) $\frac{(7 \times 100\,000) \text{ cm}}{3 \text{ cm}} = \frac{(2 \times 100\,000) \text{ cm}}{k \text{ cm}}$, $7k = 6$, $\therefore k = \frac{6}{7}$
8. Let x be the original number, new number : original number
 $= x(1 + \frac{1}{8}) : x = \frac{9x}{8} : x = \frac{9}{8} \times 8 : 1 \times 8 = 9 : 8$
9. Paul's weight : his brother's weight $= 48 : (48 - 16) = \frac{48}{16} : \frac{32}{16} = 3 : 2$
10. (a) $\text{Speed of car} = 33 \div \frac{1}{3} = 99 \text{ km/h}$; $\text{speed of train} = \frac{270}{2} = 135 \text{ km/h}$
 (b) $\text{Speed of car} : \text{speed of train} = 99 : 135 = \frac{99}{9} : \frac{135}{9} = 11 : 15$

11. Let x be the number of boys in the group, $\frac{x}{x+10} = \frac{4}{5}$, $5x = 4x + 40$, $x = 40$.
Ans. Number of boys in the group is 40.
12. Let $\$x$ be his monthly salary, $x(\frac{3}{5+3}) = 4800$, $3x = 4800 \times 8$, $x = 12800$.
Ans. His monthly salary is \$12800.
13. Let x litres be the volume of water added, $\frac{6}{4+x} = \frac{2}{3}$, $18 = 2(4+x)$, $10 = 2x$, $x = 5$.
Ans. 5 litres of water must be added.
14. New salary rate = $\$ \frac{276}{10} / h = \$27.6 / h$,
 $\therefore$ New salary : original salary = $27.6 : 24 = \frac{276}{12} : \frac{240}{12} = 23 : 20$
15. (a) $x : y = 5 \times 5 : 3 \times 5 = 25 : 15$, $y : z = 5 \times 3 : 2 \times 3 = 15 : 6$,
 $\therefore x : y : z = 25 : 15 : 6$
 (b) $y : x = 4 : 3$, $x : z = 1 \times 3 : 2 \times 3 = 3 : 6$, $\therefore x : y : z = 3 : 4 : 6$
16. (a) $4a = 3b$, $\frac{a}{b} = \frac{3}{4}$, $\therefore a : b = 3 : 4$ (b) $\frac{4}{5} = \frac{a}{b}$, $\therefore a : b = 4 : 5$
 (c) $7b = a$, $\frac{7}{1} = \frac{a}{b}$, $\therefore a : b = 7 : 1$
 (d) $4a + 2b = 21a - 7b$, $9b = 17a$, $\frac{9}{17} = \frac{a}{b}$, $\therefore a : b = 9 : 17$
17. $x : 12 = 4 : y$, $\frac{x}{12} = \frac{4}{y}$, $\therefore xy = 48$
18. $A : B = 1 : 3$, $B : C = 1 : 2 = 1 \times 3 : 2 \times 3 = 3 : 6$, $\therefore A : B : C = 1 : 3 : 6$
 $\therefore$ The amount B gets = $500 \times \frac{3}{1+3+6} = 500 \times \frac{3}{10} = \150
19. The scale = $4 \text{ cm} : 2 \text{ km} = 4 \text{ cm} : 200\,000 \text{ cm} = 1 : 50\,000$
20. Map distance of a runway = $2.5 \text{ km} \times \frac{1}{100\,000} = \frac{250\,000 \text{ cm}}{100\,000} = 2.5 \text{ cm}$
21. Actual distance = $3 \text{ mm} \times 2\,500\,000 = 7\,500\,000 \text{ mm} = 7.5 \text{ km}$
22. Length of the hall = $5 \text{ cm} \times 400 = 2\,000 \text{ cm} = 20 \text{ m}$
 Width of the hall = $4 \text{ cm} \times 400 = 1\,600 \text{ cm} = 16 \text{ m}$
 $\therefore$ Actual area of the hall = $20 \times 16 = 320 \text{ m}^2$
23. Let $\$x$ be Samuel's pocket money for 28 days. By direct proportion:
 $\frac{x}{165} = \frac{28}{3}$, $x = \frac{28}{3} \times 165 = 1540$ *Ans. The required amount is \$1540.*
24. His original average daily expenditure = $\$3\,500 \div 7 = \500
 Let $\$x$ be his new average daily expenditure. By inverse proportion:
 $\frac{x}{500} = \frac{21}{21-6}$, $x = \frac{21}{15} \times 500 = 700$
Ans. His greatest average daily expenditure will be \$700.
25. (a) We check all pairs of $\frac{x}{y}$: $\frac{4}{18} = \frac{2}{9}$; $\frac{6}{27} = \frac{2}{9}$; $\frac{10}{45} = \frac{2}{9}$
 $\therefore \frac{x_1}{y_1} = \frac{x_2}{y_2} = \text{constant}$, $\therefore \frac{x_1}{x_2} = \frac{y_1}{y_2}$ for all pairs of x, y
 $\therefore x$ and y are in direct proportion.

(b) We check all pairs of $\frac{x}{y}$: $\frac{5}{15} = \frac{1}{3}$; $\frac{15}{35} = \frac{3}{7} \neq \frac{1}{3}$

$\therefore \frac{y}{x}$ is a not constant, $\therefore x$ and y are in not direct proportion.

When x increases, y does not decrease, $\therefore x$ and y are not in inverse proportion.

Ans. x and y are neither in direct proportion nor in inverse proportion.

(c) We check all pairs of xy :

$$9 \times 28 = 252; \quad 12 \times 21 = 252; \quad 24 \times 10.5 = 252$$

$$\therefore x_1 y_1 = x_2 y_2 = \text{constant}, \quad \therefore \frac{x_1}{x_2} = \frac{y_2}{y_1} \text{ for all pairs of } x, y.$$

$\therefore x$ and y are in inverse proportion.

26. Distance travelled = $60 \times \frac{90}{60} = 90$ km,

$$\therefore \text{Volume of petrol used} = \frac{90}{6} = 15 \text{ litres}$$

27. No. of \$1 coins = $240 \times \frac{8}{8+3+4} = 128$, no. of 50¢ coins = $240 \times \frac{3}{8+3+4} = 48$,

$$\text{no. of 20¢ coins} = 240 \times \frac{4}{8+3+4} = 64$$

$$\therefore \text{Total amount} = 128 \times 1 + 48 \times 0.5 + 64 \times 0.2 = \$164.8$$

28. Average speed = $\frac{5}{1000} \text{ km} \div \frac{3}{3600} \text{ h} = \frac{5}{1000} \times \frac{3600}{3} \text{ km/h} = 6 \text{ km/h}$

29. Let $2k$ be the present age of Mary, then the present age of Lily is $3k$.

$$\frac{2k+4}{3k+4} = \frac{5}{7}, \quad 14k+28 = 15k+20, \quad k=8, \quad \therefore 2k=16, 3k=24.$$

Ans. The present ages of Mary and Lily are 16 and 24 respectively.

30. Let $3k$ be the length of the smaller part, then the length of the larger part is $7k$.

$$\therefore \text{Ratio of the three parts} = 3k : 7k \times \frac{2}{2+3} : 7k \times \frac{3}{2+3}$$

$$= 3k \times 5 : \frac{14k}{5} \times 5 : \frac{21k}{5} \times 5 = 15 : 14 : 21$$

31. $\frac{4}{a+1} \div \frac{3}{a} = \frac{7}{6}$, $\frac{4a}{3(a+1)} = \frac{7}{6}$, $24a = 21a + 21$, $3a = 21$, $\therefore a = 7$

32. $a : b = 3 \times 25 : 2 \times 25 = 75 : 50$, $a : c = \frac{5}{2} \times 30 : \frac{2}{5} \times 30 = 75 : 12$,

$$\therefore a : b : c = 75 : 50 : 12 = 25 : 16 : 4; \text{ but } b : d = 6 : 7,$$

$$\therefore b : c : d = 25 \times 6 : 16 \times 6 : 4 \times 6 = 150 : 96 : 24 = 25 : 16 : 4, \quad \therefore c : d = 36 : 175$$

33. $2p = 3q$, $p : q = 3 : 2 = 3 \times 9 : 2 \times 9 = 27 : 18$,

$$8q = 9r$$
, $q : r = 9 : 8 = 9 \times 2 : 8 \times 2 = 18 : 16$, $\therefore p : q : r = 27 : 18 : 16$

34. $\frac{1}{a} : \frac{1}{b} = 3 : 4$, $\frac{1}{a} \div \frac{1}{b} = \frac{3}{4}$, $\frac{b}{a} = \frac{3}{4}$, $\therefore a : b = 4 : 3$, $\frac{1}{b} : \frac{1}{c} = 4 : 5$, $\frac{1}{b} \div \frac{1}{c} = \frac{4}{5}$,

$$\frac{c}{b} = \frac{4}{5}, \quad \therefore b : c = 5 : 4, \quad \therefore a : b : c = 4 \times 5 : 3 \times 5 : 4 \times 3 = 20 : 15 : 12$$

$$(\text{OR: } \frac{1}{a} = 3k, \quad \frac{1}{b} = 4k, \quad \frac{1}{c} = 5k,$$

$$\therefore a : b : c = \frac{1}{3k} : \frac{1}{4k} : \frac{1}{5k} = \frac{1}{3k} \times 60k : \frac{1}{4k} \times 60k : \frac{1}{5k} \times 60k = 20 : 15 : 12$$

35. Length scale of map = 1 mm : 0.25 m = 0.1 cm : 0.25 m = 1 cm : 2.5 m

Area scale of map = $(1 \times 1) \text{ cm}^2 : (2.5 \times 2.5) \text{ m}^2 = 1 \text{ cm}^2 : 6.25 \text{ m}^2$

$$\therefore \text{Map area of the field} = 400 \times \frac{1}{6.25} = 64 \text{ cm}^2$$

36. Area scale of map = $(4 \times 4) \text{ cm}^2 : (1 \times 1) \text{ km}^2 = 16 \text{ cm}^2 : 1 \text{ km}^2$

Map area = $(2 \times 3) \text{ cm}^2 = 6 \text{ cm}^2$, $\therefore$ Actual area = $6 \times \frac{1}{16} = \frac{3}{8} \text{ km}^2$

37. $x = y(1 + 25\%)$, $\frac{x}{y} = 1.25 = \frac{5}{4}$, $\therefore x : y = 5 : 4$

$x = z(1 - 20\%)$, $\frac{x}{z} = 0.8 = \frac{4}{5}$, $\therefore x : z = 4 : 5$

$$\therefore x : y : z = 5 \times 4 : 4 \times 5 : 5 \times 5 = 20 : 16 : 25$$

38. $2a + 3b = 3a + b$, $a = 2b$

$$\therefore (4a + b) : (3a - 2b) = [4(2b) + b] : [3(2b) - 2b] = 9b : 4b = 9 : 4$$

39. Let $2a = 3b = 7c = k$, then $a = \frac{k}{2}$, $b = \frac{k}{3}$, $c = \frac{k}{7}$.

$$\therefore (a - b + c) : (a + b - c) = \left(\frac{k}{2} - \frac{k}{3} + \frac{k}{7}\right) : \left(\frac{k}{2} + \frac{k}{3} - \frac{k}{7}\right) = \frac{13}{42} : \frac{29}{42} = 13 : 29$$

40. Let $\frac{a}{b} = \frac{c}{d} = k$, then $a = bk$, $c = dk$.

$$\text{L.H.S.} = \frac{a + c}{b + d} = \frac{bk + dk}{b + d} = \frac{k(b + d)}{b + d} = k,$$

$$\text{R.H.S.} = \frac{a - c}{b - d} = \frac{bk - dk}{b - d} = \frac{k(b - d)}{b - d} = k = \text{L.H.S.}, \therefore \frac{a + c}{b + d} = \frac{a - c}{b - d}$$

41. $\therefore$ The largest angle = $180^\circ \times \frac{8}{3 + 8 + 1} = 180^\circ \times \frac{8}{12} = 120^\circ > 90^\circ$

$\therefore$ It is an obtuse-angled triangle.

42. Let $3k$ cm and $4k$ cm be the width and the length of the rectangle respectively.

$$(3k)(4k) = 432, \quad 12k^2 = 432, \quad k^2 = 36, \quad k = 6, \quad \therefore 3k = 18, \quad 4k = 24.$$

Ans. Perimeter of the rectangle is $2(18 + 24) = 84$ cm.

43. Let $r : 1$ be the ratio of the two types of tea.

$$75\left(\frac{r}{1+r}\right) + 50\left(\frac{1}{1+r}\right) = 60, \quad 75r + 50 = 60(1 + r), \quad 15r = 10, \quad r = \frac{2}{3}.$$

Ans. The ratio of the mixture is $\frac{2}{3} : 1 = 2 : 3$.

44. The cost of the mixture = $60 \div (1 + 25\%) = \$48$.

Let the cost price of coffee B be \$y.

$$15 \times \frac{2}{2+3} + y \times \frac{3}{2+3} = 48, \quad 30 + 3y = 240, \quad y = 70.$$

Ans. The cost price of coffee B is \$70/kg.

45. $150 \times \frac{m}{m+n} + 200 \times \frac{n}{m+n} = 150(1 + 10\%) \times \frac{m}{m+n} + 200(1 - 30\%) \times \frac{n}{m+n},$

$$150m + 200n = 165m + 140n, \quad 60n = 15m, \quad \frac{m}{n} = \frac{60}{15} = \frac{4}{1}, \quad \therefore m : n = 4 : 1$$

46. Let \$2k and \$k be the daily wages of a man and a woman respectively.

$$\therefore \text{Ratio of hourly wages of a man and a woman} = \frac{2k}{10} : \frac{k}{8} = \frac{1}{5} \times 40 : \frac{1}{8} \times 40 = 8 : 5$$

47. Let d m be the distance between home and school.

$$\text{Total time taken} = \frac{d}{x} + \frac{d}{y} = \frac{d(x+y)}{xy} \text{ seconds}$$

$$\therefore \text{His average speed} = 2d \div \frac{d(x+y)}{xy} = 2d \times \frac{xy}{d(x+y)} = \frac{2xy}{x+y} \text{ m/s}$$

48. Let w g be the weight of the cube with side length 7 cm.

$$\text{By direct proportion, } \frac{w}{70} = \frac{7^3}{4}, \quad w = 6002.5. \quad \text{Ans. It weighs 6002.5 g.}$$

49. Let t minutes be the time required to fill up the tank if 2 more pumps are used.

$$\text{By inverse proportion, } \frac{t}{30} = \frac{10}{10+2}, \quad t = 25. \quad \therefore \text{Time saved} = 30 - 25 = 5 \text{ min.}$$

50. (a) Let x be the required number of workers.

$$\text{By inverse proportion, } \frac{x}{6} = \frac{3}{2}, \quad x = 9. \quad \text{Ans. 9 workers are required,}$$

- (b) Let y be the required number of workers.

$$\text{By direct proportion, } \frac{y}{6} = \frac{1500 \times 0.8 \times 1}{800 \times 1 \times 0.6}, \quad y = 15. \quad \text{Ans. 15 workers are required.}$$

- (c) Let z be the required number of workers.

$$\text{Originally work done per day} = 800 \times 1 \times 0.6 \div 3 = 160 \text{ m}^3.$$

$$\text{Now work done per day} = 1200 \times 2 \times 1.8 \div 6 = 720 \text{ m}^3.$$

$$\text{By direct proportion, } \frac{z}{6} = \frac{720}{160}, \quad z = 27. \quad \text{Ans. 27 workers are required.}$$

51. Let x litres be the amount of water.

$$\text{Amount of water 1 man needs per day} = \frac{x}{6 \times 8} = \frac{x}{48} \text{ litres}$$

$$\text{Amount of water 1 boy needs per day} = \frac{x}{8 \times 10} = \frac{x}{80} \text{ litres}$$

$$\therefore \text{Time lasting} = x \div \left(\frac{x}{48} \times 12 + \frac{x}{80} \times 4 \right) = x \div \frac{3x}{10} = x \times \frac{10}{3x} = 3\frac{1}{3} \text{ days}$$

52. Work done by A : work done by B : work done by C

$$= \frac{1}{10} : \frac{1}{20} : \frac{1}{30} = \frac{60}{10} : \frac{60}{20} : \frac{60}{30} = 6 : 3 : 2$$

$$\therefore \text{The amount A will receive} = 11000 \times \frac{6}{6+3+2} = \$6000$$

53. If there are 60 chickens, the remaining food can last for 30 - 10 = 20 days.

$$\therefore \text{The number of days for 50 chickens} = \frac{60 \times 20}{50} = 24.$$

Ans. The remaining food can last for 24 - 20 = 4 days more.

54. Let BQ = x, then QC = 36 - x.

$$\frac{(18+x) \times AB}{2} : \frac{[18+(36-x)] \times AB}{2} = 3 : 2,$$

$$\frac{18+x}{54-x} = \frac{3}{2}, \quad 36 + 2x = 162 - 3x, \quad 5x = 126, \quad x = 25.2.$$

Ans. The value of BQ is 25.2.

55. Let the longer side and the shorter side of the original rectangle be b cm and a cm respectively.

The longer side of the new rectangle will be a cm, and the shorter side will be $\frac{b}{2}$ cm.

$$a : \frac{b}{2} = b : a, \quad \frac{2a}{b} = \frac{b}{a}, \quad 2a^2 = b^2, \quad \frac{a^2}{b^2} = \frac{1}{2}, \quad \frac{a}{b} = \frac{1}{\sqrt{2}}.$$

Ans. The ratio of lengths of the original rectangle is $1 : \sqrt{2}$. [OR: $\sqrt{2} : 1$]

56. Let $AB = x$, $\therefore BC = 2x$. Let $PQ = 3y$, $\therefore QR = 2y$.

$$2(x+2x) = 2(3y+2y), \quad 3x = 5y, \quad x = \frac{5y}{3}$$

$\therefore$ Area of ABCD : area of PQRS

$$= x(2x) : (3y)(2y) = x^2 : 3y^2 = \left(\frac{5y}{3}\right)^2 : 3y^2 = \frac{25}{9} : 3 = \frac{25}{9} \times 9 : 3 \times 9 = 25 : 27$$

57. (a) The exchange rate for pound to HK dollar = $49 \div 5 = \$9.8/\text{£}$

The exchange rate for pound to yen = $18 \times 9.8 = \text{¥}176.4/\text{£}$

- (b) (i) The price of the Japanese scarf = $26\,460 \div 18 = \$1\,470$

The price of the U.K. scarf = $165 \times 9.8 = \$1\,617$

- (ii) The new exchange rate be $\text{¥}n/\text{\$}$

$$26\,460 \div n = 1617, \quad n = 16.3.$$

Ans. The new exchange rate is $\text{¥}16.3/\text{\$}$.

58. (a) Richard's average cycling speed = $\frac{98 \times 1000}{\left(2 + \frac{20}{60}\right) \times 60 \times 60} = 11\frac{2}{3}$ m/s

- (b) The distance he cycles in 45 minutes = $11.7 \times 45 \times 60 = 31\,500$ m < 35 km

$\therefore$ He cannot complete 35 km in 45 minutes.

- (c) The time Steven required to finish 98 km

$$= 98\,000 \div \left(11\frac{2}{3} + 0.6\right) \text{ s} = 7\,989 \text{ s} \quad (4 \text{ sig. fig.})$$

$$2 \text{ hours } 15 \text{ minutes} = 8\,100 \text{ s} > 7\,989 \text{ s.} \quad \text{Ans. The claim is agreed.}$$

2 hours and 15 minutes = 2.25 hours > 2.2192 hours. $\therefore$ The claim is agreed.

59. (a) (i) Actual length = $12 \div 4 \times 3 = 9$ km

Actual width = $5 \div 4 \times 3 = 3.75$ km

- (ii) Area of the field on the map = $12\text{cm} \times 5\text{cm} = 60\text{cm}^2$

Actual area of the field

$$= 9 \text{ km} \times 3.75 \text{ km}$$

$$= (9 \times 1\,000 \times 100) \times (3.75 \times 1\,000 \times 100)\text{cm}^2 = 337\,500\,000\,000 \text{ cm}^2$$

$$\text{The required ratio} = 337\,500\,000\,000 : 60 = 5\,625\,000\,000 : 1$$

- (b) Longer side of the paper : longer side of the field on the original map

$$= 30\text{cm} : 12\text{cm} = 2.5 : 1$$

Shorter side of the paper : shorter side of the field on the original map

$$= 21\text{cm} : 5\text{cm} = 4.2 : 1$$

$2.5 < 4.2$, $\therefore$ we only need to consider the longer side of the field.

$$30\text{cm} : 9\text{km} = 30 : 900\,000 = 1 : 30\,000, \quad \therefore n = 30\,000$$

60. (a) $x : y = 2 : 3 = 6 : 9$, $x : z = 3 : 1 = 6 : 2$, $\therefore x : y : z = 6 : 9 : 2$

- (b) (i) From (a), apple juice : orange juice : lemon juice = $6 : 9 : 2$

$$\therefore \text{volume of orange juice required} = \frac{9}{6+9+2} \times 5.1 = \frac{9}{17} \times 5.1 \text{ L} = 2.7 \text{ L}$$

- (ii) Let the volume of apple juice, orange juice and lemon juice be $6k$ L, $9k$ L and $2k$ L respectively.

If $6k = 7.5$, $k = 1.25$. If $9k = 12$, $k = 1.33$. If $2k = 3$, $k = 1.5$.

Each type of juice would be enough for the mixture only when we take the smallest value of k , $\therefore$ we should take $k = 1.25$.

$\therefore$ The volume of mixed-fruit juice = $(6 + 9 + 2)(1.25)$ L = 21.25 L

The volume of apple juice = $6(1.25)$ L = 7.5 L

The volume of orange juice = $9(1.25)$ L = 11.25 L

The volume of lemon juice = $2(1.25)$ L = 2.5 L

61. (a) 1 h 30 min = 90 min; 2 h 40 min = 160 min

By inverse proportion, $x : y = 160 : 90 = 16 : 9$

- (b) (i) Let $x = 16k$, $y = 9k$, where $k \neq 0$.

$$(16k + 9k) \times \frac{36}{60} = 225, \quad 25k = 375, \quad k = 15.$$

Speed of pump A = $16(15) = 240$ m³/h

$\therefore$ Capacity of the tank = $240(1.5) = 360$ m³

- (ii) Speed of filling the tank when pumps A and B are used simultaneously

$$= \frac{225}{36} = 6\frac{1}{4} \text{ m}^3/\text{min} > 6 \text{ m}^3/\text{min}$$

The speed of the new pump is slower, and thus more time is required.

$\therefore$ The claim is disagreed.

Unit 10 Similar triangles

- (a) No. ($\because \angle B$ and $\angle P$ are not the included angles)

(b) Yes. $\triangle ABC \sim \triangle QRP$ ($\angle A = 180^\circ - 75^\circ - 40^\circ = 65^\circ$, $\therefore$ AAA)

(c) Yes. $\triangle ABC \sim \triangle RQP$ (ratio of 2 sides, inc. $\angle$)

(d) No. ($\because$ 3 sides are not proportional.)
- $\angle RQP = 180^\circ - 69^\circ - 37^\circ = 74^\circ$ ($\angle$ sum of Δ); $\angle EDC = \angle RQP = 74^\circ$ (corr. $\angle$ s, $\sim \Delta$ s)
- (a) $YX = QR = 13$ cm (corr. sides, $\sim \Delta$ s)

(b) $\angle YXZ = \angle QRP = 44^\circ$ (corr. $\angle$ s, $\sim \Delta$ s)

(c) $\angle PQR = 180^\circ - 47^\circ - 44^\circ = 89^\circ$ ($\angle$ sum of Δ); $\angle ZYX = \angle PQR = 89^\circ$ (corr. $\angle$ s, $\sim \Delta$ s)
- $\frac{BC}{PR} = \frac{AC}{QR} = \frac{AB}{QP}$ (corr. sides, $\sim \Delta$ s), $\frac{x}{6} = \frac{y}{7} = \frac{10}{4}$,

$\therefore x = \frac{10}{4} \times 6 = 15$; $y = \frac{10}{4} \times 7 = 17.5$; $z = \angle B = 86^\circ$ (corr. $\angle$ s, $\sim \Delta$ s)
- (a) $\frac{m}{8} = \frac{n}{6} = \frac{15}{10}$ (corr. sides, $\sim \Delta$ s), $\therefore m = \frac{15}{10} \times 8 = \underline{12}$, $n = \frac{15}{10} \times 6 = \underline{9}$

(b) $\triangle ABC \sim \triangle QPR$ (AAA), $\frac{AC}{QR} = \frac{BC}{PR} = \frac{AB}{QP}$ (corr. sides, $\sim \Delta$ s), $\frac{x}{6} = \frac{y}{7} = \frac{12}{8}$

$\therefore x = \frac{12}{8} \times 6 = 9$; $y = 6 \times \frac{8}{12} = 4$

(c) $\frac{x}{10} = \frac{y}{6} = \frac{12}{8}$ (corr. sides, $\sim \Delta$ s), $\therefore x = \frac{12}{8} \times 10 = 15$; $y = \frac{12}{8} \times 6 = 9$

(d) $\frac{7}{4} = \frac{3h+1}{5} = \frac{2k-3}{4}$ (corr. sides, $\sim \Delta$ s),

$$\therefore 35 = 12h + 4, \quad h = \frac{31}{12}; \quad 28 = 8k - 12, \quad k = \frac{30}{8} = \frac{15}{4}$$

6. (a) $\triangle EFG \sim \triangle KFH$ (AAA), $\frac{EF}{KF} = \frac{EG}{KH}$ (corr. sides, $\sim\Delta$ s),

$$\frac{m+6}{6} = \frac{28}{16}, \quad m+6 = \frac{28}{16} \times 6 = 10.5, \quad \therefore m = 4.5$$

(b) $\triangle PQR \sim \triangle SQT$ (AAA), $\frac{PQ}{QR} = \frac{SQ}{QT}$ (corr. sides, $\sim\Delta$ s),

$$\frac{z+6}{z} = \frac{3+9}{3} = 4, \quad z+6 = 4z, \quad 3z = 6, \quad \therefore z = 2$$

7. (a) $\triangle ABC \sim \triangle AED$ (AAA), $\frac{AB}{AE} = \frac{AC}{AD}$ (corr. sides, $\sim\Delta$ s),

$$\frac{y+6}{8} = \frac{8+7}{6} = \frac{5}{2}, \quad 2y+12 = 40, \quad 2y = 28, \quad \therefore y = 14$$

(b) $\triangle PQR \sim \triangle NMR$ (AAA), $\frac{NR}{PR} = \frac{NM}{PQ}$ (corr. sides, $\sim\Delta$ s),

$$\frac{r}{50} = \frac{8}{20}, \quad \therefore r = \frac{8}{20} \times 50 = 20$$

(c) $\triangle PQR \sim \triangle TQS$ (AAA), $\frac{QR}{QS} = \frac{PQ}{TQ}$ (corr. sides, $\sim\Delta$ s),

$$\frac{18+x}{28} = \frac{28+8}{18} = 2, \quad 18+x = 56, \quad \therefore x = 38$$

8. (a) In $\triangle EFG$ and $\triangle KFH$, $\angle F = \angle F$ (common),
 $\angle E = \angle HKF$ and $\angle G = \angle KHF$ (corr. $\angle$ s, $GE \parallel HK$),

$$\therefore \triangle EFG \sim \triangle KFH$$
 (AAA), $\therefore \frac{EF}{KF} = \frac{EG}{KH}$ (corr. sides, $\sim\Delta$ s),

$$\frac{2+r}{r} = \frac{20}{16} = \frac{5}{4}, \quad 8+4r = 5r, \quad \therefore r = 8$$

(b) In $\triangle ADE$ and $\triangle ABC$, $\angle A = \angle A$ (common),
 $\angle ADE = \angle ABC$ and $\angle AED = \angle ACB$ (corr. $\angle$ s, $DE \parallel BC$),

$$\therefore \triangle ADE \sim \triangle ABC$$
 (AAA), $\therefore \frac{AE}{AC} = \frac{DE}{BC}$ (corr. sides, $\sim\Delta$ s)

$$\frac{y}{y+15} = \frac{8}{20} = \frac{2}{5}, \quad 5y = 2y + 30, \quad 3y = 30, \quad \therefore y = 10$$

(c) In $\triangle PQR$ and $\triangle FGR$, $\angle R = \angle R$ (common),
 $\angle P = \angle RFG$ and $\angle Q = \angle RGF$ (corr. $\angle$ s, $PQ \parallel FG$), $\therefore \triangle PQR \sim \triangle FGR$ (AAA)

$$\frac{FG}{PQ} = \frac{FR}{PR}$$
 (corr. sides, $\sim\Delta$ s), $\frac{a}{36} = \frac{9}{9+18} = \frac{1}{3}, \quad 3a = 36, \quad \therefore a = 12$

9. (a) $\angle ACB = \angle ECD$ (vert. opp. $\angle$ s), $\angle A = \angle E$ and $\angle B = \angle D$ (alt. $\angle$ s, $AB \parallel DE$),

$$\therefore \triangle ABC \sim \triangle EDC$$
 (AAA), $\frac{AB}{ED} = \frac{BC}{DC} = \frac{AC}{EC}$ (corr. sides, $\sim\Delta$ s),

$$\frac{8}{y} = \frac{x}{6} = \frac{3}{4}, \quad \therefore x = \frac{3}{4} \times 6 = 4.5; \quad y = 8 \times \frac{4}{3} = \frac{32}{3}$$

(b) $\angle PRQ = \angle TRS$ (vert. opp. $\angle$ s), $\angle P = \angle T$ and $\angle Q = \angle S$ (alt. $\angle$ s, $PQ \parallel ST$),

$$\therefore \triangle PQR \sim \triangle TSR$$
 (AAA), $\frac{PQ}{TS} = \frac{QR}{SR} = \frac{PR}{TR}$ (corr. sides, $\sim\Delta$ s),

$$\frac{5}{k} = \frac{4}{10} = \frac{h}{12}, \quad \therefore h = \frac{4}{10} \times 12 = 4.8; \quad k = 5 \times \frac{10}{4} = 12.5$$

10. Let y m be the actual height. $\frac{y \text{ m}}{6 \text{ cm}} = \frac{60 \text{ m}}{8 \text{ cm}}, \quad y = \frac{60}{8} \times 6 = 45$

Ans. The actual height is 45 m.

11. (a) $\frac{XZ}{XM} = \frac{XY}{XN}$ (corr. sides, $\sim \Delta$ s), $\therefore \frac{a}{18} = \frac{6}{12}, \quad a = \frac{6}{12} \times 18 = 9$

(b) $\therefore \angle Z = \angle M$ (corr. $\angle$ s, $\sim \Delta$ s), $\therefore YZ \parallel MN$ (alt. $\angle$ s eq.)

12. (a) No. Their corr. $\angle$ s are equal, but their sides may not be proportional.

(b) Yes. Their corr. $\angle$ s are equal, and their sides are proportional.

(c) No. Their sides are proportional, but their corr. $\angle$ s may not be equal.

(d) Yes. Their corr. $\angle$ s are equal, and their sides are proportional (1:1).

13. $\frac{\text{Perimeter of the second } \Delta}{\text{Perimeter of the first } \Delta} = \frac{96}{5+3+4} = \frac{96}{12} = 8$

$\therefore$ the longest side $= 5 \times 8 = 40$ cm

14. $\Delta AEC \sim \Delta ADB$ (AAA), $\therefore \frac{AC}{1.8} = \frac{40}{2.6}, \quad AC = 27.7 \text{ m}$ (3 sig. fig.)

Ans. The distance between the tree and the building $= 27.7 - 1.8 = 25.9$ m.

15. $\angle BAC = \angle AED = 77^\circ$ (corr. $\angle$ s, $\sim \Delta$ s), $\angle ADE = \angle BCA = 44^\circ$ (corr. $\angle$ s, $\sim \Delta$ s),
 $\therefore \angle DAE = 180^\circ - 77^\circ - 44^\circ = 59^\circ$ ($\angle$ sum of Δ), $\therefore \angle BAD = 77^\circ - 59^\circ = 18^\circ$

16. (a) $\angle Q = 180^\circ - 80^\circ - 32^\circ = 68^\circ = \angle B$ ($\angle$ sum of Δ),
 $\angle A = 180^\circ - 68^\circ - 80^\circ = 32^\circ = \angle R$ ($\angle$ sum of Δ), $\angle P = \angle C = 80^\circ$ (given),
 $\therefore \Delta PQR \sim \Delta CBA$ (AAA)

(b) $\frac{AB}{RQ} = \frac{AC}{RP} = \frac{BC}{QP}$ (corr. sides, $\sim \Delta$ s), $\frac{m}{9} = \frac{n}{13} = \frac{3}{6},$
 $\therefore m = \frac{3}{6} \times 9 = 4.5, \quad n = \frac{3}{6} \times 13 = 6.5$

17. (a) $\angle BCA + \angle BCD = \angle CDE + \angle CED$ (ext. $\angle$ of Δ),
 $\angle BCA + 90^\circ = \angle CDE + 90^\circ, \quad \therefore \angle BCA = \angle CDE$

(b) $\angle A = \angle E = 90^\circ$ (given), $\angle BCA = \angle CDE$ (proved),
 $\angle ABC = \angle ECD$ ($\angle$ sum of Δ), $\therefore \Delta ABC \sim \Delta ECD$ (AAA)

(c) $\frac{CE}{BA} = \frac{CD}{BC}$ (corr. sides, $\sim \Delta$ s), $\frac{CE}{10} = \frac{20}{26}, \quad CE = \frac{20}{26} \times 10 = \frac{100}{13}$

18. (a) In ΔADB and ΔBDC , $\angle ADB = \angle BDC = 90^\circ$ (given);
 $\angle ABD = 90^\circ - \angle CBD, \quad \angle BCD = 180^\circ - 90^\circ - \angle CBD = 90^\circ - \angle CBD$ ($\angle$ sum of Δ),
 $\therefore \angle ABD = \angle BCD; \quad \angle A = \angle CBD$ ($\angle$ sum of Δ); $\therefore \Delta ADB \sim \Delta BDC$ (AAA)

(b) $\therefore \Delta ADB \sim \Delta BDC$ (proved), $\therefore \frac{CD}{BD} = \frac{BD}{AD}$ (corr. sides, $\sim \Delta$ s), $CD = \frac{BD^2}{AD} = \frac{6^2}{4} = 9$

19. (a) In ΔMNX and ΔYZX , $\frac{XM}{XY} = \frac{15}{9} = \frac{5}{3}, \quad \frac{XN}{XZ} = \frac{20}{12} = \frac{5}{3},$

$\angle MXN = \angle YXZ$ (vert. opp. $\angle$ s), $\therefore \Delta MNX \sim \Delta YZX$ (ratio of 2 sides, inc. $\angle$)

(b) $\frac{YZ}{MN} = \frac{3}{5}$ (corr. sides, $\sim \Delta$ s), $\frac{y}{18} = \frac{3}{5}, \quad y = \frac{3}{5} \times 18 = 10.8$

(c) $\angle Y = \angle M$ and $\angle Z = \angle N$ (corr. $\angle$ s, $\sim \Delta$ s)

To claim that $YZ \parallel MN$, what is required is $\angle Y = \angle N$ or $\angle Z = \angle M$, but such information is not provided, $\therefore$ the claim is disagreed.

20. (a) In $\triangle BAD$ and $\triangle CDB$, $\angle BAD = \angle BDC$ (given), $\frac{AD}{DB} = \frac{4}{8} = \frac{1}{2}$, $\frac{AB}{DC} = \frac{6}{12} = \frac{1}{2}$,
 $\therefore \triangle BAD \sim \triangle CDB$ (ratio of 2 sides, inc. $\angle$)
- (b) $\frac{DB}{BC} = \frac{1}{2}$ (corr. sides, $\sim\Delta$ s), $\frac{8}{BC} = \frac{1}{2}$, $\therefore BC = 8 \times 2 = 16$
21. (a) (i) $\angle PMN = \angle PQR$ (corr. $\angle$ s, $MN \parallel QR$), $\angle PNM = \angle PRQ$ (corr. $\angle$ s, $MN \parallel QR$),
 $\angle MPN = \angle QPR$ (common $\angle$), $\therefore \triangle PMN \sim \triangle PQR$ (AAA)
- (ii) $\angle NMT = \angle QRT$ (alt. $\angle$ s, $MN \parallel QR$), $\angle MNT = \angle RQT$ (alt. $\angle$ s, $MN \parallel QR$),
 $\angle MTN = \angle RTQ$ (vert. opp. $\angle$ s), $\therefore \triangle MNT \sim \triangle RQT$ (AAA)
- (b) (i) $\because \triangle MNT \sim \triangle RQT$ (proved), $\therefore \frac{QR}{NM} = \frac{QT}{NT}$ (corr. sides, $\sim\Delta$ s)
- $$\frac{QR}{7} = \frac{6}{4}, \therefore QR = \frac{6}{4} \times 7 = 10.5$$
- (ii) $\because \triangle PMN \sim \triangle PQR$ (proved), $\therefore \frac{PM}{PQ} = \frac{MN}{QR}$ (corr. sides, $\sim\Delta$ s),
- $$\frac{5}{5+MQ} = \frac{7}{10.5}, 52.5 = 35 + 7MQ, 7MQ = 17.5, \therefore MQ = 2.5$$
22. (a) In $\triangle ADE$ and $\triangle ACB$, $\angle A = \angle A$ (common angle), $\angle DE = \angle ACB = 90^\circ$ (given),
 $\angle AED = \angle ABC$ ($\angle$ sum of Δ), $\therefore \triangle ADE \sim \triangle ACB$ (AAA)
- (b) $\frac{AE}{AB} = \frac{DE}{BC}$ (corr. sides, $\sim\Delta$ s), but $DE = EC$ (given) and $EC = AC - AE = 24 - AE$
 $\therefore \frac{AE}{AB} = \frac{24 - AE}{BC}$, $\frac{AE}{25} = \frac{24 - AE}{7}$, $7AE = 25(24 - AE)$,
 $7AE = 25 \times 24 - 25AE$, $32AE = 25 \times 24$, $AE = \frac{25 \times 24}{32} = 18.75$
23. (a) Yes. In $\triangle BCA$ and $\triangle DCB$, $\angle A = \angle DBC$ (given), $\angle C = \angle C$ (common),
 $\angle ABC = \angle BDC$ ($\angle$ sum of Δ), $\therefore \triangle BCA \sim \triangle DCB$ (AAA)
- (b) $\frac{BC}{DC} = \frac{AC}{BC}$ (corr. sides, $\sim\Delta$ s), $\frac{15}{12} = \frac{12 + AD}{15}$, $225 = 144 + 12AD$, $\therefore AD = 6.75$
24. In $\triangle ABC$ and $\triangle CDB$, $\frac{AB}{CD} = \frac{14}{21} = \frac{2}{3}$, $\frac{BC}{DB} = \frac{12}{18} = \frac{2}{3}$, $\frac{AC}{CB} = \frac{8}{12} = \frac{2}{3}$,
 $\therefore \triangle ABC \sim \triangle CDB$ (3 sides prop.), $\therefore \angle ACB = \angle CBD$ (corr. $\angle$ s, $\sim\Delta$ s),
 $\therefore AC \parallel BD$ (alt. $\angle$ s, eq.)
25. (a) In $\triangle ADB$ and $\triangle ABC$, $\frac{AD}{AB} = \frac{12}{18} = \frac{2}{3}$, $\frac{AB}{AC} = \frac{18}{15+12} = \frac{2}{3}$,
 $\angle DAB = \angle BAC$ (common), $\therefore \triangle ABC \sim \triangle ADB$ (ratio of 2 sides, inc. $\angle$),
- (b) $\therefore \frac{BD}{CB} = \frac{2}{3}$ (corr. sides, $\sim\Delta$ s), $\frac{10}{x} = \frac{2}{3}$, $\therefore x = 10 \times \frac{3}{2} = 15$
26. (a) In $\triangle PQR$ and $\triangle PRS$, $\frac{PR}{PS} = \frac{PQ}{PR}$ (given), $\angle QPR = \angle RPS$ (common),
 $\therefore \triangle PQR \sim \triangle PRS$ (ratio of 2 sides, inc. $\angle$)
- (b) $\angle PSR = 180^\circ - 116^\circ = 64^\circ$ (adj. $\angle$ s on st. line),
 $\angle PRQ = \angle PSR = 64^\circ$ (corr. $\angle$ s, $\sim\Delta$ s), $\therefore \angle PRS = 64^\circ - \theta$
In $\triangle PSR$, $84^\circ + 64^\circ + (64^\circ - \theta) = 180^\circ$ ($\angle$ sum of Δ), $\therefore \theta = 32^\circ$
27. (a) $\triangle FDA \sim \triangle FEB \sim \triangle CEA \sim \triangle CDB$
- (b) In $\triangle CDB$ and $\triangle CEA$, $\angle C = \angle C$ (common angle), $\angle BDC = \angle AEC = 90^\circ$ (given),

$$\angle DBC = \angle EAC \text{ (}\angle \text{ sum of } \Delta), \therefore \Delta CDB \sim \Delta CEA \text{ (AAA),}$$

$$\therefore \frac{DC}{EC} = \frac{BC}{AC} \text{ (corr. sides, } \sim \Delta\text{s)}, \quad \frac{DC}{18} = \frac{6+18}{36} = \frac{2}{3},$$

$$\therefore DC = \frac{2}{3} \times 18 = 12 \text{ cm}, \quad \therefore AD = 36 - 12 = 24 \text{ cm.}$$

28. (a) $\Delta LPN \sim \Delta DEN$ (AAA), $\Delta LCD \sim \Delta LMN$ (AAA).

(b) $\frac{DN}{LN} = \frac{DE}{LP}$ (corr. sides, $\sim \Delta\text{s}$), $\frac{m}{m+6} = \frac{5}{8}$, $8m = 5m + 30$, $\therefore m = 10$.

$$\frac{LD}{LN} = \frac{DC}{NM} \text{ (corr. sides, } \sim \Delta\text{s)}, \quad \frac{6}{6+m} = \frac{n}{12}, \quad \therefore n = \frac{6}{6+10} \times 12 = 4.5$$

29. (a) $\therefore \Delta ABC \sim \Delta AEF$ (AAA), $\therefore \frac{8}{8+12} = \frac{x}{x+4+5}$ (corr. sides, $\sim \Delta\text{s}$)

$$\frac{2}{5} = \frac{x}{x+9}, \quad 2(x+9) = 5x, \quad x = 6$$

$$\therefore \Delta ABD \sim \Delta AEG \text{ (AAA)}, \quad \therefore \frac{8}{8+12} = \frac{x+4}{x+4+5+y} \text{ (corr. sides, } \sim \Delta\text{s)}$$

$$\frac{2}{5} = \frac{10}{15+y}, \quad 15+y = 25, \quad y = 10$$

(b) $\therefore \Delta PQR \sim \Delta PST$ (AAA), $\therefore \frac{a}{a+b} = \frac{10}{10+5} = \frac{2}{3}$ (corr. sides, $\sim \Delta\text{s}$),

$$3a = 2a + 2b, \quad a = 2b \dots (1)$$

$$\therefore \Delta PQT \sim \Delta PSU \text{ (AAA)}, \quad \therefore \frac{a+b}{a+b+6} = \frac{10}{10+5} = \frac{2}{3} \text{ (corr. sides, } \sim \Delta\text{s)}$$

$$3a + 3b = 2a + 2b + 12, \quad a + b = 12 \dots (2)$$

$$\text{Solving (1) and (2), } a = 8, \quad b = 4$$

30. (a) $\Delta HEF \sim \Delta HCD$ (AAA), $\Delta DEF \sim \Delta DGH$ (AAA), $\Delta CDE \sim \Delta HGE$ (AAA)

(b) $\Delta CDE \sim \Delta HGE$, $\therefore \frac{x}{5} = \frac{8}{4}$, $x = 10$

$$\Delta HEF \sim \Delta HCD, \quad \therefore \frac{HF}{HD} = \frac{EF}{CD} = \frac{HE}{HC} \text{ (corr. sides, } \sim \Delta\text{s)}, \quad \frac{y}{y+3} = \frac{z}{5} = \frac{8}{8+4} = \frac{2}{3}$$

$$\therefore z = \frac{2}{3} \times 5 = \frac{10}{3}; \quad 3y = 2y + 6, \quad \therefore y = 6$$

31. Join BE, and mark the intersection of BE and CD as G. $\Delta ECG \sim \Delta EAB$ (AAA),

$$\therefore \frac{CG}{AB} = \frac{EC}{EA} = \frac{EG}{EB} \text{ (corr. sides, } \sim \Delta\text{s)}, \quad \therefore \frac{CG}{8} = \frac{3}{3+6} = \frac{1}{3}, \quad CG = \frac{1}{3} \times 8 = \frac{8}{3}$$

$$\therefore EG : EB = 1 : 3, \quad \therefore EG : GB : EB = 1 : (3-1) : 3 = 1 : 2 : 3, \quad \therefore \frac{BG}{BE} = \frac{2}{3}$$

$$\Delta BDG \sim \Delta BFE \text{ (AAA)}, \quad \therefore \frac{DG}{FE} = \frac{BG}{BE} \text{ (corr. sides, } \sim \Delta\text{s)}$$

$$\therefore \frac{DG}{6} = \frac{2}{3}, \quad DG = \frac{2}{3} \times 6 = 4, \quad \therefore CD = \frac{8}{3} + 4 = 6\frac{2}{3}$$

32. (a) $\therefore \Delta ADE \sim \Delta AFG$ (AAA), $\therefore \frac{DE}{FG} = \frac{5+5}{5+5+5} = \frac{2}{3}$ (corr. sides, $\sim \Delta\text{s}$),

$$\therefore \Delta BDH \sim \Delta BFG \text{ (AAA)}, \quad \frac{DH}{FG} = \frac{5}{5+5} = \frac{1}{2} \text{ (corr. sides, } \sim \Delta\text{s)},$$

$$\therefore DE : FG = 2 : 3 = 4 : 6, \quad DH : FG = 1 : 2 = 3 : 6, \quad \therefore DH : DE = 3 : 4$$

$$(b) \frac{DH}{DE} = \frac{3}{4} \text{ (proved), } \frac{DH}{24} = \frac{3}{4}, \quad DH = \frac{3}{4} \times 24 = 18, \quad \therefore HE = 24 - 18 = 6 \text{ cm}$$

$$33. \therefore \triangle BFG \sim \triangle BAC \text{ (AAA), } \therefore \frac{BF}{BA} = \frac{BG}{BC} \text{ (corr. sides, } \sim \Delta s)$$

$$\frac{42}{BA} = \frac{4+3}{4+3+1} = \frac{7}{8}, \quad 336 = 7 BA, \quad BA = 48 \text{ (corr. sides, } \sim \Delta s)$$

$$\therefore \triangle CDE \sim \triangle CBA \text{ (AAA), } \frac{DE}{BA} = \frac{CD}{CB} \text{ (corr. sides, } \sim \Delta s)$$

$$\therefore \frac{DE}{48} = \frac{3+1}{4+3+1} = \frac{1}{2}, \quad \therefore DE = \frac{1}{2} \times 48 = 24$$

$$34. \triangle ABE \sim \triangle ACF \sim \triangle ADG \text{ (AAA), } \therefore \frac{AE}{AG} = \frac{BE}{DG} \text{ and } \frac{BE}{CF} = \frac{AE}{AF} \text{ (corr. sides, } \sim \Delta s)$$

$$\frac{6}{6+EF+3} = \frac{3}{5}, \quad 30 = 27 + 3 EF, \quad 3 = 3 EF, \quad EF = 1$$

$$\text{Also } \frac{3}{CF} = \frac{6}{6+EF}, \quad \frac{3}{CF} = \frac{6}{6+1} = \frac{6}{7}, \quad 21 = 6CF, \quad \therefore CF = \frac{21}{6} = 3.5$$

$$35. (a) \text{ In } \triangle CDF \text{ and } \triangle GEF, \angle CFD = \angle GFE \text{ (vert. opp. } \angle s), \\ \angle CDF = \angle GEF \text{ and } \angle DCF = \angle EGF \text{ (alt. } \angle s, AD \parallel EG), \\ \therefore \triangle CDF \sim \triangle GEF \text{ (AAA)}$$

$$(b) \therefore \triangle CDF \sim \triangle GEF \text{ (proved), } \therefore \frac{EG}{CD} = \frac{EF}{FD} \text{ (corr. sides, } \sim \Delta s), \therefore \frac{EG}{CD} = \frac{3}{4}$$

$$EG : CD = 3 : 4 \text{ and } AC : CD = 3 : 2 = 6 : 4, \therefore EG : AC = 3 : 6 = 1 : 2$$

$$\therefore \triangle BEG \sim \triangle BAC \text{ (AAA), } \therefore \frac{BE}{AB} = \frac{EG}{AC} \text{ (corr. sides, } \sim \Delta s)$$

$$\therefore BE : AB = 1 : 2, \quad BE : AB : AE = 1 : 2 : (2 - 1) = 1 : 2 : 1$$

$$\therefore AE : EB = 1 : 1$$

$$36. \text{ In } \triangle BCE \text{ and } \triangle CHF, \angle BEC = \angle CFH = 90^\circ \text{ (square),} \\ \angle BCE = \angle CHF \text{ (corr. sides, } CD \parallel HG), \angle CBE = \angle HCF \text{ (} \angle \text{ sum of } \Delta),$$

$$\therefore \triangle BCE \sim \triangle CHF \text{ (AAA), } \therefore \frac{HF}{CE} = \frac{CF}{BE} \text{ (corr. sides, } \sim \Delta s)$$

$$CF = DG = 3 \text{ cm and } BE = AD = 1 \text{ cm (squares)}$$

$$\therefore \frac{HF}{3-1} = \frac{3}{1}, \quad HF = 6. \quad GJ = HG = 6 + 3 = 9 \text{ cm}$$

$$37. \therefore \triangle ECD \sim \triangle EAB \text{ (AAA), } \therefore \frac{r}{p} = \frac{CE}{AE} \text{ (corr. sides, } \sim \Delta s),$$

$$\therefore \triangle ACD \sim \triangle AEF \text{ (AAA), } \therefore \frac{r}{q} = \frac{AC}{AE} \text{ (corr. sides, } \sim \Delta s),$$

$$\frac{r}{q} = \frac{AE-CE}{AE} = 1 - \frac{CE}{AE} = 1 - \frac{r}{p}, \quad \frac{r}{p} + \frac{r}{q} = 1, \quad \therefore \frac{1}{p} + \frac{1}{q} = \frac{1}{r}$$

$$38. (a) \angle AEC = \angle AFD \text{ (corr. } \angle s, CE \parallel DF), \\ \angle CAE = \angle DAF \text{ (common } \angle), \angle ACE = \angle ADF \text{ (} \angle \text{ sum of } \Delta),$$

$$\therefore \triangle ACE \sim \triangle ADF \text{ (AAA), } \therefore \frac{CE}{DF} = \frac{AE}{AF} \text{ (corr. sides, } \sim \Delta s)$$

$$\frac{AE}{AF} = \frac{AE}{AE+EF} = \frac{1}{1+2} = \frac{1}{2}, \quad \therefore CE : DF = 1 : 2$$

$$(b) \angle BEC = \angle BFG \text{ and } \angle BCE = \angle BGF \text{ (corr. } \angle s, CE \parallel DF),$$

- $\angle CBE = \angle GBF$ (common $\angle$), $\therefore \triangle BCE \sim \triangle BGF$ (AAA),
 $\therefore \frac{CE}{GF} = \frac{BE}{BF}$ (corr. sides, $\sim\Delta$ s), $\frac{BE}{BF} = \frac{BF+FE}{BF} = \frac{3+1}{3} = \frac{4}{3}$, $\therefore CE : GF = 4 : 3$
- (c) $CE : GF = 4 : 3$ and $CE : DF = 1 : 2 = 4 : 8$,
 $\therefore CE : GF : DF = 4 : 3 : 8$, $\therefore CE : DG = CE : (DF - GF) = 4 : (8 - 3) = 4 : 5$
39. (a) $BQ = DQ$ and $AQ = EQ$ (mid-points),
 $\angle AQB = \angle EQD$ (vert. opp. $\angle$ s), $\therefore \triangle ABQ \sim \triangle EDQ$ (SAS),
 $\therefore \angle BAQ = \angle DEQ$ (corr. $\angle$ s, $\cong\Delta$ s), $\therefore AB \parallel DE$ (alt. $\angle$ s equal)
- (b) $EP = PB$ (mid-point), $\therefore EP : EB = 1 : (1 + 1) = 1 : 2$
 $EQ = QA$ (mid-point), $\therefore EQ : EA = 1 : (1 + 1) = 1 : 2$
 $\angle PEQ = \angle BEA$ (common $\angle$), $\therefore \triangle QPE \sim \triangle ABE$ (ratios of two sides, inc. $\angle$)
- (c) $\therefore \angle EPQ = \angle EBA$ (corr. $\angle$ s, $\sim\Delta$ s), $\therefore QP \parallel AB$ (corr. $\angle$ s equal)
 But $AB \parallel DE$ (from (a)), $\therefore AB$ and DE are parallel to QP .
40. (a) (i) $\frac{PU}{RU} = \frac{8}{5}$, $\frac{QU}{SU} = \frac{6.4}{4} = \frac{64}{40} = \frac{8}{5}$,
 $\angle PUQ = \angle SUR$ (vert. opp. $\angle$ s), $\therefore \triangle PQU \sim \triangle RSU$ (ratios of two sides, inc. $\angle$)
 (ii) $RU \neq SU$, $\therefore \angle URS \neq \angle USR$, $\angle URS = \angle UPQ$ (corr. $\angle$ s, $\sim\Delta$ s),
 $\therefore \angle UPQ \neq \angle USR$, $\therefore PQ$ and RS are not parallel (alt. $\angle$ s not eq.)
Ans. The claim is disagreed.
- (b) $\frac{RS}{PQ} = \frac{RU}{PU}$ (corr. sides, $\sim\Delta$ s), $\therefore \frac{RS}{12} = \frac{5}{8}$, $RS = 7.5\text{cm}$
- (c) $\therefore \triangle RSU \sim \triangle PQU$ (proved), $\therefore \angle SRU = \angle QPU = 40^\circ$ (corr. $\angle$ s, $\sim\Delta$ s)
 $\angle PRQ = 180^\circ - \angle SRU - \angle SRT$ (adj. $\angle$ s on st. line)
 $= 180^\circ - 40^\circ - 60^\circ = 80^\circ$
41. (a) $\angle ADG = \angle CEG = 90^\circ$ (given), $\angle AGD = \angle CGE$ (vert. opp. $\angle$ s),
 $\angle DAG = \angle ECG$ ($\angle$ sum of Δ), $\therefore \triangle ADG \sim \triangle CEG$ (AAA)
- (b) $AD : DF = 1 : 2$, $\therefore AD : AF = 1 : (1 + 2) = 1 : 3$
 $AG : GE = 1 : 2$, $\therefore AG : AE = 1 : (1 + 2) = 1 : 3$
 $\angle DAG = \angle FAE$ (common $\angle$), $\therefore \triangle ADG \sim \triangle AFE$ (ratio of 2 sides, inc. $\angle$)
 $\therefore \angle ADG = \angle AFE$ (corr. $\angle$ s, $\sim\Delta$ s), $\therefore CD \parallel EF$ (corr. $\angle$ s eq.)
- (c) (i) $\triangle ADG \sim \triangle AFE$ (proved), $\therefore \frac{FE}{DG} = \frac{AF}{AD}$ (corr. sides, $\sim\Delta$ s)
 $AD : AF = 1 : 3$, $\therefore AF = 3AD = 3(8) = 24\text{cm}$, and $FE = 3(DG) = 3(6) = 18\text{cm}$
 Area of $\triangle AFE = (AF)(FE) \div 2 = (24)(18) \div 2 = 216\text{ cm}^2$
 (ii) $GE : GA = 2 : 1$, $\therefore GE = 2(10) = 20\text{cm}$, $\triangle ADG \sim \triangle CEG$ (proved),
 $\therefore \frac{CE}{AD} = \frac{GE}{GD}$ (corr. sides, $\sim\Delta$ s), $\frac{CE}{8} = \frac{20}{6}$, $CE = \frac{80}{3}$
 Area of $\triangle CEG = (GE)(CE) \div 2 = (20)(\frac{80}{3}) \div 2 = \frac{800}{3}\text{ cm}^2 = 266\frac{2}{3}\text{ cm}^2$
 (iii) $\triangle ADG \sim \triangle AFE$ (proved) and $FE : DG = AF : AD = 3 : 1$ (corr. sides, $\sim\Delta$ s)
 $\therefore FE = 3(DG) = 3(6) = 18\text{cm}$
 In $\triangle ABE$, $\angle FBE = 180^\circ - \angle AEB - \angle DAG$ ($\angle$ sum of Δ)
 $= 180 - 90^\circ - \angle DAG = 90^\circ - \angle DAG$
 In $\triangle ADG$, $\angle DGA = 180^\circ - \angle ADG - \angle DAG$ ($\angle$ sum of Δ)
 $= 180 - 90^\circ - \angle DAG = 90^\circ - \angle DAG$
 $\therefore \angle FBE = \angle DGA$; $\angle BFE = \angle GDA = 90^\circ$,

$$\therefore \angle FEB = \angle DGA \text{ (}\angle\text{sum of } \Delta\text{)}, \quad \therefore \triangle EFB \sim \triangle ADG$$

$$\therefore \frac{FB}{DG} = \frac{FE}{AD} \text{ (corr. sides, } \sim\Delta\text{s)}, \quad \frac{FB}{6} = \frac{18}{8}, \quad FB = 13.5\text{cm}$$

$$\text{Area of } \triangle EFB = (FB)(FE) \div 2 = (13.5)(18) \div 2 = 121.5\text{cm}^2$$

42. (a) $\angle NTQ = \angle PTS$ (vert. opp. $\angle$ s)

$$\therefore QN \parallel PS \text{ (given)}, \quad \therefore \angle QNT = \angle SPT \text{ (alt. } \angle\text{s, } QN \parallel PS)$$

$$\angle NQT = \angle PST \text{ (}\angle\text{sum of } \Delta\text{)}, \quad \therefore \triangle NQT \sim \triangle PST \text{ (AAA)}$$

(b) $\angle MTP = \angle QTR$ (vert. opp. $\angle$ s)

$$\therefore PM \parallel QR \text{ (given)}, \quad \therefore \angle PMT = \angle RQT \text{ (alt. } \angle\text{s, } PM \parallel QR)$$

$$\angle MPT = \angle QRT \text{ (}\angle\text{sum of } \Delta\text{)}, \quad \therefore \triangle RTQ \sim \triangle PTM \text{ (AAA)}$$

(c) $\therefore \triangle NQT \sim \triangle PST$ (proved), $\therefore \frac{TN}{TP} = \frac{TQ}{TS}$ (corr. sides, $\sim\Delta$ s), $TN \times TS = TP \times TQ$.

$$\therefore \triangle RTQ \sim \triangle PTM \text{ (proved)}, \quad \therefore \frac{TR}{TP} = \frac{TQ}{TM} \text{ (corr. sides, } \sim\Delta\text{s)}, \quad TM \times TR = TP \times TQ.$$

$$\therefore TN \times TS = TM \times TR$$

(d) $TN \times TS = TM \times TR$ (proved), $\therefore \frac{TM}{TS} = \frac{TN}{TR}$, $\angle MTN = \angle STR$ (common $\angle$),

$$\therefore \triangle MTN \sim \triangle STR \text{ (ratio of 2 sides, inc. } \angle\text{)}, \quad \therefore \angle TMN = \angle TSR \text{ (corr. } \angle\text{s, } \sim\Delta\text{s)},$$

$$\therefore MN \parallel SR \text{ (corr. } \angle\text{s equal)}$$

43. (a) (i) Let $\angle A = k$. In $\triangle AGF$, $\angle AFG = 180^\circ - 90^\circ - k = 90^\circ - k$ ($\angle$ sum of Δ)

$$\angle AFD = 90^\circ \text{ (given)}, \quad \therefore \angle GFD = 90^\circ - \angle AFG = 90^\circ - (90^\circ - k) = k.$$

$$\text{In } \triangle FGD, \angle FDG = 180^\circ - 90^\circ - \angle GFD = 90^\circ - k \text{ (}\angle\text{sum of } \Delta\text{)} = \angle AFG$$

$$\text{The three angles of } \triangle AGF \text{ and } \triangle FGD \text{ are } k, 90^\circ, 90^\circ - k,$$

$$\therefore \triangle AGF \sim \triangle FGD \text{ (AAA)}$$

(ii) $AG : GC : CD = 1 : 1 : 2$, $AG = a$, $\therefore GC = a$, $CD = 2a$

$$DG = GC + CD = a + 2a = 3a, \quad \frac{AG}{FG} = \frac{FG}{DG} \text{ (corr. sides, } \sim\Delta\text{s)}$$

$$FG^2 = (AG)(DG) = (a)(3a) = 3a^2$$

(b) (i) In $\triangle ABC$, $\angle B = 180^\circ - 90^\circ - \angle A = 90^\circ - k$

$$\text{In } \triangle EFB, \angle BEF = 180^\circ - 90^\circ - \angle B = 90^\circ - (90^\circ - k) = k$$

$$\text{The three angles of } \triangle FGD \text{ and } \triangle EFB \text{ are } k, 90^\circ, 90^\circ - k,$$

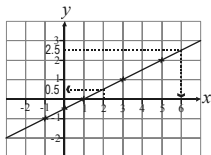
$$\therefore \triangle FGD \sim \triangle EFB \text{ (AAA)}$$

(ii) $\frac{EF}{FG} = \frac{BF}{DG}$ (corr. sides, $\sim\Delta$ s), $\therefore \frac{EF}{BF} = \frac{FG}{DG}$, $\therefore \frac{EF^2}{BF^2} = \frac{FG^2}{DG^2}$.

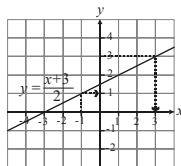
$$FG^2 = 3a^2 \text{ (from a(ii))}, \quad DG = 3a, \quad DG^2 = (3a)^2 = 9a^2, \quad \therefore \frac{EF^2}{BF^2} = \frac{3a^2}{9a^2} = \frac{1}{3}$$

Unit 11 Graphs of linear equations in two unknowns

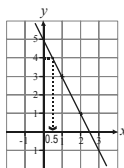
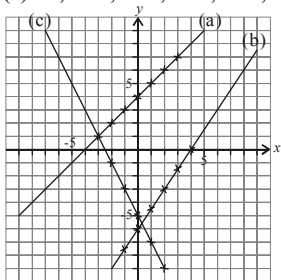
1. (a) $-1, -0.5, 0, 1, 2$
 (b) $m = 0.5, n = 6$



2. (a) 1
 (b) 3



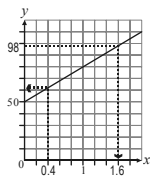
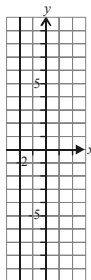
3. (a) 3, 1, -1
 (b) x-axis: (2.5, 0), y-axis: (0, 5)
 (c) From the graph, $k = 0.5$
 (d) $2(k) + 4 = 5$, $2k = 1$, $\therefore k = 0.5$
4. (a) 2, 3, 4, 5, 6, 7
 (b) -7.5, -6, -4.5, -3, -1.5, 0
 (c) 1, -1, -3, -5, -7, -9



Q3

5. (a) $y = hx + 50$
 (b) \$62
 (c) 1.6 kg
 (d) When $x = 1$,
 $y = 80$.
 $80 = h(1) + 50$,
 $\therefore h = 30$

6. $x + 2 = 0$, $x = -2$



7. (a) $2(-7) + k(-6) - 4 = 0$, $-6k = 18$, $\therefore k = -3$
 (b) $2h - 3(\frac{5}{2}) - 4 = 0$, $4h - 15 - 8 = 0$, $4h = 23$, $\therefore h = 5\frac{3}{4}$
 (c) Let M be $(x, 0)$, $2x - 3(0) - 4 = 0$, $2x = 4$, $\therefore x = 2$.
 Let N be $(0, y)$, $2(0) - 3y - 4 = 0$, $-3y = 4$, $\therefore y = -1\frac{1}{3}$.

Ans. The coordinates of M and N are (2, 0) and $(0, -1\frac{1}{3})$ respectively.

8. On the x-axis, $y = 0$, $\frac{3}{2}x - \frac{1}{3}(0) = \frac{3}{8}$, $\therefore x = \frac{1}{4}$. Ans. The point is $(\frac{1}{4}, 0)$.
 On the y-axis, $x = 0$, $\frac{3}{2}(0) - \frac{1}{3}y = \frac{3}{8}$, $\therefore y = -1\frac{1}{8}$. Ans. The point is $(0, -1\frac{1}{8})$.

9. (a) $a(2) - 3(4) + 2 = 0$, $2a = 10$, $\therefore a = 5$

(b) When $x = 0$, $5(0) - 3y + 2 = 0$, $y = \frac{2}{3}$

When $y = 0$, $5x - 3(0) + 2 = 0$, $x = -\frac{2}{5}$

$\therefore$ The shaded area = $\frac{1}{2} \times \frac{2}{3} \times \frac{2}{5} = \frac{2}{15}$ sq. units

10. (a) $4 = 3(0) + k$, $\therefore k = 4$

(b) $0 = 3x + 4$, $x = -\frac{4}{3}$, $\therefore$ Coordinates of Q = $(-\frac{4}{3}, 0)$

$\therefore$ OQ = OR, $\therefore$ Coordinates of R = $(0, \frac{4}{3})$ or $(0, -\frac{4}{3})$

When R = $(0, \frac{4}{3})$, $PR = 4 - \frac{4}{3} = \frac{8}{3}$,

$\therefore$ Area of $\Delta PQR = \frac{1}{2} \times \frac{8}{3} \times \frac{4}{3} = \frac{16}{9} = 1\frac{7}{9}$ sq. units

When $R = (0, -\frac{4}{3})$, $PR = 4 + \frac{4}{3} = \frac{16}{3}$,

$\therefore$ Area of $\triangle PQR = \frac{1}{2} \times \frac{16}{3} \times \frac{4}{3} = \frac{32}{9} = 3\frac{5}{9}$ sq. units

11. $3(\frac{1}{2}) + 4(p) - 5 = 0$, $3 + 8p - 10 = 0$, $8p = 7$, $\therefore p = \frac{7}{8}$

$3(\frac{1}{5}) + 4(q) - 5 = 0$, $3 + 20q - 25 = 0$, $20q = 22$, $\therefore q = \frac{11}{10}$

$\therefore 2p + q = 2(\frac{7}{8}) + \frac{11}{10} = \frac{35}{20} + \frac{22}{20} = 2\frac{17}{20}$

12. (a) $3(2r) + pr = 0$, $pr = -6r$, $\therefore p = -6$ ($\because r \neq 0$)

(b) Put $(-2s, s)$ into $3x - 6y = 0$,

L.H.S. = $3(-2s) - 6s = -12s$, R.H.S. = 0,

If $s = 0$, L.H.S. = R.H.S., $\therefore$ D lies on the graph when $s = 0$.

If $s \neq 0$, L.H.S. $\neq$ R.H.S., $\therefore$ D does not lie on the graph when $s \neq 0$.

13. $\because PR \parallel y$ -axis, $\therefore$ Coordinates of R are $(-10, 0)$.

Let Q be $(0, q)$, $3(q) + 5(0) = 4$, $3q = 4$, $\therefore q = \frac{4}{3}$,

$\therefore$ Area of PROQ = $\frac{(PR + OQ) \times OR}{2} = \frac{1}{2} \times (18 + \frac{4}{3}) \times 10 = 96\frac{2}{3}$ sq. units

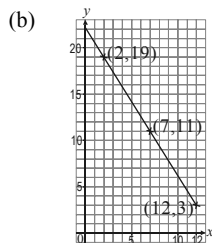
14. (a) $2.4x + 1.5y = 33.3$, $24x + 15y = 333$,

$\therefore 8x + 5y = 111$

(c) $\because x$ and y must be integers,

$\therefore$ From the graph, the possible answers of (x, y) are $(2, 19)$
or $(7, 11)$ or $(12, 3)$.

Ans. 2 \$2.4 stamps and 19 \$1.5 stamps; 7 \$2.4 stamps and 11 \$1.5 stamps; or 12 \$2.4 stamps and 3 \$1.5 stamps.



Unit 12 Simultaneous linear equations in two unknowns

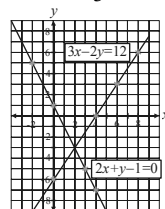
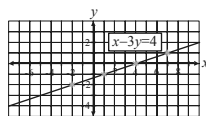
1	x	-2	1	4	7
	y	-2	-1	0	1

2. (a) When $x = 5$, $y = -7$, $3(5) + k(-7) - 1 = 0$,
 $14 = 7k$, $\therefore k = 2$

(b) When $x = h$, $y = \frac{5}{2}$, $3(h) + 2(\frac{5}{2}) - 1 = 0$, $3h + 4 = 0$, $\therefore h = -\frac{4}{3}$

3	(a)	x	-2	0	3	4
		y	5	1	-5	-7
	(b)	x	0	4	6	8
		y	-6	0	3	6

(c) From the graph, $x = 2$, $y = -3$



4. (a) $\begin{cases} y = 8x - 19 \dots (1) \\ y = 6x - 15 \dots (2) \end{cases}$ Put (1) into (2), $8x - 19 = 6x - 15$, $x = 2 \dots (3)$
Put (3) into (1), $y = 8 \times 2 - 19 = -3$. *Ans.* $x = 2, y = -3$.
- (b) $\begin{cases} x = 2y + 13 \dots (1) \\ 3x + y = 4 \dots (2) \end{cases}$ Put (1) into (2), $3(2y + 13) + y = 4$, $6y + 39 + y = 4$,
 $y = -5 \dots (3)$. Put (3) into (1), $x = 2(-5) + 13 = 3$. *Ans.* $x = 3, y = -5$.
- (c) $\begin{cases} 3y - 7x = 1 \dots (1) \\ y + 2x = 9 \dots (2) \end{cases}$ From (1), $3y = 1 + 7x$, $y = \frac{1+7x}{3} \dots (3)$
Put (3) into (2), $\frac{1+7x}{3} + 2x = 9$, $1 + 7x + 6x = 27$, $x = 2 \dots (4)$
Put (4) into (3), $y = \frac{1+7 \times 2}{3} = \frac{15}{3} = 5$. *Ans.* $x = 2, y = 5$.
- (d) $\begin{cases} 2y = 5x + 5 \dots (1) \\ 2y = 6x + 8 \dots (2) \end{cases}$ Put (1) into (2), $5x + 5 = 6x + 8$, $x = -3 \dots (3)$
Put (3) into (1), $2y = 5(-3) + 5$, $y = \frac{-10}{2} = -5$. *Ans.* $x = -3, y = -5$.
- (e) $\begin{cases} 8x + 8 = 3y \dots (1) \\ 2y = 11 - 6x \dots (2) \end{cases}$ From (1), $y = \frac{8x+8}{3} \dots (3)$. Put (3) into (2),
 $2\left(\frac{8x+8}{3}\right) = 11 - 6x$, $16x + 16 = 33 - 18x$, $34x = 17$, $x = \frac{1}{2} \dots (4)$
Put (4) into (3), $y = \frac{8 \times \frac{1}{2} + 8}{3} = \frac{4+8}{3} = \frac{12}{3} = 4$ *Ans.* $x = \frac{1}{2}, y = 4$.
- (f) $\begin{cases} 5x + 3y = 7 \dots (1) \\ 5y - 3x = 6 \dots (2) \end{cases}$ From (1), $y = \frac{7-5x}{3} \dots (3)$. Put (3) into (2),
 $5\left(\frac{7-5x}{3}\right) - 3x = 6$, $35 - 25x - 9x = 18$, $-34x = -17$, $x = \frac{1}{2} \dots (4)$
Put (4) into (3), $y = \frac{1}{3}\left(7 - 5 \times \frac{1}{2}\right) = \frac{1}{3} \times \frac{9}{2} = \frac{3}{2}$ *Ans.* $x = \frac{1}{2}, y = \frac{3}{2}$.
5. (a) $\begin{cases} x + y = 11 \dots (1) \\ x - y = 7 \dots (2) \end{cases}$ (1) + (2), $2x = 18$, $x = 9 \dots (3)$
Put (3) into (1), $9 + y = 11$, $y = 2$
Ans. $x = 9, y = 2$.
- (b) $\begin{cases} x + 2y = 3 \dots (1) \\ x - y = 9 \dots (2) \end{cases}$ (2) $\times 2$, $2x - 2y = 18 \dots (3)$
(1) + (3), $3x = 21$, $x = 7 \dots (4)$
Put (4) into (2), $7 - y = 9$, $7 - 9 = y$, $y = -2$. *Ans.* $x = 7, y = -2$
- (c) $\begin{cases} 4x - y = 18 \dots (1) \\ x + 2y = -9 \dots (2) \end{cases}$ (1) $\times 2$, $8x - 2y = 36 \dots (3)$
(2) + (3), $9x = 27$, $x = 3 \dots (4)$
Put (4) into (2), $3 + 2y = -9$, $2y = -12$, $y = -6$. *Ans.* $x = 3, y = -6$
- (d) $\begin{cases} 5x + 3y = 2 \dots (1) \\ 3x + 9y = -6 \dots (2) \end{cases}$ (1) $\times 3$, $15x + 9y = 6 \dots (3)$
(3) - (2), $12x = 12$, $x = 1 \dots (4)$
Put (4) into (1), $5 + 3y = 2$, $3y = -3$, $y = -1$. *Ans.* $x = 1, y = -1$
- (e) $\begin{cases} 3y + 2x = 2 \dots (1) \\ 2y - 3x = -16 \dots (2) \end{cases}$ (1) $\times 3$, $9y + 6x = 6 \dots (3)$
(2) $\times 2$, $4y - 6x = -32 \dots (4)$
(3) + (4), $13y = -26$, $y = -2 \dots (5)$
Put (5) into (1), $-6 + 2x = 2$, $2x = 8$, $x = 4$. *Ans.* $x = 4, y = -2$

$$(f) \begin{cases} 5x + 5y = 1 \dots (1) & (1) \times 2, \quad 10x + 10y = 2 \dots (3) \\ 3x - 2y = 5 \dots (2) & (2) \times 5, \quad 15x - 10y = 25 \dots (4) \\ & (3) + (4), \quad 25x = 27, x = \frac{27}{25} \dots (5) \end{cases}$$

Put (5) into (1), $\frac{5 \times 27}{25} + 5y = 1$, $5y = 1 - \frac{27}{5}$, $5y = \frac{-22}{5}$, $y = \frac{-22}{25}$

Ans. $x = \frac{27}{25}$, $y = \frac{-22}{25}$

6. (a) $3x + y - 120 = 3x - 2y$, $y - 120 = -2y$, $3y = 120$, $y = 40$, $3x - 2y = 10 + 2x$, $3x - 2 \times 40 = 10 + 2x$, $x = 10 + 80$, $x = 90$. Ans. (90, 40)

(b) $9x + 3y + 5 = x - 2y + 7$, $8x + 5y = 2 \dots (1)$

$x - 2y + 7 = 7x - 3y - 4$, $6x - y = 11 \dots (2)$

$(2) \times 5$, $30x - 5y = 55 \dots (3)$.

$(1) + (3)$, $38x = 57$, $x = \frac{3}{2} \dots (4)$

Put (4) into (2), $9 - y = 11$, $y = -2$. Ans. $(\frac{3}{2}, -2)$

(c) $\frac{5x}{3} - y = 6 \dots (1)$, $\frac{x}{3} + y = 6 \dots (2)$.

$(1) + (2)$, $\frac{5x}{3} + \frac{x}{3} = 12$, $5x + x = 36$,

$6x = 36$, $x = 6 \dots (3)$.

Put (3) into (2), $\frac{6}{3} + y = 6$, $2 + y = 6$, $y = 4$. Ans. (6, 4)

(d) $\frac{6x - 3y}{4} = 6 \dots (1)$, $\frac{12x + 5y}{30} = 6 \dots (2)$.

From (1), $6x - 3y = 24$, $12x - 6y = 48 \dots (3)$

From (2), $12x + 5y = 180 \dots (4)$

$(4) - (3)$, $\therefore 5y + 6y = 180 - 48$, $11y = 132$, $y = 12 \dots (5)$

Put (5) into (4), $12x + 5(12) = 180$, $12x = 120$, $x = 10$. Ans. (10, 12)

7. (a) $\begin{cases} 9x - 5y - 11 = 0 \dots (1) \\ 3x - 5y + 13 = 0 \dots (2) \end{cases}$

$(1) - (2)$, $6x - 24 = 0$, $6x = 24$, $x = 4 \dots (3)$

Put (3) into (1), $36 - 5y - 11 = 0$, $-5y = -25$, $y = 5$. Ans. (4, 5)

(b) $\begin{cases} 0.35x - 0.15y = 1 \dots (1) & (1) \times 2, \quad 0.7x - 0.3y = 2 \dots (3) \\ 0.2x + 0.1y = 1.5 \dots (2) & (2) \times 3, \quad 0.6x + 0.3y = 4.5 \dots (4) \end{cases}$

$(3) + (4)$, $1.3x = 6.5$, $x = 5 \dots (5)$

Put (5) into (2), $0.2(5) + 0.1y = 1.5$, $0.1y = 0.5$, $y = 5$. Ans. (5, 5)

(c) $\begin{cases} 10x + y = 3(x + y), \quad 10x + y = 3x + 3y, \quad 7x - 2y = 0 \dots (1) \\ 9(x - y) = x - y - 32, \quad 9x - 9y = x - y - 32, \quad x - y = -4 \dots (2) \end{cases}$

From (2), $x = y - 4 \dots (3)$.

Put (3) into (1), $7(y - 4) - 2y = 0$, $5y = 28$, $y = \frac{28}{5} \dots (4)$.

Put (4) into (3), $x = \frac{28}{5} - 4 = \frac{28 - 20}{5} = \frac{8}{5}$. Ans. $(\frac{8}{5}, \frac{28}{5})$

$$(d) \begin{cases} 3(x - y) = -(7 + 2y), & 3x - 3y = -7 - 2y, & 3x - y = -7, & y = 3x + 7 \dots (1) \\ 3(2x - y) = 2 + y, & 6x - 3y = 2 + y, & 6x - 4y = 2 \dots (2) \end{cases}$$

Put (1) into (2), $6x - 4(3x + 7) = 2$, $6x - 12x - 28 = 2$, $-6x = 30$, $x = -5 \dots (3)$.

Put (3) into (1), $y = 3(-5) + 7 = -8$. *Ans.* $(-5, -8)$

$$(e) \begin{cases} 3x + 2y = 4(4 + x) + 1, & 3x + 2y = 16 + 4x + 1, & x = 2y - 17 \dots (1) \\ 5x + 3y = 7(4 + x) - 2, & 5x + 3y = 28 + 7x - 2, & 2x - 3y = -26 \dots (2) \end{cases}$$

Put (1) into (2), $2(2y - 17) - 3y = -26$, $4y - 34 - 3y = -26$, $y = 8 \dots (3)$

Put (3) into (1), $x = 2(8) - 17 = -1$. *Ans.* $(-1, 8)$

$$(f) \begin{cases} -2(2 + 4y) = 5x - 5y, & -4 - 8y = 5x - 5y, & 5x + 3y = -4 \dots (1) \\ 3(1 - x) + 2(y + 1) = y - x, & 3 - 3x + 2y + 2 = y - x, & y = 2x - 5 \dots (2) \end{cases}$$

Put (2) into (1), $5x + 3(2x - 5) = -4$, $5x + 6x - 15 = -4$, $11x = 11$, $x = 1 \dots (3)$.

Put (3) into (2), $y = 2(1) - 5 = -3$. *Ans.* $(1, -3)$

$$(g) \begin{cases} 4(x - y) + 2(x + y) = -(19 + y), & 6x - 2y = -19 - y, & 6x + 19 = y \dots (1) \\ 5(x + y) - (2x + y) = -5, & 5x + 5y - 2x - y = -5, & 3x + 4y = -5 \dots (2) \end{cases}$$

Put (1) into (2),

$3x + 4(6x + 19) = -5$, $3x + 24x + 76 = -5$, $27x = -81$, $x = -3 \dots (3)$.

Put (3) into (1), $y = 6(-3) + 19 = -1$. *Ans.* $(-3, -1)$

$$8. (a) \begin{cases} x = 2y + 8 \dots (1) & \text{Put (2) into (1), } x = 2\left(\frac{x}{3} + 4\right) + 8 = \frac{2x}{3} + 16, \\ \frac{x}{3} + 4 = y \dots (2) & 3x = 2x + 48, x = 48 \dots (3) \end{cases}$$

Put (3) into (1), $48 = 2y + 8$, $2y = 40$, $y = 20$. *Ans.* $(48, 20)$

$$(b) \begin{cases} y = \frac{3}{2}x - 4 \dots (1) & \text{Put (1) into (2), } x = -\frac{2}{3}\left(\frac{3}{2}x - 4\right) = -x + \frac{8}{3}, \\ x = -\frac{2}{3}y \dots (2) & 2x = \frac{8}{3}, x = \frac{4}{3} \dots (3) \end{cases}$$

Put (3) into (2), $-\frac{4}{3} = -\frac{2}{3}y$, $y = \frac{4}{3} \times \left(-\frac{3}{2}\right) = -2$. *Ans.* $\left(\frac{4}{3}, -2\right)$

$$(c) \begin{cases} \frac{2y + x}{3} = 3 \dots (1) & \text{From (1), } 2y + x = 9 \dots (3) \\ y - \frac{2}{3}x = 1 \dots (2) & \text{From (2), } 3y - 2x = 3 \dots (4) \end{cases}$$

$(3) \times 2 + (4)$, $2(2y + x) + (3y - 2x) = 9 \times 2 + 3$, $7y = 21$, $y = 3$

$\therefore 2(3) + x = 9$, $x = 3$. *Ans.* $(3, 3)$

$$(d) \begin{cases} \frac{x}{5} - \frac{3y}{2} = 3 \dots (1) & (1) \times 10, 2x - 15y = 30 \dots (3) \\ \frac{3x}{8} - \frac{5y}{2} = 5 \dots (2) & (2) \times 8, 3x - 20y = 40 \dots (4) \end{cases}$$

$(3) \times 3 - (4) \times 2$, $3(2x - 15y) - 2(3x - 20y) = 90 - 80$,

$-45y + 40y = 10$, $-5y = 10$, $y = -2$

$\therefore 2x - 15(-2) = 30$, $2x = 0$, $x = 0$. *Ans.* $(0, -2)$

$$(e) \begin{cases} \frac{5x}{3} - y = -17 \dots (1) & (1) \times 3, 5x - 3y = -51 \dots (3) \\ \frac{x}{3} - \frac{y}{2} = -4 \dots (2) & (2) \times 6, 2x - 3y = -24 \dots (4) \end{cases}$$

$(3) - (4)$, $3x = -27$, $x = -9 \dots (5)$

Put (5) into (3), $5(-9) - 3y = -51$, $3y = 6$, $y = 2$ *Ans.* $(-9, 2)$

$$(f) \begin{cases} \frac{x}{3} - y = -5 \dots (1) \\ \frac{3y}{2} + \frac{2x}{3} = \frac{26}{3} \dots (2) \end{cases} \quad \text{From (1), } y = \frac{x}{3} + 5 \dots (3)$$

Put (3) into (2),

$$\frac{3}{2} \left(\frac{x}{3} + 5 \right) + \frac{2x}{3} = \frac{26}{3}, \quad \frac{x}{2} + \frac{2x}{3} = \frac{26}{3} - \frac{15}{2}, \quad \frac{7x}{6} = \frac{52-45}{6}, \quad \frac{7x}{6} = \frac{7}{6}, \quad x = 1$$

$$\therefore y = \frac{1}{3} + 5 = \frac{16}{3} \quad \text{Ans. } \left(1, \frac{16}{3}\right)$$

$$9. (a) \text{ Let } m = \frac{1}{x}, n = \frac{1}{y} \quad \therefore \begin{cases} 2m + n = 2 \dots (1) \\ 3m - 2n = 10 \dots (2) \end{cases}$$

$$(1) \times 2 + (2), \quad \therefore 2(2m + n) + (3m - 2n) = 4 + 10, \quad 7m = 14, \quad m = 2 \dots (3)$$

Put (3) into (1), $n = 2 - 4 = -2 \dots (4)$

$$x = \frac{1}{m} = \frac{1}{2}, \quad y = \frac{1}{n} = -\frac{1}{2} \quad \text{Ans. } \left(\frac{1}{2}, -\frac{1}{2}\right)$$

$$(b) \begin{cases} \frac{2}{x} - \frac{1}{y} = \frac{3}{2} \dots (1) \\ \frac{1}{x} + \frac{3}{y} = 2 \dots (2) \end{cases} \quad \begin{aligned} (1) \times 3, \quad \frac{6}{x} - \frac{3}{y} &= \frac{9}{2} \dots (3) \\ (2) + (3), \quad \frac{6}{x} + \frac{1}{y} &= \frac{9}{2} + 2, \quad \frac{7}{x} = \frac{13}{2}, \quad x = \frac{14}{13} \dots (4) \end{aligned}$$

$$\text{Put (4) into (2), } \frac{1}{\frac{14}{13}} + \frac{3}{y} = 2, \quad \frac{13}{14} + \frac{3}{y} = 2, \quad \frac{3}{y} = \frac{15}{14}, \quad y = \frac{14}{5} \quad \text{Ans. } \left(\frac{14}{13}, \frac{14}{5}\right)$$

10. Let the bigger number be y and the smaller number be x .

$$\therefore x + y = 91 \text{ and } y - x = 17. \quad \text{Solving the equations, we get } x = 37 \text{ and } y = 54.$$

Ans. The numbers are 37 and 54.

11. Let the bigger number be y and the smaller number be x .

$$\therefore \frac{1}{4}(x + y) = 16 \text{ and } 2(y - x) = 36.$$

Solving the equations, we get $x = 23$ and $y = 41$. Ans. The numbers are 23 and 41.

12. Let the bigger number be y and the smaller number be x .

$$\therefore \begin{cases} y - x = \frac{3}{5}(x + y), \quad 8x - 2y = 0, \quad 4x - y = 0 \\ y = 3x + 14 \end{cases}$$

Solving the equations, we get $x = 14$ and $y = 56$. Ans. The numbers are 14 and 56.

$$13. \text{ Let the numerator be } y \text{ and the denominator be } x. \quad \therefore \begin{cases} \frac{y+9}{2x} = \frac{2}{3}, \quad 3y + 27 = 4x \\ \frac{y-1}{x} = \frac{1}{2}, \quad 2y - 2 = x \end{cases}$$

Solving the equations, we get $x = 12$ and $y = 7$. Ans. The fraction is $\frac{7}{12}$.

14. Let the prices of a table and a chair be x and y respectively.

$$\therefore 3x + 4y = 2800 \text{ and } 4x + 9y = 4650.$$

Solving the equations, we get $x = 600$ and $y = 250$.

Ans. The prices of a table and a chair are \$600 and \$250 respectively.

15. $2m - 3n + 10 = 4m - 5n + 6 = m + n - 9$.

Solving the equations, we get $m = 9$ and $n = 7$.

$$\therefore \text{The perimeter} = 3(9 + 7 - 9) = 3(7) = 21 \text{ units.}$$

16. (a) Opposite sides of a rectangle are equal,
$$\begin{cases} x + 3 = y + 5, & x - y = 2 \\ x - y + 4 = 3x - 4y + 3, & 2x - 3y = 1 \end{cases}$$

Solving the equations, we get $x = 5$ and $y = 3$.
- (b) $AB = 5 + 3 = 8$, $BC = 5 - 3 + 4 = 6$, $\therefore$ perimeter $= 2(8 + 6) = 28$ units.
- (c) Area $= 8 \times 6 = 48$ square units.
17. Let the numbers of students and pencils be x and y respectively.
 $\therefore y = 6x + 5$ and $y = 7x - 8$. Solving the equations, we get $x = 13$ and $y = 83$.
Ans. The number of students and pencils are 13 and 83 respectively.
18. Let the numbers of \$2 coins and \$5 coins be x and y respectively.
 $\therefore x + y = 30$ and $2x + 5y = 90$. Solving the equations, we get $x = 20$ and $y = 10$.
Ans. The numbers of \$2 and \$5 coins are 20 and 10 respectively.
19. Let the tens' digit and the units' digit be x and y respectively.
The original no. $= 10x + y$,
the reversed no. $= 10y + x$
 $\therefore \begin{cases} 10x + y = 8(x + y), & 2x - 7y = 0 \\ 10y + x = \frac{1}{3}(10x + y) + 3, & 7x - 29y + 9 = 0 \end{cases}$
Solving the equations, we get $x = 7$ and $y = 2$. *Ans. The number is 72.*
20. $x^2 + Ax - A \equiv x^2 + 3x - Bx - 3B - 11 \equiv x^2 + x(3 - B) - (3B + 11)$
Comparing the coefficients, we get: $A = 3 - B$ and $A = 3B + 11$.
Solving the equations, we get $A = 5$ and $B = -2$.
21. Let the present ages of David and Bobby be x and y respectively.
 $\therefore \begin{cases} x = y + 13 \\ y - 11 = \frac{2}{3}(x - 11), & 2x - 3y = -11 \end{cases}$
Solving the equations, we get: $x = 50$ and $y = 37$.
Ans. The present ages of David and Bobby are 50 and 37 respectively.
22. Let the present ages of A and B be x and y respectively.
 $\therefore \begin{cases} \frac{x}{y} = \frac{5}{3}, & 3x = 5y \\ \frac{x + 9}{y + 9} = \frac{4}{3}, & 3x - 4y = 9 \end{cases}$
Solving the equations, we get $x = 15$ and $y = 9$.
Ans. The present ages of A and B are 15 and 9 respectively.
23. Let the speeds of the cars be x km/h and y km/h. $\therefore \begin{cases} y = x + 12 \\ 0.5(x + y) = 79, & x + y = 158 \end{cases}$
Solving the equations, we get $x = 73$ and $y = 85$.
Ans. The speeds of the cars are 73 km/h and 85 km/h.
24. Let the speeds of the cars be x km/h and y km/h. $\therefore \begin{cases} 4(x + y) = 280, & x + y = 70 \\ 14(y - x) = 280, & y - x = 20 \end{cases}$
Solving the equations, we get $x = 25$ and $y = 45$.
Ans. The speeds of two cars are 25 km/h and 45 km/h.
25. Let the speeds of the boat and the water current be x km/h and y km/h respectively.
 $\therefore \begin{cases} 6(x + y) = 108, & x + y = 18 \\ 9(x - y) = 108, & x - y = 12 \end{cases}$
Solving the equations, we get $x = 15$ and $y = 3$.
Ans. The speeds of the boat and the current are 15 km/h and 3 km/h respectively.
26. Let x km/h and y km/h be the speeds of the ship in still water and the current of water respectively.
 $22.5(x - y) = 360$, $x - y = 16$(1)
 $20(x + y) = 360$, $x + y = 18$(2)

Solving (1) and (2), we get $x = 17$ and $y = 1$.

Ans. The speed of the ship against the current = $17 - 1 = 16$ km/h.

$$27. \frac{4x+3y}{10} - \frac{2x-y}{5} = \frac{x-y}{2}, \quad 4x+3y-4x+2y=5x-5y, \quad x-2y=0 \dots (1)$$

$$8y - \frac{5x-2}{3} = 2x+y, \quad 24y-5x+2=6x+3y, \quad 11x-21y=2 \dots (2)$$

Solving (1) and (2), we have: $x = 4$ and $y = 2$.

$$28. \text{ Let } x = \frac{1}{u} \text{ and } y = \frac{1}{v}. \quad \therefore 12x-5y+\frac{1}{10}=0, \text{ and } 2x+6y-\frac{34}{10}=0$$

Solving the equations, we get $x = \frac{1}{5}$ and $y = \frac{1}{2}$. Ans. $u = \frac{1}{x} = 5$, $v = \frac{1}{y} = 2$.

$$29. \begin{cases} \frac{6}{x} - \frac{10x}{y} = -5 \dots (1), & (1) \times 2, \frac{12}{x} - \frac{20x}{y} = -10 \dots (3) \\ \frac{9}{x} - \frac{4x}{y} = 9 \dots (2), & (2) \times 5, \frac{45}{x} - \frac{20x}{y} = 45 \dots (4) \end{cases}$$

$$(4) - (3), \quad \therefore \frac{45}{x} - \frac{12}{x} = 45 + 10, \quad \frac{33}{x} = 55, \quad x = \frac{33}{55} = \frac{3}{5} \dots (5)$$

$$\text{Put (5) into (1), } 6\left(\frac{5}{3}\right) - \frac{1}{y}(10)\left(\frac{3}{5}\right) = -5, \quad 10 - \frac{6}{y} = -5, \quad 15 = \frac{6}{y}, \quad y = \frac{6}{15} = \frac{2}{5}$$

[Or: Let $\frac{1}{x} = a$, $\frac{x}{y} = b$, then solve for a and b first.] Ans. $\left(\frac{3}{5}, \frac{2}{5}\right)$

$$30. \text{ Let the equations be (1) and (2) respectively. From (1), } x = -2y \dots (3)$$

$$\text{Sub. (3) into (2), } \therefore -2y = \frac{1}{1 - \frac{1}{1 - \frac{2y}{y}}} = \frac{1}{1 - \frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2},$$

$$\therefore y = -\frac{3}{4}, \quad \therefore x = -2\left(-\frac{3}{4}\right) = \frac{3}{2}. \quad \text{Ans. } \left(\frac{3}{2}, -\frac{3}{4}\right)$$

$$31. (a) \quad x - (7 + 3x) + 1 = 0, \quad -2x = 6, \quad \therefore x = -3, \quad \therefore y = 7 + 3(-3) = -2$$

$$(b) \text{ Let } y = \frac{1}{p+q}, x = \frac{1}{p}.$$

$$\text{From (a), } -3 = \frac{1}{p}, \quad \therefore p = -\frac{1}{3}$$

$$-2 = \frac{1}{p+q}, \quad -2p - 2q = 1, \quad -2\left(-\frac{1}{3}\right) - 2q = 1, \quad -2q = \frac{1}{3}, \quad \therefore q = -\frac{1}{6}$$

$$32. (a) \quad A(2) + B(1) = 18, \quad 2A + B = 18 \dots (1)$$

$$A(-6) + B(15) = 18, \quad -2A + 5B = 6 \dots (2)$$

Solving (1) and (2), we have $A = 7$ and $B = 4$.

$$(b) \quad 7(5) + 4(k) = 18, \quad 4k = -17, \quad \therefore k = -\frac{17}{4}$$

$$33. (a) \text{ Two straight lines will never intersect when they are parallel to each other.}$$

$$(b) \quad 6x - 3y + 2 = 0, \quad 12x - 6y + 4 = 0 \dots (1)$$

$$4x + ry + 4 = 0, \quad 12x + 3ry + 12 = 0 \dots (2)$$

They are parallel when $3r = -6$, $\therefore r = -2$

34. (a) $5x + y - 4 = 0$(1); $2x + y - 10 = 0$(2)
Solving (1) and (2), we have $x = -2$ and $y = 14$.
Ans. The coordinates of P are $(-2, 14)$.
- (b) $a(-2) + b(14) + 16 = 0$, $a - 7b = 8$(1)
 $7a(-2) - 2b(14) - 14 = 0$, $a + 2b = -1$(2)
Solving (1) and (2), we get $a = 1$ and $b = -1$.
35. $a + b = 2m$(1); $a - b = 2n$(2)
(1) + (2), $2a = 2m + 2n$, $a = m + n$(3)
Put (3) into (1), $(m + n) + b = 2m$, $b = m - n$, $\therefore \frac{a}{b} = \frac{m+n}{m-n}$
36. (a) Let the equations be (1), (2), (3) and (4) respectively.
(1) + (2) + (3), $\therefore (x_1 + x_2) + (x_2 + x_3) + (x_3 + x_1) = 10 + (-3) + 9$,
 $2(x_1 + x_2 + x_3) = 16$, $x_1 + x_2 + x_3 = 8$(5).
Put (5) into (4), $y = 8$.
- (b) Put (2) into (5), $\therefore x_1 + (-3) = 8$, $x_1 = 11$.
Put (3) into (5), $\therefore x_2 + 9 = 8$, $x_2 = -1$.
Put (1) into (5), $10 + x_3 = 8$, $x_3 = -2$.
Ans. $x_1 = 11$, $x_2 = -1$, $x_3 = -2$.
37. (a) $16\,500 = a + k(1\,000)$, $a + 1\,000k = 16\,500$(1)
 $34\,500 = a + k(5\,000)$, $a + 5\,000k = 34\,500$(2)
Solving (1) and (2), we get $a = 12\,000$ and $k = 4.5$
- (b) When $n = 40\,000$, $C = 12\,000 + 4.5(40\,000) = 192\,000$
Ans. The average printing cost of 1 copy = $192\,000 \div 40\,000 = \4.8 .
38. Let \$x and \$y be the amounts that A and B originally had.
 $2(x - 240) = y + 240$, $2x - y = 720$(1)
 $x + 1\,010 = 3(y - 1\,010)$, $x - 3y = -4\,040$(2)
Solving (1) and (2), we get $x = 1\,240$ and $y = 1\,760$.
Ans. The amount they have altogether = $1\,240 + 1\,760 = \$3\,000$
39. Let x and y be the numbers of coins in P and Q respectively.
 $2(x - 22) = y + 22$, $2x - y = 66$(1)
 $2(x - 22) + 46 = 5y + 2(22)$, $2x - 5y = 42$(2)
Solving (1) and (2), we get $x = 36$ and $y = 6$.
Ans. There are 36 coins in Box P and 6 coins in Box Q originally.
40. Let x and y be the present ages of Adam and his sister respectively.
 $x - 4 = 4(y - 4)$, $x - 4y = -12$(1)
 $x + 8 = 2(y + 8)$, $x - 2y = 8$(2)
Solving (1) and (2), we get $x = 28$ and $y = 10$.
Ans. The sum of their present ages = $28 + 10 = 38$.
41. Let x and y be the present ages of the elder daughter and the woman respectively,
 $\therefore$ the present age of the younger daughter is $(x - 4)$.
 $x + (x - 4) + y = 50$, $2x + y = 54$(1)
 $y - x = 63 - y$, $x - 2y = -63$(2)
Solving (1) and (2), we get $x = 9$ and $y = 36$; $\therefore x - 4 = 9 - 4 = 5$.
Ans. The woman is 36 years old and the daughters are 9 and 5 years old.
42. Let x m/s and y m/s be A's speed and B's speed respectively,
 $\therefore$ the speed of C is $(y + 1.5)$ m/s.
 $x \times 12 \times 60 = y \times 10 \times 60$, $6x - 5y = 0$(1)

$$x \times 18 \times 60 = (y + 1.5) \times (18 - 6) \times 60, \quad 3x - 2y = 3 \dots\dots\dots(2)$$

Solving (1) and (2), we get $x = 5$ and $y = 6$; $\therefore y + 1.5 = 6 + 1.5 = 7.5$.

Ans. The speed of A, B and C are 5 m/s, 6 m/s and 7.5 m/s respectively.

43. The speeds of the boat against the current and along the current are $(x - y)$ km/h and $(x + y)$ km/h respectively.

$$\therefore \frac{315}{x - y} - \frac{315}{x + y} = 2 \dots\dots(1), \quad \text{and} \quad \frac{28}{x - y} = \frac{36}{x + y} \dots\dots(2)$$

$$\text{Let } u = \frac{1}{x - y} \text{ and } v = \frac{1}{x + y}.$$

$$\text{From (1), } 315u - 315v = 2 \dots\dots\dots(3), \quad \text{from (2) } 7u - 9v = 0 \dots\dots\dots(4)$$

$$\text{Solving (3) and (4), we get } u = \frac{1}{35} \text{ and } v = \frac{1}{45}.$$

$$\therefore \frac{1}{x - y} = \frac{1}{35}, \quad x - y = 35 \dots\dots\dots(5), \quad \text{and} \quad \frac{1}{x + y} = \frac{1}{45}, \quad x + y = 45 \dots\dots\dots(6)$$

Solving (5) and (6), we get $x = 40$ and $y = 5$.

44. Let x h and y h be the time taken by A and B respectively.

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{3} \dots\dots\dots(1), \quad \frac{1}{x} \times 4 + \left(\frac{1}{x} + \frac{1}{y}\right) \times 2 = 1 \dots\dots\dots(2)$$

$$\text{Put (1) into (2), } \frac{4}{x} + \frac{2}{3} = 1, \quad \frac{1}{x} = \frac{1}{12}, \quad \therefore x = 12.$$

$$\frac{1}{12} + \frac{1}{y} = \frac{1}{3}; \quad \frac{1}{y} = \frac{1}{4}, \quad \therefore y = 4$$

Ans. A and B take 12 h and 4 h respectively to fill up the pool alone.

45. Let x days and y days be the time taken by A and B respectively.

$$x = 2y \dots\dots\dots(1); \quad \frac{1}{x} + \frac{1}{y} = \frac{1}{14} \dots\dots\dots(2)$$

$$\text{Put (1) into (2), } \therefore \frac{1}{2y} + \frac{1}{y} = \frac{1}{14}, \quad \frac{3}{2y} = \frac{1}{14}, \quad y = \frac{3}{2} \times 14 = 21; \quad \therefore x = 2(21) = 42$$

Ans. A and B take 42 days and 21 days respectively.

46. Let x and y be the original numbers of workers and days required respectively,

$$(x + 8)(y - 10) = xy, \quad -10x + 8y = 80 \dots\dots\dots(1)$$

$$(x - 8)(y + 20) = xy, \quad 20x - 8y = 160 \dots\dots\dots(2)$$

Solving (1) and (2), we get $x = 24$ and $y = 40$.

Ans. The numbers of workers and days required are 24 and 40 respectively.

47. (a) Let $\$x$, $\$y$ and $\$z$ be the costs of an apple, an orange and a pear respectively.

$$x + 3y + 2z = 22.6 \dots\dots\dots(1); \quad 5x + 8y + 3z = 62.6 \dots\dots\dots(2)$$

$$(1) \times 5 - (2), \quad 7y + 7z = 50.4, \quad y + z = 7.2 \dots\dots\dots(3)$$

Ans. The total cost of 1 orange and 1 pear is \$7.2.

- (b) $5y = 4z$, $5y - 4z = 0 \dots\dots\dots(4)$.

Solving, (3) and (4), we have $y = 3.2$, $z = 4$.

$$\text{From (1), } x + 3(3.2) + 2(4) = 22.6, \quad x = 5$$

Ans. The costs of an apple, an orange and a pear are \$5, \$3.2 and \$4 respectively.

48. Let x and y be the numbers of males and females in project A respectively,

$\therefore$ in project B, the no. of males = $181 - x$, and the no. of females = $154 - y$.

$$x = 2y + 3, \quad x - 2y = 3 \dots\dots\dots(1);$$

$$3(181 - x) = 154 - y, \quad 3x - y = 389 \dots\dots\dots(2)$$

Solving (1) and (2), we get $x = 155$ and $y = 76$.

Ans. There are 155 males and 76 females in project A, while there are 26 males and 78 females in project B.

49. (a) $\begin{cases} 5x - y = 2 & \dots\dots(1) \\ 10x + 2y = 12 & \dots\dots(2) \end{cases}$, $(2) - (1) \times 2$, $4y = 8$, $y = 2$.

Sub. $y = 2$ into (1), $x = \frac{2+2}{5} = \frac{4}{5}$. $\therefore x = \frac{4}{5}$, $y = 2$

(b) $\begin{cases} \frac{5}{2m+5n} - \frac{m}{2} + n = 2 \\ \frac{10}{2m+5n} + m - 2n = 12 \end{cases}$, $\begin{cases} 5\left(\frac{1}{2m+5n}\right) - \frac{m-2n}{2} = 2 \\ 10\left(\frac{1}{2m+5n}\right) + 2\left(\frac{m-2n}{2}\right) = 12 \end{cases}$

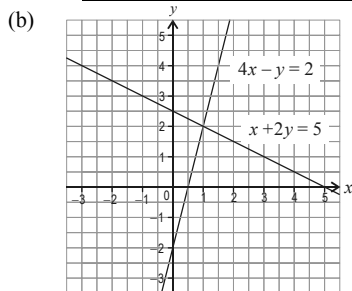
From (a), $\begin{cases} \frac{1}{2m+5n} = \frac{4}{5} \\ \frac{m-2n}{2} = 2 \end{cases}$, $\therefore \begin{cases} 2m+5n = \frac{5}{4} & \dots\dots(3) \\ m-2n = 4 & \dots\dots(4) \end{cases}$

$(3) - (4) \times 2$, $9n = -\frac{27}{4}$, $n = -\frac{3}{4}$.

Sub. $n = -\frac{3}{4}$ into (4), $m = 4 + 2\left(-\frac{3}{4}\right) = \frac{5}{2}$. $\therefore m = \frac{5}{2}$, $n = -\frac{3}{4}$

50. (a)

x	-3	0	4
y	4	2.5	0.5



(c) (i) From the graph, the solutions are $(x, y) = (1, 2)$.

(ii) $\begin{cases} \frac{1}{2a} + \frac{2}{b} = 5 \\ \frac{2}{a} - \frac{1}{b} = 2 \end{cases}$, $\begin{cases} \frac{1}{2a} + 2\left(\frac{1}{b}\right) = 5 \\ 4\left(\frac{1}{2a}\right) - \frac{1}{b} = 2 \end{cases}$

From (c)(i), $\frac{1}{2a} = 1$, $\frac{1}{b} = 2$,

$\therefore a = \frac{1}{2}$, $b = \frac{1}{2}$

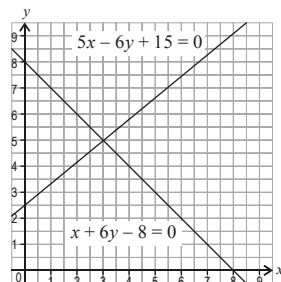
51. (a)

x	0	6	9
y	2.5	7.5	10

(b) (i) $\begin{cases} \frac{x+3}{y} = \frac{6}{5} \\ \frac{x}{y-8} = -1 \end{cases}$ $\therefore \begin{cases} 5x - 6y + 15 = 0 \\ x + y - 8 = 0 \end{cases}$

(ii) From (a), the solutions of $\begin{cases} 5x - 6y + 15 = 0 \\ x + y - 8 = 0 \end{cases}$ are $(x, y) = (3, 5)$.

$\therefore$ the fraction is $\frac{3}{5}$.



52. (a) $\begin{cases} a - 2b = -0.4 & \dots\dots(1) \\ a + b = 3.2 & \dots\dots(2) \end{cases}$

$(2) - (1)$, $3b = 3.6$, $b = 1.2$

Sub. $b = 1.2$ into (2), $a = 3.2 - 1.2 = 2$,
 $\therefore a = 2$, $b = 1.2$.

- (b) (i) Let
- w
- hours be the driving time from city A to city B.

$$\begin{cases} x = 2w - 0.4 \\ x + w = 3.2 \end{cases}, \quad \begin{cases} x - 2w = -0.4 \\ x + w = 3.2 \end{cases} \quad \text{From (a), } x = 2, \quad w = 1.2. \\ y = \frac{96}{w} = \frac{96}{1.2} = 80. \quad \text{Ans. } x = 2, y = 80.$$

$$(ii) \text{ Speed from home to City A} = \frac{96}{2} = 48 \text{ km/h}$$

$$\text{Speed from home to City B} = \frac{96}{1.2} = 80 \text{ km/h}$$

$$\text{The required difference} = 80 - 48 = 32 \text{ km/h}$$

$$(iii) \text{ Time for his whole journey} = 2 + 2 = 4 \text{ hours.}$$

$\therefore$ The claim is disagreed.

Unit 13 Rational & irrational numbers

- 5.20
 - 3.48
 - 63.2
 - $= \sqrt{65} = 8.06$
- $= \sqrt{99} = 9.95$
 - $= \sqrt{1600} = 40$
- $= \sqrt{12^2} = 12$
 - $= \sqrt{13^2} = 13$
 - $= -\sqrt{4 \times 144} = -2 \times 12 = -24$
 - $= \frac{5}{9}$
 - $= -\sqrt{\frac{64}{49}} = -\frac{8}{7}$
 - $= \sqrt{\frac{225}{10\,000}} = \sqrt{\frac{9}{400}} = \frac{3}{20}$
- $\because 25 < 35 < 36, \quad \sqrt{25} < \sqrt{35} < \sqrt{36}, \quad 5 < \sqrt{35} < 6, \quad \therefore$ the integral part of $\sqrt{35}$ is 5.
 - $\because 81 < 92 < 100, \quad \sqrt{81} < \sqrt{92} < \sqrt{100}, \quad 9 < \sqrt{92} < 10, \quad \therefore$ the integral part of $\sqrt{92}$ is 9.
 - $\because 100 < 106 < 121, \quad \sqrt{100} < \sqrt{106} < \sqrt{121}, \quad 10 < \sqrt{106} < 11, \quad \therefore$ the integral part of $\sqrt{106}$ is 10.
- 4.0
 - 4.6
 - 2
- $= 1 - 8 = -7, \quad \therefore$ rational
 - $= \sqrt{3 \times 81} = \sqrt{3} \times 9, \quad \therefore$ irrational
 - recurring decimal, $\therefore$ rational
- $= 2\sqrt{10}, \quad \therefore$ yes
 - $= \frac{16}{17}, \quad \therefore$ no
 - $= 30\sqrt{10}, \quad \therefore$ yes
 - $= \sqrt{\frac{36}{3}} = \sqrt{12} = 2\sqrt{3}, \quad \therefore$ yes
 - $= \sqrt{45} = 3\sqrt{5}, \quad \therefore$ yes
 - $= 3, \quad \therefore$ no
- $= \sqrt{4 \times 2} = 2\sqrt{2}$
 - $= \sqrt{9 \times 2} = 3\sqrt{2}$
 - $= \sqrt{9 \times 3} = 3\sqrt{3}$
 - $= \sqrt{36 \times 2} = 6\sqrt{2}$
 - $= \sqrt{9 \times 5} = 3\sqrt{5}$
 - $= \sqrt{16 \times 3} = 4\sqrt{3}$
 - $= \sqrt{49 \times 2} = 7\sqrt{2}$
 - $= \sqrt{25 \times 3} = 5\sqrt{3}$
 - $= \sqrt{121 \times 3} = 11\sqrt{3}$
 - $= \sqrt{81 \times 3} = 9\sqrt{3}$
 - $= \sqrt{81 \times 5} = 9\sqrt{5}$
 - $= \sqrt{25 \times 5} = 5\sqrt{5}$
 - $= \sqrt{2} \times \sqrt{5^2} \times \sqrt{m^2} \times \sqrt{n^4} = 5mn^2\sqrt{2}$
 - $= \sqrt{2 \times 3^2 \times ab \times b^4} = 3b^2\sqrt{2ab}$
- $5\sqrt{3} - 8\sqrt{5}$
 - $7\sqrt{7} - 2\sqrt{11}$
 - $3\sqrt{w}$
 - $5\sqrt{y} - 2\sqrt{x}$
- $= 2\sqrt{2} + 4\sqrt{2} = 6\sqrt{2}$
 - $= 3\sqrt{3} - \sqrt{3} = 2\sqrt{3}$
 - $= 3\sqrt{2} - 4\sqrt{2} + 5\sqrt{2} = 4\sqrt{2}$
 - $= 2\sqrt{3} + 5\sqrt{3} - 3\sqrt{2} = 7\sqrt{3} - 3\sqrt{2}$
 - $= 2\sqrt{3} + 6\sqrt{3} - 5\sqrt{3} = 3\sqrt{3}$
 - $= 2\sqrt{7} - 5\sqrt{7} - 3\sqrt{7} = -6\sqrt{7}$

- (g) $= 8\sqrt{x} - 7\sqrt{x} = \sqrt{x}$ (h) $= 2m\sqrt{2mn} + 3m\sqrt{2mn} = 5m\sqrt{2mn}$
10. (a) $= 24 \times 2 = 48$ (b) $= 10 \times 3 \times \sqrt{3} = 30\sqrt{3}$
- (c) $= 36 \times 2 = 72$ (d) $= 9 \times 5 = 45$
- (e) $= (2 \times \sqrt{2}) \times (\sqrt{2} \times \sqrt{13}) = 4\sqrt{13}$ (f) $= (\sqrt{6} \times \sqrt{10}) \times \sqrt{10} = 10\sqrt{6}$
- (g) $= (6 \times \sqrt{3}) \times (5 \times \sqrt{3} \times \sqrt{2}) = 90\sqrt{2}$ (h) $= 15\sqrt{6}$
- (i) $= 2\sqrt{33} \times \sqrt{33} = 66$ (j) $= 3\sqrt{17} \times 2\sqrt{17} = 6 \times 17 = 102$
- (k) $= 3\sqrt{7ab} \times (a\sqrt{7ab} \times \sqrt{2}) = 3a \times 7ab \times \sqrt{2} = 21a^2b\sqrt{2}$
- (l) $= (\sqrt{3n} \times \sqrt{7mn}) \times \sqrt{7mn} = 7mn\sqrt{3n}$
11. (a) 6 (b) $= \sqrt{10} \times 2 = 2\sqrt{10}$
- (c) $= \frac{\sqrt{2} \times 3\sqrt{7}}{3 \times 3\sqrt{7}} = \frac{\sqrt{2}}{3}$ (d) $= \frac{5 \times \sqrt{3}}{2 \times \sqrt{3}} = \frac{5}{2}$
- (e) $= \frac{2\sqrt{2} \times \sqrt{14}}{3 \times \sqrt{14}} = \frac{2\sqrt{2}}{3}$ (f) $= \frac{(4 \times \sqrt{3} \times \sqrt{5}) \times (2 \times 3 \times \sqrt{3})}{4 \times 3 \times \sqrt{5}} = 2 \times 3 = 6$
12. (a) $= 2 - 4\sqrt{2} - 5 = -3 - 4\sqrt{2}$
- (b) $= 30 - 17\sqrt{5} + 12 = 42 - 17\sqrt{5}$
- (c) $= (7 - 3\sqrt{3})(\sqrt{3} - 2) = 7\sqrt{3} - 14 - 9 + 6\sqrt{3} = 13\sqrt{3} - 23$
- (d) $= \sqrt{14} \cdot \sqrt{7} - \sqrt{14} \cdot \sqrt{2} + 5\sqrt{7} - 5\sqrt{2} = 7\sqrt{2} - 2\sqrt{7} + 5\sqrt{7} - 5\sqrt{2} = 2\sqrt{2} + 3\sqrt{7}$
- (e) $= 5y - 6 + 7\sqrt{y}$
- (f) $= 3x - 4\sqrt{xy} + 6\sqrt{xy} - 8y = 3x - 8y + 2\sqrt{xy}$
13. (a) $= 7 - 2\sqrt{35} + 5 = 12 - 2\sqrt{35}$ (b) $= 18 + 12\sqrt{2} + 4 = 22 + 12\sqrt{2}$
- (c) $= 48 + 8\sqrt{15} + 5 = 53 + 8\sqrt{15}$ (d) $= 24 - 2\sqrt{24 \times 12} + 12 = 36 - 24\sqrt{2}$
- (e) $= 25x + 4 - 20\sqrt{x}$ (f) $= 9x + 49y + 42\sqrt{xy}$
14. (a) $= 5 - 4 = 1$ (b) $= 9 - 20 = -11$ (c) $x - 4y^2$
- (d) $= (4\sqrt{a} - 3\sqrt{b})(4\sqrt{a} + 3\sqrt{b}) = 16a - 9b$
15. (a) $= \frac{2\sqrt{7} \times \sqrt{7}}{\sqrt{7}} = 2\sqrt{7}$ (b) $= \frac{8 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{8\sqrt{5}}{5}$ (c) $= \frac{4\sqrt{3} \times \sqrt{3}}{\sqrt{3}} = 4\sqrt{3}$
- (d) $= \frac{4}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$ (e) $= \frac{24}{4\sqrt{3}} = \frac{2 \times 3}{\sqrt{3}} = 2\sqrt{3}$ (f) $= \frac{6\sqrt{7} \times \sqrt{7}}{2\sqrt{7}} = 3\sqrt{7}$
- (g) $= \frac{5 \times 2}{\sqrt{5} \times \sqrt{3}} = \frac{\sqrt{5} \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{15}}{3}$
- (h) $= \frac{\sqrt{4x} \times \sqrt{a}}{\sqrt{a} \times \sqrt{a}} = \frac{2\sqrt{ax}}{a}$ (i) $= \frac{2m \times \sqrt{n}}{3\sqrt{n} \times \sqrt{n}} = \frac{2m\sqrt{n}}{3n}$
16. $= \frac{1}{4}(10)\sqrt{3} + \frac{\sqrt{3}}{2} - 2\sqrt{3} + \frac{\sqrt{3}}{3} = \sqrt{3}(\frac{5}{2} + \frac{1}{2} - 2 + \frac{1}{3}) = \frac{4\sqrt{3}}{3}$
17. (a) $12\,100 = 121 \times 100 = (11 \times 10)^2 = 110^2$. Also, note that $(-110)^2 = 12\,100$,
 $\therefore$ the square roots of 12100 are 110 and -110.
- (b) $-8\,000 = -(8 \times 1\,000) = (-1)^3(2 \times 10)^3 = (-20)^3$, $\therefore$ the cube root of -8 000 is -20.
- (c) $0.0625 = \frac{625}{10\,000} = \frac{5^4}{10^4} = \left(\frac{1}{2}\right)^4 = 0.5^4$. Also, note that $(-0.5)^4 = 0.0625$,
 $\therefore$ the fourth roots of 0.0625 are 0.5 and -0.5.

18. $x = \sqrt{3}, y = \sqrt{27} = \sqrt{3^3} = (\sqrt{3})^3, \therefore y = x^3$; but $y = \sqrt{27} = \sqrt{3^3} = 3\sqrt{3}, \therefore y = 3x$.

Ans. Both of them are correct.

19. (a) $\sqrt{12} = 2\sqrt{3} \approx 2 \times 1.732 = 3.464$ (b) $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \approx \frac{1.732}{3} = 0.577$ (3 sig. fig.)

20. (a) $\sqrt{1.8} = \sqrt{\frac{18}{10}} = \sqrt{\frac{3^2}{5}} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5} = \frac{3 \times 2.236}{5} = 1.3416$

(b) $\sqrt{12} - \sqrt{8} = 2\sqrt{3} - 2\sqrt{2} = 2(\sqrt{3} - \sqrt{2}) = 2(1.732 - 1.414) = 0.636$

21. (a) $= (10) - 1 = 9$

(b) $9 = (\sqrt{10} - 1)(\sqrt{10} + 1), \therefore 3 = \frac{(\sqrt{10} - 1)(\sqrt{10} + 1)}{3}, \frac{3}{\sqrt{10} + 1} = \frac{\sqrt{10} - 1}{3}$

(c) L.H.S. $= \frac{3}{\sqrt{10} + 1} \cdot \frac{\sqrt{10} - 1}{\sqrt{10} - 1} = \frac{3(\sqrt{10} - 1)}{10 - 1} = \frac{3(\sqrt{10} - 1)}{9} = \frac{\sqrt{10} - 1}{3} = \text{R.H.S.}$

22. $= \frac{(\sqrt{a})^2}{\sqrt{a}\sqrt{a+1}} - \frac{(\sqrt{a+1})^2}{\sqrt{a}\sqrt{a+1}} - \frac{1}{\sqrt{a}\sqrt{a+1}} = \frac{a - a - 1 - 1}{\sqrt{a}\sqrt{a+1}} = \frac{-2}{\sqrt{a(a+1)}}$

23. $(\sqrt{3} + 1)\sqrt{x} = 2, \sqrt{x} = \frac{2}{\sqrt{3} + 1} = \frac{2(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)} = \frac{2(\sqrt{3} - 1)}{3 - 1} = \sqrt{3} - 1$

$\therefore x = (\sqrt{3} - 1)^2 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}, \therefore a = 4, b = -2.$

24. $(x - \frac{1}{x})^2 = (3\sqrt{5} - 2)^2, x^2 - 2 + \frac{1}{x^2} = 45 - 12\sqrt{5} + 4, \therefore x^2 + \frac{1}{x^2} = 51 - 12\sqrt{5}$

25. $AB = \sqrt{27} \text{ cm} = 3\sqrt{3} \text{ cm}, BC = \sqrt{75} \text{ cm} = 5\sqrt{3} \text{ cm}, AC = 12 \text{ cm} = 2\sqrt{3} \text{ cm}$

$AB + AC = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3} = BC, \therefore A, B, C$ form a straight line (line segment).

26. $\sqrt{32} + \sqrt{18} = 4\sqrt{2} + 3\sqrt{2} = 7\sqrt{2}, \text{ but } \sqrt{147} = \sqrt{7 \times 7 \times 3} = 7\sqrt{3}, \therefore \sqrt{32} + \sqrt{18} < \sqrt{147}$

Ans. Since the sum of 2 sides is smaller than the third side, they cannot form a triangle.

27. $(2\sqrt[3]{3})^3 = (\sqrt{x})^3, 2^3 \cdot 3 = \sqrt{x^3}, \therefore 2^3 \cdot 3 = \sqrt{y}, \sqrt{y} = 24, y = 24^2 = 576$

28. (a) $\frac{m}{n} = (\sqrt{5} - 2) \div \frac{1}{\sqrt{5} + 2} = (\sqrt{5} - 2)(\sqrt{5} + 2) = 5 - 4 = 1$

(b) $\frac{m^{2005}}{n^{2003}} = (\frac{m}{n})^{2003} \times m^2 = (1)^{2003} \times (\sqrt{5} - 2)^2 = 5 - 4\sqrt{5} + 4 = 9 - 4\sqrt{5}$

29. $= \left[(\sqrt{3} - \sqrt{2} + 1) (\sqrt{3} + \sqrt{2} - 1) \right]^5$
 $= \left\{ \left[\sqrt{3} - (\sqrt{2} - 1) \right] \left[\sqrt{3} + (\sqrt{2} - 1) \right] \right\}^5$
 $= \left[(\sqrt{3})^2 - (\sqrt{2} - 1)^2 \right]^5 = \left[3 - (2 - 2\sqrt{2} + 1) \right]^5 = (2\sqrt{2})^5$
 $= 2^5 \times (\sqrt{2})^5 = 32 \times (\sqrt{2})^4 \sqrt{2} = 32 \times 2^2 \times \sqrt{2} = 128\sqrt{2}$

30. (a) L.H.S. $= \frac{a - b}{\sqrt{a} - \sqrt{b}} = \frac{(\sqrt{a})^2 - (\sqrt{b})^2}{\sqrt{a} - \sqrt{b}} = \frac{(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})}{\sqrt{a} - \sqrt{b}} = \sqrt{a} + \sqrt{b}$

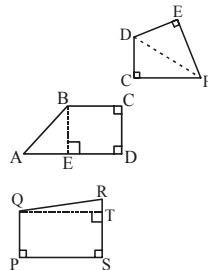
R.H.S. $= \sqrt{a} + \sqrt{b} = \text{L.H.S.} \therefore \text{It is an identity.}$

(b) From (a), $\frac{a-b}{\sqrt{a}-\sqrt{b}} \equiv \sqrt{a} + \sqrt{b}$, $\therefore \frac{1}{\sqrt{a}+\sqrt{b}} = \frac{\sqrt{a}-\sqrt{b}}{a-b}$ (*)

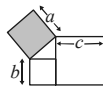
$$\begin{aligned} & \frac{1}{\sqrt{2}+\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \dots + \frac{1}{\sqrt{100}+\sqrt{99}} \\ &= \left(\frac{\sqrt{2}-\sqrt{1}}{2-1}\right) + \left(\frac{\sqrt{3}-\sqrt{2}}{3-2}\right) + \left(\frac{\sqrt{4}-\sqrt{3}}{4-3}\right) + \dots + \frac{\sqrt{100}-\sqrt{99}}{100-99} \quad [\text{From (*)}] \\ &= (\sqrt{2}-\sqrt{1}) + (\sqrt{3}-\sqrt{2}) + (\sqrt{4}-\sqrt{3}) + \dots + (\sqrt{100}-\sqrt{99}) \\ &= -\sqrt{1} + \sqrt{100} = -1 + 10 = 9 \end{aligned}$$

Unit 14 Pythagoras' theorem

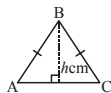
- $x^2 = 10^2 + 24^2$, $x = \sqrt{100 + 576} = \sqrt{676} = 26$
 - $17^2 = y^2 + 8^2$, $y^2 = 17^2 - 8^2$, $y = \sqrt{289 - 64} = \sqrt{225} = 15$
 - $x^2 = 15^2 - 10^2 = 225 - 100 = 125$, $x = \sqrt{125} = 11.2$
 $y^2 = x^2 - 8^2 = 125 - 64 = 61$, $y = \sqrt{61} = 7.8$
 - $20^2 = h^2 + \left(\frac{24}{2}\right)^2$, $h = \sqrt{400 - 144} = \sqrt{256} = 16$ cm
- $PS = \sqrt{10^2 - 6^2} = \sqrt{64} = 8$; $SR = \sqrt{8^2 - 6^2} = \sqrt{28}$, $a = PS + SR = 8 + \sqrt{28} = 13.3$
 - $AC = 3 + 6 = 9$, $BC^2 = 10^2 - 9^2 = 19$,
 $m^2 = BC^2 + 6^2 = 19 + 36 = 55$, $m = \sqrt{55} = 7.42$
 - $GF = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5$, $EF = \sqrt{15^2 - 12^2} = \sqrt{225 - 144} = \sqrt{81} = 9$
 $x = EF - GF = 9 - 5 = 4$
 - $x = \sqrt{39^2 - 36^2} = \sqrt{225} = 15$, $ML = \sqrt{45^2 - 36^2} = \sqrt{729} = 27$
 $y = ML - x = 27 - 15 = 12$
 - In ΔPQR , $PQ^2 + RQ^2 = PR^2$
 $(n+8)^2 + 10^2 = 26^2$, $(n+8)^2 = 26^2 - 10^2$,
 - Join DF. $DF^2 = 6^2 + 10^2$, and $DF^2 = 7^2 + y^2$,
 $\therefore y^2 + 49 = 36 + 100$, $y^2 = 87$, $\therefore y = \sqrt{87} = 9.33$
- Draw $BE \perp AD$. $BE = CD = 8$, $ED = BC = 7$,
 $AE = AD - ED = 13 - 7 = 6$
 $y = \sqrt{BE^2 + AE^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$
 - Draw $QT \perp RS$. $QT = PS = 40$, $TS = QP = 15$
 $RT = \sqrt{QR^2 - QT^2} = \sqrt{41^2 + 40^2} = \sqrt{81} = 9$
 $x = RT + TS = 9 + 15 = 24$
- $(x+2)^2 = x^2 + 12^2$, $x^2 + 4x + 4 = x^2 + 144$, $4x = 140$, $\therefore x = 35$
 - Let $BC = x$ cm. $\therefore AB = 2x$ cm.
 $AB^2 = 4^2 + BC^2$, $(2x)^2 = 16 + x^2$, $3x^2 = 16$, $x = \sqrt{\frac{16}{3}}$; $AB = 2x = 2\left(\sqrt{\frac{16}{3}}\right) = 4.62$ cm
- $BD = 14 + 10 = 24$; $AB = \sqrt{25^2 - BD^2} = \sqrt{25^2 - 24^2} = \sqrt{49} = 7$
 Area of $\Delta ACD = \frac{1}{2}(AB)(CD) = \frac{1}{2}(7)(10) = 35$ sq. units



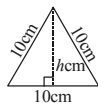
7. $b = \sqrt{49} = 7$; $c = \sqrt{169} = 13$;
 $a^2 = b^2 + (c - b)^2 = 7^2 + (13 - 7)^2 = 49 + 36 = 85$
 The shaded area $= a^2 = 85 \text{ cm}^2$



8. Let the altitude from B to AC be h cm. $\frac{8h}{2} = 24, h = 6$
 $AB^2 = h^2 + \left(\frac{8}{2}\right)^2 = 6^2 + 4^2 = 52$, $AB = \sqrt{52} = 7.21 \text{ cm}$



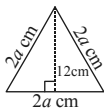
9. Let the height of the equilateral triangle be h cm.
 $10^2 = h^2 + \left(\frac{10}{2}\right)^2$, $h^2 = 100 - 25 = 75$, $h = \sqrt{75} \text{ cm}$
 $\therefore \text{Area} = \frac{1}{2}(10)(\sqrt{75}) = 43.3 \text{ cm}^2$



10. Let the side of the equilateral triangle be $2a$ cm.

$$(2a)^2 = 12^2 + a^2, \quad 4a^2 = 144 + a^2, \quad 3a^2 = 144, \quad a = \sqrt{\frac{144}{3}} = \sqrt{48}$$

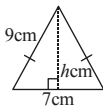
$$\therefore \text{Area} = \frac{1}{2}(2\sqrt{48})(12) = 8.31 \text{ cm}^2$$



11. Let h cm be the altitude.

$$9^2 = h^2 + \left(\frac{7}{2}\right)^2, \quad h^2 = 81 - \frac{49}{4} = \frac{275}{4}, \quad h = \sqrt{\frac{275}{4}}$$

$$\therefore \text{Area} = \frac{1}{2}(7)\left(\sqrt{\frac{275}{4}}\right) = 29.0 \text{ cm}^2$$



12. Let its sides be x m. $x^2 + x^2 = 30$, $2x^2 = 30$, $x^2 = 15$, $x = \sqrt{15}$

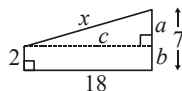
$$\therefore \text{Area} = x^2 = 15 \text{ m}^2; \quad \text{perimeter} = 4x = 4\sqrt{15} = 15.5 \text{ m}$$

13. Let x m be the distance between the tops of the two posts.

$$c = 18, \quad b = 2, \quad a = 7 - b = 7 - 2 = 5$$

$$x = \sqrt{a^2 + c^2} = \sqrt{5^2 + 18^2} = 18.7$$

Ans. The distance between the tops is 18.7 m.



14. (a) $AC = \sqrt{15^2 + 20^2} = \sqrt{625} = 25 \text{ cm}$

$$(b) AS = \sqrt{60^2 + AC^2} = \sqrt{3600 + 625} = \sqrt{4225} = 65 \text{ cm}$$

15. The diagonal of the rectangle with dimensions $8 \text{ cm} \times 10 \text{ cm} = b\sqrt{8^2 + 10^2} = \sqrt{164} \text{ cm}$.

$$\therefore \text{The longest pencil} = \sqrt{4^2 + 164} = \sqrt{180} = 13.4 \text{ cm}$$

16. $BC = \sqrt{50^2 - 48^2} = \sqrt{196} = 14 \text{ cm}$. Area $= (48)(14) = 672 \text{ cm}^2$

17. Let his speed be x km/h. The distance he traveled in 40 mins $= x \times \frac{40}{60} = \frac{2}{3}x \text{ km}$

$$\left(\frac{2}{3}x\right)^2 + 35^2 = 37^2, \quad \frac{4}{9}x^2 = 37^2 - 35^2 = 144, \quad x = \sqrt{\frac{144 \times 9}{4}} = 18$$

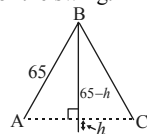
Ans. His speed was 18 km/h.

18. Let h cm be the distance between the highest and the lowest point of the swing.

$$AC = 25 \text{ cm}, \quad \therefore (65 - h)^2 + \left(\frac{25}{2}\right)^2 = 65^2,$$

$$(65 - h)^2 = \sqrt{65^2 - 12.5^2}, \quad 65 - h = 63.79, \quad h = 1.21$$

Ans. The distance is 1.21 cm.



19. (a) $AB^2 = 52^2 = 2704$, $AC^2 + BC^2 = 48^2 + 20^2 = 2704$
 $\therefore AB^2 = AC^2 + BC^2$, $\therefore \triangle ABC$ is right-angled.

(b) $PR^2 = 41^2 = 1681$, $PQ^2 + QR^2 = 38^2 + 15^2 = 1669$

$\therefore PR^2 \neq PQ^2 + QR^2$, $\therefore \triangle PQR$ is not right-angled.

(c) $CE^2 = 137^2 = 18769$, $CD^2 + DE^2 = 88^2 + 105^2 = 18769$

$\therefore CE^2 = CD^2 + DE^2$, $\therefore \triangle CDE$ is right-angled.

20. (a) $PQ^2 = 93$, $PR^2 + QR^2 = (3\sqrt{5})^2 + (4\sqrt{3})^2 = 45 + 48 = 93$

$\therefore PR^2 + QR^2 = PQ^2$, $\therefore \triangle PQR$ is a right-angled triangle.

(b) $\therefore PR \perp QR$, $\therefore \text{area} = \frac{1}{2}(3\sqrt{5})(4\sqrt{3}) = 6\sqrt{15} \text{ cm}^2$

21. No, the longest side is 175. He should compare: “ $49^2 + 168^2$ ” with 175^2 .

$\therefore 49^2 + 168^2 = 2401 + 28224 = 30625 = 175^2$

$\therefore$ It is a right-angled triangle (converse of Pyth. Thm.)

$\therefore$ I disagree with Simon.

22. $AC = PQ = 24$, $CB = CQ + QB = AP + QB = 3 + 4 = 7$

$\therefore AB = \sqrt{24^2 + 7^2} = \sqrt{625} = 25 \text{ m}$

23. $OQ = OC + CQ = 9 + 6 = 15 \text{ (cm)}$,

$OB = OQ = 15 \text{ (radii)}$, $OC^2 + BC^2 = OB^2$,

$9^2 + BC^2 = 15^2$, $BC^2 = 15^2 - 9^2$, $BC = \sqrt{225 - 81} = 12 \text{ cm}$

$\therefore$ The area of the rectangle = $OC \times BC = 9 \times 12 = 108 \text{ cm}^2$

24. The nearer vertical distance = $12 - 4 = 8 \text{ cm}$

$\therefore$ The distance the foot of the ladder will slide

$= \sqrt{20^2 - 8^2} - \sqrt{20^2 - 12^2} = \sqrt{336} - 16 = 2.33 \text{ cm}$

25. The horizontal distance between P and Q = $112 + 14 + 14 = 140$

The vertical distance between P and Q = $16 + 16 + 16 = 48$

The shortest distance between P and Q = $\sqrt{140^2 + 48^2} = \sqrt{21904} = 148 \text{ (Pyth. Thm.)}$

26. $AC = \sqrt{AB^2 + BC^2} = \sqrt{7^2 + 24^2} = 25$

Area of $\triangle ABC = \frac{1}{2}(AC)(BM) = \frac{1}{2}(AB)(BC)$, $25BM = (7)(24)$, $BM = 6.72 \text{ cm}$

27. Let $BC = x \text{ cm}$, $\therefore CD = (24 - x) \text{ cm}$.

$AC^2 = 16^2 + x^2$, and $CE^2 = 12^2 + (24 - x)^2$

$\therefore AC = CE$, $\therefore 16^2 + x^2 = 12^2 + (24 - x)^2 = 12^2 + 24^2 - 48x + x^2$

$48x = 12^2 + 24^2 - 16^2$, $x = \frac{464}{48} = 9\frac{2}{3}$, $\therefore BC = 9\frac{2}{3} \text{ cm}$

28. Join QS.

$QS = \sqrt{8^2 + 6^2} = \sqrt{100} = 10 \text{ cm (Pyth. Thm.)}$

In $\triangle QSR$, $QS^2 + SR^2 = 10^2 + 24^2 = 100 + 576 = 676$

$QR^2 = 26^2 = 676$, $\therefore QS^2 + SR^2 = QR^2$,

$\therefore \angle QSR = 90^\circ$ (converse of Pyth. Thm.)

$\therefore \text{Area of PQRS} = \frac{1}{2}(PQ)(PS) + \frac{1}{2}(QS)(SR) = \frac{1}{2}(6)(8) + \frac{1}{2}(10)(24) = 144 \text{ cm}^2$

29. $AB = 72 \times \frac{6}{60} = 7.2 \text{ km}$, $DE = BC = 72 \times \frac{10}{60} = 12 \text{ km}$,

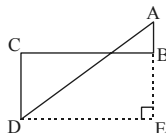
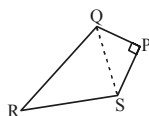
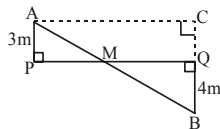
$BE = CD = 8 \text{ km}$. $\therefore AE = 7.2 + 8 = 15.2 \text{ km}$

$AD = \sqrt{DE^2 + AE^2} = \sqrt{12^2 + 15.2^2} = 19.4 \text{ km}$

Ans. He is 19.4 km from the starting point.

30. $DQ = \sqrt{24^2 + 18^2} = \sqrt{900} = 30 \text{ cm}$.

Let $h \text{ cm}$ be the distance between PB and DQ.



$$\text{Area of ABCD} = 2 \times \frac{1}{2}(24)(18) + (DQ)(h) = 24^2$$

$$432 + 30h = 576, \quad h = \frac{144}{30} = 4.8. \quad \text{Ans. The distance between PB and DQ is 4.8 cm.}$$

$$31. \text{ In } \triangle ABC, \quad AB^2 + BC^2 = 24^2 + 10^2 = 676, \quad AC^2 = 26^2 = 676$$

$$\therefore AC^2 + BC^2 = AB^2, \quad \therefore \angle B = 90^\circ \text{ (Converse of Pyth. Thm.)}$$

$$\angle B = \angle Q = 90^\circ \text{ (proved), } AB = PQ \text{ and } BC = QR \text{ (given), } \therefore \triangle ABC \cong \triangle PQR \text{ (S.A.S.)}$$

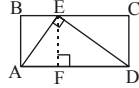
$$32. \quad ED^2 = \sqrt{25^2 - 7^2} = \sqrt{576} = 24.$$

Let EF be the height from E to AD.

$$\text{Area of } \triangle AED = \frac{1}{2}(AE)(ED) = \frac{1}{2}(AD)(EF)$$

$$\therefore 25EF = (7)(24), \quad EF = 6.72; \quad AB = EF = 6.72$$

$$\therefore \text{Area of ABCD} = (25)(6.72) = 168 \text{ sq. units}$$

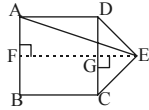


$$33. \text{ From E, draw } EF \perp AB. \quad DG = AF = \frac{1}{2}(12) = 6 \text{ cm.}$$

$$GE = \sqrt{DE^2 - DG^2} = \sqrt{10^2 - 6^2} = \sqrt{64} = 8 \text{ cm}$$

$$FG = BC = 12 \text{ cm; } FE = FG + GE = 12 + 8 = 20 \text{ cm}$$

$$\text{In } \triangle AFE, \quad AE = \sqrt{AF^2 + FE^2} = \sqrt{6^2 + 20^2} = \sqrt{436} = 20.9 \text{ cm}$$



$$34. \text{ (a) } DQ = 18 - 16 = 2 \text{ cm; } BQ = \sqrt{16^2 + 12^2} = 20 \text{ cm (Pyth. Thm.)}$$

$$\text{(b) In } \triangle DPQ, \quad PQ^2 = PD^2 + DQ^2 = x^2 + 2^2 = x^2 + 4$$

$$\text{In } \triangle PQB, \quad y^2 = PQ^2 + BQ^2 = (x^2 + 4) + 20^2 = x^2 + 404$$

$$\text{(c) In } \triangle APB, \quad AP = (12 - x) \text{ cm, } \therefore y^2 = AP^2 + 18^2 = (12 - x)^2 + 18^2$$

$$\therefore (12 - x)^2 + 18^2 = x^2 + 404,$$

$$144 - 24x + x^2 + 324 = x^2 + 404, \quad 64 = 24x, \quad \therefore x = \frac{8}{3}$$

$$\text{From (b), } y^2 = \left(\frac{8}{3}\right)^2 + 404 = \frac{3700}{9}, \quad y = \sqrt{\frac{3700}{9}} = \frac{10\sqrt{37}}{3}$$

$$35. \quad AC = EG = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ (Pyth. Thm.)}$$

$$\text{In } \triangle DEG \text{ and } \triangle ACG, \quad \angle DEG = \angle ACG = 90^\circ + 45^\circ = 135^\circ \text{ (square),}$$

$$\frac{DE}{AC} = \frac{1}{\sqrt{2}}, \quad \frac{EG}{CG} = \frac{\sqrt{2}}{1+1} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}, \quad \frac{DE}{AC} = \frac{EG}{CG},$$

$$\therefore \triangle DEG \sim \triangle ACG \text{ (ratio of 2 sides, inc. } \angle)$$

$$36. \text{ Let } QD = x \text{ cm. } QC = AQ = AD - QD = (16 - x) \text{ cm; } CD = AB = 10 \text{ cm}$$

$$\text{In } \triangle QCD, \quad QC^2 = QD^2 + CD^2,$$

$$(16 - x)^2 = x^2 + 10^2, \quad 256 - 32x + x^2 = x^2 + 100, \quad 156 = 32x, \quad x = 4.875, \quad \therefore QD = 4.875 \text{ cm}$$

$$37. \text{ Join BQ and CQ.}$$

$$BQ^2 = QP^2 + BP^2, \text{ and } BQ^2 = AB^2 + 3^2$$

$$\therefore AB^2 + 9 = QP^2 + BP^2 \dots\dots (i)$$

$$CQ^2 = QP^2 + PC^2 = QP^2 + BP^2, \text{ and } CQ^2 = 5^2 + 4^2 = 41$$

$$\therefore QP^2 + BP^2 = 41 \dots\dots (ii)$$

$$\text{From (i) and (ii), } AB^2 + 9 = 41, \quad AB^2 = 32, \quad AB = 5.66$$

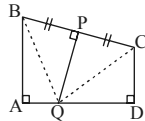
$$38. \quad DE^2 = EA^2 - AD^2, \text{ and } DE^2 = EC^2 - DC^2$$

$$\therefore EA^2 - AD^2 = EC^2 - DC^2, \quad EA^2 - EC^2 = AD^2 - DC^2 \dots\dots (i)$$

$$BD^2 = AB^2 - AD^2, \text{ and } BD^2 = BC^2 - DC^2$$

$$\therefore AB^2 - AD^2 = BC^2 - DC^2, \quad AB^2 - BC^2 = AD^2 - DC^2 \dots\dots (ii)$$

$$\text{From (i) and (ii), } EA^2 - EC^2 = AB^2 - BC^2$$



39. (a) $PR^2 + QR^2 = (m^2 - n^2)^2 + (2mn)^2 = m^4 - 2m^2n^2 + n^4 + 4m^2n^2 = m^4 + 2m^2n^2 + n^4$.
 $PR^2 = (m^2 + n^2)^2 = m^4 + 2m^2n^2 + n^4$
 $\therefore PR^2 + QR^2 = PQ^2$, $\therefore$ PQR is a right-angled triangle.

- (b) Let $24 = 2mn$.

The table below shows the possible values of m , n , $m^2 - n^2$ and $m^2 + n^2$.

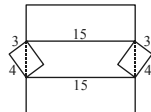
$m = 4, n = 3$	$m^2 - n^2 = 4^2 - 3^2 = 7$	$m^2 + n^2 = 4^2 + 3^2 = 25$
$m = 6, n = 2$	$m^2 - n^2 = 6^2 - 2^2 = 32$	$m^2 + n^2 = 6^2 + 2^2 = 40$
$m = 12, n = 1$	$m^2 - n^2 = 12^2 - 1^2 = 143$	$m^2 + n^2 = 12^2 + 1^2 = 145$

Ans. Three possible sets of values of the side:

24 cm, 7 cm, 25 cm; 24 cm, 32 cm, 40 cm; 24 cm, 143 cm, 145 cm.

40. The diagonal of the small rectangular board = $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$ cm

The new enclosed area = $15 \times 5 - 2 \times \left(\frac{1}{2} \times 3 \times 4\right) = 75 - 12 = 63$ cm²



41. (a) The shortest distance = AQ. $AP^2 = 10^2 + 20^2 = 500$

$$AQ = \sqrt{QP^2 + AP^2} = \sqrt{10^2 + 500} = \sqrt{600} = 24.5 \text{ cm}$$

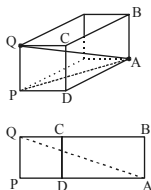
Ans. The shortest distance to fly from Q to A is 24.5 cm.

- (b) Imagine cutting the box along QP and BA so that QPDBCQ becomes a rectangle. The shortest distance from A to Q is the diagonal AQ in the rectangle.

$$AP = AD + DP = 20 + 10 = 30 \text{ cm},$$

$$AQ = \sqrt{10^2 + 30^2} = \sqrt{1000} = 31.6 \text{ cm}$$

Ans. The shortest distance to crawl from A to Q is 31.6 cm.



42. (a) $\frac{AB}{BE} = \frac{20}{15} = \frac{4}{3}$, $\frac{BC}{BD} = \frac{4}{3}$, $\angle ABC = \angle EBD$ (common $\angle$),

$\therefore \triangle ABC \sim \triangle EBD$ (ratios of two sides, inc. $\angle$ s)

- (b) (i) $\angle BDE = \angle BCA = 90^\circ$ (corr. $\angle$ s, $\sim \Delta$ s)

$$(ii) BD^2 = BE^2 - DE^2 \text{ (Pyth. Thm.)},$$

$$BD = \sqrt{15^2 - 9^2} = 12, \quad AD = AB - BD = 20 - 12 = 8$$

43. (a) $\angle PQT = \angle RST = 90^\circ$ and $PQ = RS$ (rectangle)
 $\angle PTQ = \angle RTS$ (vert. opp. $\angle$ s), $\therefore \triangle PQT \cong \triangle RST$ (AAS)

- (b) Let $ST = x$ cm. $QT = ST = x$ cm (corr. sides, $\cong \Delta$ s)

$$RT = QR - QT = (24 - x) \text{ cm}; \quad RS = PQ = 12 \text{ cm}$$

$$\text{In } \triangle RST, RT^2 = RS^2 + ST^2 \text{ (Pyth. Thm.)},$$

$$(24 - x)^2 = 12^2 + x^2, \quad 24^2 - 48x + x^2 = 12^2 + x^2, \quad 48x = 432, \quad x = 9, \quad \therefore ST = 9 \text{ cm}$$

44. (a) $PR^2 = PQ^2 + QR^2$ (Pyth. Thm.), $PR = \sqrt{16^2 + 30^2} = 34$ cm

- (b) (i) $PQ + QR + RS + PS = (78 + 2\sqrt{33})$ cm,

$$\therefore RS = (78 + 2\sqrt{33}) - 16 - 30 - 32 = 2\sqrt{33} \text{ cm}$$

$$\therefore PS^2 + RS^2 = [32^2 + (2\sqrt{33})^2] \text{ cm}^2 = 1156 \text{ cm}^2 = 34^2 \text{ cm}^2 = PR^2,$$

$\therefore \triangle PRS$ is a right-angled triangle. (Converse of Pyth. Thm.)

- (ii) Let N be a point on PR such that $SN \perp PR$.

$$\text{When T moves to N, area of } \triangle PRS = \frac{(PS)(RS)}{2} = \frac{(PR)(SN)}{2}$$

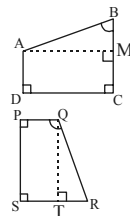
$$\therefore SN = \frac{(PS)(RS)}{PR} = \frac{(32)(2\sqrt{33})}{34} \text{ cm} = 10.8 \text{ cm (corr. to 3 sig fig.)}$$

$\therefore SN < 11$ cm, $\therefore$ the claim is disagreed.

45. (a) $CD = AB$. $CQ = CD - DQ = (15 - t)$ cm
 (b) $BQ = DQ = t$ cm, $BC = AD = 6$ cm
 In $\triangle BCQ$, $BQ^2 = BC^2 + CQ^2$,
 $t^2 = 6^2 + (15 - t)^2$, $t^2 = 36 + 15^2 - 30t + t^2$, $30t = 261$, $t = 8.7$
 (c) (i) In figure (b):
 $\angle BAP = 90^\circ = \angle BCQ$, and $BA = BC = 6$ cm (rectangle)
 $\angle ABQ = \angle PBC = 90^\circ$ (rectangle),
 i.e. $\angle ABP + \angle PBQ = \angle PBQ + \angle CBQ$,
 $\therefore \angle ABP = \angle CBQ$, $\therefore \triangle BAP \cong \triangle BCQ$ (ASA)
 (ii) $CQ = 15 - t = 15 - 8.7 = 6.3$ cm.
 $AP = CQ$ (corr. sides, $\cong \triangle$ s), $\therefore AP = 6.3$ cm

Unit 15 Introduction to trigonometric ratios

1. (a) $\theta = 70.3^\circ$ (b) $\cos \theta = \frac{2}{3}$, $\theta = 48.2^\circ$ (c) $\sin \theta = \frac{4}{5}$, $\theta = 53.1^\circ$
 2. (a) $\cos \theta = \frac{10}{12}$, $\theta = 33.6^\circ$ (b) $\sin x = \frac{21}{25}$, $x = 57.1^\circ$ (c) $\tan y = \frac{6}{17}$, $y = 19.4^\circ$
 3. (a) $a = 13 \tan 36^\circ = 9.45$ (b) $y = \frac{24}{\cos 55^\circ} = 41.8$
 (c) $k = \frac{37}{\tan 61^\circ} = 20.5$ (d) $x = 8 \sin 26^\circ = 3.51$
 4. (a) $\tan \theta = \frac{14}{20}$, $\theta = 35.0^\circ$; $x = \sqrt{20^2 + 14^2} = \sqrt{596} = 24.4$
 (b) $y = 12 \cos 56^\circ = 6.71$ cm; $\sin \theta = \frac{5}{y} = \frac{5}{6.71}$, $\theta = 48.2^\circ$
 (c) In $\triangle ABD$, $x = \frac{11}{\sin 27^\circ} = 24.2$.
 In $\triangle ABC$, $\frac{y}{x} = \tan 27^\circ$, $y = x \tan 27^\circ = 24.2 \tan 27^\circ = 12.3$
 (d) In $\triangle ABD$, $BD = 16 \sin(30^\circ + 20^\circ) = 16 \sin 50^\circ$; $AD = 16 \cos 50^\circ$
 In $\triangle ACD$, $CD = AD \tan 20^\circ = 16 \cos 50^\circ \tan 20^\circ$
 $x = BD - CD = 16 \sin 50^\circ - 16 \cos 50^\circ \tan 20^\circ = 8.51$
 (e) $x = 8 \tan 17^\circ$; $\tan(17^\circ + \theta) = \frac{2x}{8} = \frac{2 \times 8 \tan 17^\circ}{8} = 0.61146$
 $\therefore 17^\circ + \theta = 31.4^\circ$, $\theta = 14.4^\circ$
 (f) $a = 60 \sin 33^\circ = 32.7$ cm;
 $c = \frac{a}{\cos 44^\circ} = \frac{60 \sin 33^\circ}{\cos 44^\circ} = 45.4$ cm
 $b = a \tan 44^\circ = 60 \sin 33^\circ \tan 44^\circ \approx 31.557 \approx 31.6$ cm
 $PR = 60 \cos 33^\circ$; $d = PR - b = 60 \cos 33^\circ - 31.557 = 18.8$ cm
 5. (a) Draw $AM \perp BC$. In $\triangle ABM$, $a = \frac{BM}{\cos 50^\circ} = \frac{7-4}{\cos 50^\circ} = 4.67$
 $b = AM = BM \tan 50^\circ = 3 \tan 50^\circ = 3.58$
 (b) Draw $QT \perp SR$. $\cos \angle RQT = \frac{QT}{18} = \frac{15}{18}$, $\angle RQT = 33.6^\circ$
 $x = 90^\circ + \angle RQT = 90^\circ + 33.6^\circ = 123.6^\circ$



6. (a) $\cos 32^\circ = \frac{0.5y}{17}$, $y = \frac{17 \cos 32^\circ}{0.5} = 28.8$

(b) $\sin\left(\frac{50^\circ}{2}\right) = \frac{0.5x}{8}$, $x = \frac{8 \sin 25^\circ}{0.5} = 6.76$

(c) $\sin \frac{\theta}{2} = \frac{9 \div 2}{15} = \frac{3}{10}$, $\frac{\theta}{2} = 17.4576^\circ$, $\theta = 34.9^\circ$

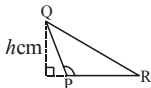
7. (a) $DE = 8 \sin 56^\circ$, $CE = 8 \cos 56^\circ$,

$$\text{area} = \frac{1}{2}(DE)(CE) = \frac{1}{2}(8 \sin 56^\circ)(8 \cos 56^\circ) = 14.8 \text{ cm}^2$$

(b) Let h cm be the height from Q to PR.

$$h = 5 \sin(180^\circ - 110^\circ) = 5 \sin 70^\circ$$

$$\text{Area} = \frac{1}{2}(\text{PR})(h) = \frac{1}{2}(7)(5 \sin 70^\circ) = 16.4 \text{ cm}^2$$

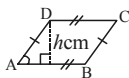


(c) Let h cm be the height from D to AB.

$$h = 10 \sin 60^\circ; \text{ area} = 12 \times (10 \sin 60^\circ) = 103.9 \text{ cm}^2$$

(d) $LM = LN = 7$ cm; $\angle L = 60^\circ$; the height $= 7 \sin 67^\circ$

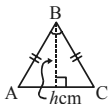
$$\text{Area} = \frac{1}{2}(7)(7 \sin 60^\circ) = 21.2 \text{ cm}^2$$



(e) Let h cm be the height from B to AC.

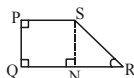
$$h = 10 \cos \frac{46^\circ}{2} = 10 \cos 23^\circ; \quad \frac{0.5AC}{10} = \sin \frac{46^\circ}{2}, \quad AC = 20 \sin 23^\circ$$

$$\text{Area} = \frac{1}{2}(AC)(h) = \frac{1}{2}(20 \sin 23^\circ)(10 \cos 23^\circ) = 36.0 \text{ cm}^2$$



(f) Draw $SN \perp QR$. $SN = PQ = 3$ cm; $\tan 58^\circ = \frac{SN}{NR}$, $NR = \frac{3}{\tan 58^\circ}$

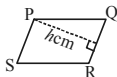
$$\therefore QR = 4 + \frac{3}{\tan 58^\circ} = 5.8746; \quad \text{area} = \frac{1}{2}(4 + 5.8746)(3) = 14.8 \text{ cm}^2$$



8. $h = 3.5 \tan 30^\circ$, $\text{area} = \frac{1}{2}(7)(3.5 \tan 30^\circ) = 7.07 \text{ cm}^2$

9. Let h cm be the height from P to QR. $h = 26 \sin \angle Q$,

$$\therefore 14 \times 26 \sin \angle Q = 160, \quad \sin \angle Q = \frac{160}{14 \times 26} = 0.43956, \quad \angle Q = 26.1^\circ$$



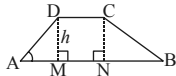
10. (a) $BD = 18 \tan 43^\circ$, $\text{area of } \triangle ABC = \frac{1}{2}(18 + 40)(18 \tan 43^\circ) = 486.8 \text{ sq. units}$

(b) $a = \frac{18}{\cos 43^\circ} = 24.6$; $b = \sqrt{BD^2 + 40^2} = \sqrt{(18 \tan 43^\circ)^2 + 40^2} = 43.4$

$$\tan \theta = \frac{40}{BD} = \frac{40}{18 \tan 43^\circ} = 2.383, \quad \therefore \theta = 67.2^\circ$$

11. (a) Let $h = DM = CN$. $h = 8 \sin 54^\circ = 13 \sin \angle B$,

$$\therefore \sin \angle B = \frac{8 \sin 54^\circ}{13} = 0.49786, \quad \angle B = 29.86^\circ$$



(b) $AM = 8 \cos 54^\circ$, $NB = 13 \cos 29.86^\circ$,

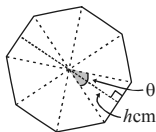
$$\therefore AB = 8 \cos 54^\circ + 5 + 13 \cos 29.86^\circ = 21.0 \text{ cm}$$

(c) $\text{Area of } ABCD = \frac{1}{2}(5 + 20.976)(8 \sin 54^\circ) = 84.1 \text{ cm}^2$

12. Let
- h
- cm be the height from the centre to a circle.

$$\theta = \frac{360^\circ}{8} = 45^\circ, \quad \frac{6 \div 2}{h} = \tan \frac{45^\circ}{2}, \quad h = \frac{3}{\tan 22.5^\circ}$$

$$\text{Area of the octagon} = 8 \times \frac{1}{2} (6) \left(\frac{3}{\tan 22.5^\circ} \right) = 173.8 \text{ cm}^2$$



13. Let the length of the rope be
- x
- cm.
- $\cos 72^\circ = \frac{4-2.5}{x}$
- ,
- $x = \frac{1.5}{\cos 72^\circ} = 4.85$

Ans. The rope is 4.85 m long.

- 14.
- $\sin \frac{\theta}{2} = \frac{8 \div 2}{10} = \frac{4}{10}$
- ,
- $\therefore \frac{\theta}{2} = 23.578^\circ$
- ,
- $\theta = 47.2^\circ$

15. Let
- $\angle POQ = \theta$
- .
- $\cos \frac{\theta}{2} = \frac{2.6}{3}$
- ,
- $\therefore \frac{\theta}{2} \approx 29.9264^\circ$
- ,
- $\theta = 60^\circ$
- (to the nearest degree)

16. Let
- h
- cm be the greatest height.

$$\cos 28^\circ = \frac{PM}{PC} = \frac{24-h}{24}, \quad h = 24 - 24 \cos 28^\circ = 2.81 \text{ m}$$



17. The vertical distance he has risen =
- $450 \sin 11^\circ + 200 \sin 34^\circ = 198 \text{ m}$

18. The distance between the two ends of the ropes =
- $\frac{19}{\tan 38^\circ} + \frac{19}{\tan 48^\circ} = 41.4 \text{ m}$

19. Let the distance the plane traveled be
- x
- km.

$$\sin 32^\circ = \frac{1.5 \times 1000}{x}, \quad x = \frac{1500}{\sin 32^\circ}$$

$$\text{The time taken} = x \div 900 = \frac{1500}{\sin 32^\circ} \div 900 = 3.15 \text{ minutes (2 d.p.)}$$

20. Let
- x
- m be the length of the post.
- $\frac{x}{3.8} = \tan 40^\circ$
- ,
- $x = 3.8 \tan 40^\circ$

Let y m be the length of the new shadow.

$$\frac{x}{y} = \tan 28^\circ, \quad y = \frac{x}{\tan 28^\circ} = \frac{3.8 \tan 40^\circ}{\tan 28^\circ} = 6.00 \text{ m}$$

Ans. The new length of the shadow is 6.00 m.

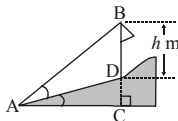
21. Let the length of the pole be
- h
- m.

$$DC = 4 \sin 15^\circ, \quad AC = 4 \cos 15^\circ.$$

$$\text{In } \triangle ABC, \quad BC = AC \tan (27^\circ + 15^\circ),$$

$$h + 4 \sin 15^\circ = 4 \cos 15^\circ \tan 42^\circ, \quad h = 2.44$$

Ans. The vertical pole is 2.44 m long.



22. (a)
- $DE = \frac{8}{\tan 30^\circ}$
- ,
- $\therefore \text{area of } \triangle CDE = \frac{1}{2} (8) \left(\frac{8}{\tan 30^\circ} \right) = 55.4 \text{ cm}^2$

$$(b) \quad DC = \frac{8}{\sin 30^\circ} = 16, \quad \therefore \text{area of } ABCD = 16^2 = 256 \text{ cm}^2$$

$$(c) \quad \angle ADF = 180^\circ - 30^\circ - 90^\circ = 60^\circ, \quad AD = DC = 16 \text{ cm},$$

$$\text{The vertical distance from A to DE} = AD \sin \angle ADF = 16 \sin 60^\circ = 13.9 \text{ cm}$$

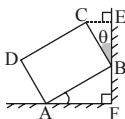
23. Draw
- $CE \perp EF$
- . In
- $\triangle ABF$
- ,
- $BF = AB \sin 30^\circ = 30 \sin 30^\circ$
- .

$$\angle ABE = 30^\circ + 90^\circ \text{ (ext. } \angle \text{ of } \triangle), \quad \therefore \theta + 90^\circ = 30^\circ + 90^\circ, \quad \theta = 30^\circ$$

$$\text{In } \triangle CBE, \quad BE = BC \cos \theta = 20 \cos 30^\circ$$

$$\text{The vertical distance from C to the horizontal}$$

$$= BF + BE = 30 \sin 30^\circ + 20 \cos 30^\circ = 3.23 \text{ cm}$$



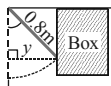
24. In
- $\triangle BEC$
- ,
- $\frac{6}{EC} = \tan \frac{50^\circ}{2}$
- ,
- $EC = \frac{6}{\tan 25^\circ}$

$$\text{The distance from the vertex to the bottom of the cylinder} = 15 - \frac{6}{\tan 25^\circ} = 2.13 \text{ cm}$$

25. Let y m be the original distance between the box and the closed door.

$$\frac{y}{0.8} = \sin 50^\circ, \quad y = 0.8 \sin 50^\circ = 0.613$$

$$x = 0.8 - y = 0.8 - 0.613 = 0.187$$



26. (a) $PR = 3 + 4 = 7$, $PQ = \sqrt{12^2 - 7^2} = \sqrt{95}$. $\therefore \cos \angle PQR = \frac{\sqrt{95}}{12}$, $\angle PQR = 35.7^\circ$

$$\tan \angle PQS = \frac{3}{\sqrt{95}}, \quad \angle PQS = 17.1^\circ, \quad \therefore n = 35.7^\circ - 17.1^\circ = 18.6^\circ$$

- (b) $QS = \sqrt{8^2 + 6^2} = 10$, $RQ = QS = 10$, $\therefore PR = 10 + 8 = 18$

$$\tan \angle R = \frac{6}{18}, \quad \angle R = 18.4^\circ, \quad \theta = \angle R = 18.4^\circ \text{ (base } \angle \text{s, isos. } \Delta \text{)}$$

- (c) $EF = \frac{5}{\tan 35^\circ}$, $CE = \sqrt{13^2 - 5^2} = 12$, $\therefore m = 12 - \frac{5}{\tan 35^\circ} = 4.86$

$$\sin \angle DCE = \frac{5}{13}, \quad \angle DCE = 22.6^\circ; \quad \therefore \theta = 35^\circ - 22.6^\circ = 12.4^\circ \text{ (ext. } \angle \text{ of } \Delta \text{)}$$

- (d) $\sin \theta = \frac{12}{13}$, $\theta = 67.4^\circ$, $\theta + \angle F + 90^\circ = 180^\circ$, $\therefore \angle D = \theta = 67.4^\circ$

$$\cos \angle D = \frac{7}{y + 12}, \quad \cos 67.4^\circ (y + 12) = 7, \quad y = \frac{7}{\cos 67.4^\circ} - 12 = 6.22$$

27. (a) $3\theta = 64.158^\circ$, $\theta = 21.4^\circ$

$$(b) \tan(\theta + 18^\circ) = 1.37374, \quad \theta + 18^\circ = 53.9^\circ, \quad \theta = 53.9^\circ - 18^\circ = 35.9^\circ$$

$$(c) \cos(2\theta - 15^\circ) = \frac{1}{5}, \quad 2\theta - 15^\circ = 78.462^\circ, \quad \theta = \frac{78.462^\circ + 15^\circ}{2} \approx 46.7^\circ$$

28. No. Because we don't know whether the triangle is right-angled.

29. (a) From the graph, $\sin 20^\circ = 0.35$; $\sin 38^\circ = 0.60$;

$$\sin 64^\circ = 0.90 \text{ (correct to the nearest 0.05)}$$

- (b) From the graph, $\cos 20^\circ = 0.95$; $\cos 38^\circ = 0.80$;

$$\cos 64^\circ = 0.45 \text{ (correct to the nearest 0.05)}$$

- (c) When $\theta = 45^\circ$; $\cos \theta = \sin \theta$.

When θ increases, $\cos \theta$ decreases but $\sin \theta$ increases.

$\therefore$ If $\cos \theta > \sin \theta$, $\theta < 45^\circ$. (OR: $0^\circ < \theta < 45^\circ$)

30. $\angle B = 180^\circ - 45^\circ - 90^\circ = 45^\circ$. $\therefore \triangle ABC$ is isosceles, and $CA = CB$.

$$\text{Let } CA = CB = 1 \text{ unit, } \therefore AB = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ units, } \frac{AC}{AB} = \frac{1}{\sqrt{2}}, \quad \therefore \cos 45^\circ = \frac{1}{\sqrt{2}}$$

31. $PQ = RQ$ (equil. Δ), $\angle QLP = \angle QLR = 90^\circ$ (given), $QL = QL$ (common),

$$\therefore \triangle QLP \cong \triangle QLR \text{ (R.H.S.)}, \quad \therefore PL = LR \text{ (corr. sides, } \cong \Delta \text{s)}$$

$$\text{Let } PQ = 2 \text{ units, } PL = \frac{1}{2} PR = \frac{1}{2}(2) = 1 \text{ unit, } \therefore QL = \sqrt{2^2 - 1^2} = \sqrt{3},$$

$$\angle P = 60^\circ \text{ (equil. } \Delta \text{)}, \quad \angle PQL = 180^\circ - 90^\circ - 60^\circ = 30^\circ,$$

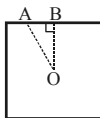
$$\tan 60^\circ = \frac{QL}{PL} = \frac{\sqrt{3}}{1} = \sqrt{3}, \quad \sin 30^\circ = \frac{PL}{PQ} = \frac{1}{2}$$

32. In $\triangle ABC$, $\sin \angle CBD = \frac{12}{20}$, $\angle CBD = 36.9^\circ$;

$$\angle DCB = \angle CBD \text{ (base } \angle \text{s, isos. } \Delta \text{)}, \quad \therefore \angle DCB = 36.9^\circ$$

$$33. \angle AOB = \frac{360^\circ}{12} = 30^\circ, \quad OB = \frac{15}{2} = 7.5 \text{ cm}$$

The distance between the two marks
 $= AB = OB \tan 30^\circ = 7.5 \tan 30^\circ = 4.33 \text{ cm}$



34. (a) BCED is a rectangle.

$$\angle ECG = 180^\circ - 90^\circ - 57^\circ = 33^\circ; \quad CE = BD = 28 \text{ cm.}$$

The vertical distance from E to the ground = EG

$$= 28 \sin 33^\circ = 15.2 \text{ cm}$$

- (b) $\angle ECG + \angle CGE = \angle CEF = \angle CED + \angle DEF$ (ext. $\angle$ of Δ)

$$\therefore 33^\circ + 90^\circ = 90^\circ + \angle DEF, \quad \angle DEF = 33^\circ; \quad EF = 40 \cos 33^\circ$$

Vertical distance from D to the ground = FE + EG

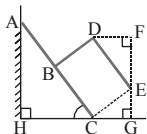
$$= 40 \cos 33^\circ + 28 \sin 33^\circ = 48.8 \text{ cm}$$

- (c) $AC = 50 + 40 = 90$, $\therefore HC = 90 \cos 57^\circ$

$$DF = 40 \sin 33^\circ; \quad CG = 28 \cos 33^\circ$$

Horizontal distance from D to the wall = $HC + CG - DF$

$$= 90 \cos 57^\circ + 28 \cos 33^\circ - 40 \sin 33^\circ = 50.7 \text{ cm}$$



$$35. (a) PS = 16 \text{ cm}, \quad \therefore PB = \frac{16}{\sin 30^\circ}.$$

$$QP = 20 \text{ cm}, \quad \angle APQ = 30^\circ, \quad \therefore AP = \frac{20}{\cos 30^\circ}$$

$$\text{The length of the rod} = AP + PB = \frac{20}{\cos 30^\circ} + \frac{16}{\sin 30^\circ} = 55.1 \text{ cm}$$

$$(b) SB = \frac{PS}{\tan 30^\circ} = \frac{16}{\tan 30^\circ}.$$

Let x cm be the new horizontal distance.

$$\therefore \text{the length of the rod} = 55.1 \text{ cm}, \quad \therefore x = 55.1 \cos 20^\circ$$

$$\text{The distance top B slides} = x - (PS + SB) = 55.1 \cos 20^\circ - (20 + \frac{16}{\tan 30^\circ}) = 4.06 \text{ cm}$$

36. Let r cm be the radius of the sphere.

$$\text{In } \triangle DAE, \quad DE = r, \quad \angle DAE = \frac{52^\circ}{2} = 26^\circ, \quad \therefore DA = \frac{r}{\sin 26^\circ}$$

$$BD + DA = 8, \quad \therefore r + \frac{r}{\sin 26^\circ} = 8, \quad r = 8 \div (1 + \frac{1}{\sin 26^\circ}) = 2.44$$

Ans. The radius of the sphere is 2.44 cm.

37. (a) In $\triangle PQS$, $PS = QS \tan 24^\circ = (3 + 5) \tan 24^\circ = 8 \tan 24^\circ$

$$\text{Area of } \triangle PQT = \frac{1}{2} (QT)(PS) = \frac{1}{2} (3)(8 \tan 24^\circ) = 5.34 \text{ cm}^2$$

- (b) In $\triangle TSR$, $SR = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$

$$\tan \angle QRS = \frac{3+5}{12} = \frac{2}{3}, \quad \angle QRS = 33.69^\circ$$

$$\sin \angle TRS = \frac{5}{13}, \quad \angle TRS = 22.62^\circ; \quad \angle QRT = \angle QRS - \angle TRS = 33.69^\circ - 22.62^\circ \approx 11.1^\circ$$

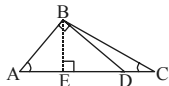
38. (a) In $\triangle ABD$, $BD = 12 \sin 48^\circ = 8.92$

- (b) Draw $BE \perp AC$. $\angle ADB = 180^\circ - 90^\circ - 48^\circ = 42^\circ$

$$\text{In } \triangle BED, \quad BE = BD \sin 42^\circ = (12 \sin 48^\circ)(\sin 42^\circ) = 5.967$$

$$\therefore EC = 12 \sin 48^\circ \cos 42^\circ + 4 = 10.627$$

$$\tan \theta = \frac{BE}{EC} = \frac{5.967}{10.627} = 0.561494, \quad \theta = 29.3^\circ$$



39. In $\triangle BAC$, $AC = x \cos \theta$. $\therefore \angle BCD + \angle ADC = 90^\circ + 90^\circ = 180^\circ$,
 $\therefore BC \parallel AD$ (int. $\angle$ s supp.). $\angle CAD = \angle ACB = \theta$ (alt. $\angle$ s, $BC \parallel AD$)

$$\text{In } \triangle ACD, \sin \angle CAD = \frac{CD}{AC}, \therefore \sin \theta = \frac{CD}{x \cos \theta}, \quad CD = x \cos \theta \sin \theta$$

40. $p^2 = q^2 + 12$, $\therefore p^2 - q^2 = 12$.

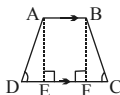
Draw AE and $BF \perp DC$. Let $h = AE = BF$.

$$DE = FC = \frac{p-q}{2}, \quad AE = DE \tan \theta = \left(\frac{p-q}{2}\right) \tan \theta$$

Area of the trapezium

$$= \frac{1}{2}(p+q)(AE) = \frac{1}{2}(p+q)\left(\frac{p-q}{2}\right) \tan \theta$$

$$= \frac{1}{4}(p^2 - q^2) \tan \theta = \frac{1}{4}(12) \tan \theta = 3 \tan \theta.$$



41. (a) $\angle F = \frac{180^\circ \times 4}{6} = 120^\circ$, $AF = EF = 6$ cm, $\frac{0.5AE}{AF} = \sin \frac{120^\circ}{2}$,

$$\therefore AE = 2(6) \sin 60^\circ = 10.4$$
 cm

- (b) $\angle BAF = \angle F = 120^\circ$, $\angle EAF = \frac{180^\circ - 120^\circ}{2} = 30^\circ$ (base $\angle$ s, isos. $\triangle$)

$$\therefore \angle BAE = \angle BAF - \angle EAF = 120^\circ - 30^\circ = 90^\circ$$

$\therefore ABE$ is a right-angled triangle.

- (c) Area of $\triangle ABE = \frac{1}{2}(AB)(AE) = \frac{1}{2}(6)(10.4) = 31.2$ cm²

42. (a) In $\triangle ABD$, $\angle ABD = 180^\circ - 90^\circ - \theta = 90^\circ - \theta$ ($\angle$ sum of $\triangle$)

In $\triangle ABC$, $\angle BCD = 180^\circ - 90^\circ - \theta = 90^\circ - \theta$ ($\angle$ sum of $\triangle$)

$$\therefore \angle ABD = \angle BCD;$$

$$\angle DBC = 90^\circ - \angle ABD = 90^\circ - (90^\circ - \theta) = \theta, \quad \therefore \angle A = \angle DBC;$$

$$\angle ADB = \angle BDC = 90^\circ, \quad \therefore \triangle ABD \sim \triangle BCD \text{ (AAA)}$$

- (b) $\frac{BD}{CD} = \frac{AD}{BD}$ (corr. sides, $\sim \triangle$ s), $\therefore \frac{BD}{27} = \frac{48}{BD}$, $BD = \sqrt{48 \times 27} = 36$

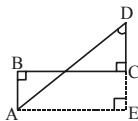
$$\text{In } \triangle ABD, \tan \theta = \frac{36}{48}, \quad \theta = 36.9^\circ$$

43. Produce DC to E , so that $AE \perp DE$.

$$AE = BC = 56, \quad CE = AB = 15.$$

$$DE = DC + CE = 18 + 15 = 33.$$

$$\text{In } \triangle ADE, \tan \theta = \frac{AE}{DE} = \frac{56}{33}, \quad \theta = 59.5^\circ$$



44. Let $y = AB = CD$. $\therefore DE = y \sin 70^\circ$, $BE = y \sin 40^\circ$, $DE - BE = 4$,

$$\therefore y \sin 70^\circ - y \sin 40^\circ = 4, \quad y = \frac{4}{\sin 70^\circ - \sin 40^\circ} = 13.47.$$

$$AE = y \cos 40^\circ, \quad CE = y \cos 70^\circ$$

$$x = AE - CE = y(\cos 40^\circ - \cos 70^\circ) = (13.47)(\cos 40^\circ - \cos 70^\circ) = 5.71$$

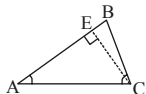
45. Let $PB = x$. $\therefore AP = 2x$; $AB = x + 2x = 3x$

$$\text{In } \triangle ABC, AC = AB \tan 45^\circ = 3x(1) = 3x.$$

$$\tan \angle APC = \frac{AC}{AP} = \frac{3x}{2x} = \frac{3}{2}, \quad \therefore \angle APC = 56.3^\circ$$

$$\angle CPQ + 56.3^\circ = 45^\circ + 90^\circ \text{ (ext. } \angle \text{ of } \triangle), \quad \therefore \angle CPQ = 78.7^\circ$$

46. (a) Let $x = PC = CB$. $\tan 56^\circ = \frac{2x}{AB}$, $\therefore \frac{x}{AB} = \frac{\tan 56^\circ}{2}$, but $\tan \theta = \frac{x}{AB}$,
 $\therefore \tan \theta = \frac{\tan 56^\circ}{2} = 0.74128$, $\theta \approx 36.549^\circ \approx 36.5^\circ$
- (b) $PC = 10 \sin \theta = 10 \sin 36.549^\circ = 5.955$, $\frac{2PC}{PA} = \sin 56^\circ$, $PA = \frac{2(5.955)}{\sin 56^\circ} = 14.4$ cm
47. (a) Let $a = AE = ED = DC$. $\tan 30^\circ = \frac{BC}{3a}$, $\tan \angle BDC = \frac{BC}{a}$
 $\therefore \tan 30^\circ = \frac{1}{3}(\tan \angle BDC)$, $\tan \angle BDC = 3 \tan 30^\circ = 1.7321$, $\angle BDC = 60^\circ$
- (b) $\angle DBA = 60^\circ - 30^\circ$ (ext. $\angle$ of Δ)
 $\therefore \angle BAD = \angle DBA = 30^\circ$, $\therefore DA = DB$ (sides opp. equal $\angle$ s)
48. (a) In ΔABD , let $\angle B = x$. $\angle BAD = \angle B = x$ (base $\angle$ s, isos. Δ)
 In ΔADC , let $\angle C = y$. $\angle CAD = \angle C = y$ (base $\angle$ s, isos. Δ)
 In ΔABC , let $\angle B + \angle BAD + \angle CAD + \angle C = 180^\circ$ ($\angle$ sum of Δ)
 $x + x + y + y = 180^\circ$, $x + y = 90^\circ$, $\angle BAC = x + y$, $\therefore \angle BAC = 90^\circ$
- (b) $\sin \angle C = \frac{12}{10+10} = \frac{12}{20}$, $\therefore \angle C = 36.9^\circ$
49. (a) $PQ^2 + QR^2 = 10.5^2 + 10^2 = 210.25$, $PR^2 = 14.5^2 = 210.25$
 $\therefore PQ^2 + QR^2 = PR^2$, $\therefore \angle Q = 90^\circ$ (converse of Pyth. Thm.)
 i.e. PQR is a right-angled triangle.
- (b) $\angle Q = 90^\circ$ (proved); $\cos \angle P = \frac{10.5}{14.5}$, $\angle P = 43.6^\circ$, $\angle R = 180^\circ - 90^\circ - 43.6^\circ = 46.4^\circ$
50. (a) $AB^2 + BC^2 = 13^2 + 15^2 = 394$, $AC^2 = 18^2 = 324$
 $\therefore AB^2 + BC^2 \neq AC^2$, $\therefore ABC$ is not a right-angled triangle.
- (b) $BP^2 = 13^2 - x^2$, and $BP^2 = 15^2 - PC^2 = 15^2 - (18 - x)^2$
 $\therefore 169 - x^2 = 225 - (18 - x)^2$,
 $169 - x^2 = 225 - 324 + 36x - x^2$, $36x = 169 + 324 - 225 = 268$, $x = \frac{268}{36} = 7.44$
 $\cos \angle A = \frac{x}{13} = \frac{268}{36} \cdot \frac{1}{13} = \frac{67}{117}$, $\therefore \angle A = 55.1^\circ$
51. (a) $\tan 41^\circ = \frac{h}{AD}$, $AD = \frac{h}{\tan 41^\circ}$, $\tan 29^\circ = \frac{h}{CD}$, $CD = \frac{h}{\tan 29^\circ}$
- (b) $AD + CD = 30$, $\frac{h}{\tan 41^\circ} + \frac{h}{\tan 29^\circ} = 30$, $h = 30 \div \left(\frac{1}{\tan 41^\circ} + \frac{1}{\tan 29^\circ} \right) = 10.2$ cm
52. Draw $CE \perp AB$.
 $\angle B = 180^\circ - 30^\circ - 70^\circ = 80^\circ$, $AE = x \cos 30^\circ$, $BE = y \cos 80^\circ$
 $AB = AE + EB = x \cos 30^\circ + y \cos 80^\circ$
53. (a) $PR = \frac{QR}{\tan 25^\circ} = 2.1445QR$, $SR = \frac{QR}{\tan 50^\circ} = 0.8391QR$
 $\therefore PS = 2.1445QR - 0.8391QR = 1.3054QR$
 $PS : SR = 1.3054QR : 0.8391QR = \frac{1.3054}{0.8391} : 1 = 1.556 : 1$, $\therefore n = 1.556$
- (b) Area of ΔPQS : Area of $\Delta QRS = \frac{1}{2}(PS)(QR) : \frac{1}{2}(SR)(QR) = PS : SR$
 $\therefore \text{area of } \Delta PQS = \frac{1.536}{1} \times 30 \approx 46.7$ cm²



54. (a) $\therefore$ Half of the water remained, $\therefore$ the cross-sectional area of the water is half of ABCD.
The water level is parallel to the horizontal, $\therefore \angle BAC = x^\circ$ (alt. $\angle$ s, $\parallel$ lines)
 $\tan \angle BAC = \frac{BC}{AB}$, $\tan x^\circ = \frac{50}{40}$, $x = 51.3$
- (b) $\therefore$ one-third of the water remained,
 $\therefore$ the cross-sectional area of the water $= \frac{1}{3}(40 \times 50) = \frac{2000}{3}$
 $\therefore \frac{1}{2}(BE)(40) = \frac{2000}{3}$, $BE = \frac{100}{3}$
The water level is parallel to the horizontal, $\therefore \angle EAB = y^\circ$ (alt. $\angle$ s, $\parallel$ lines),
 $\tan y^\circ = \frac{BE}{AB} = \frac{\left(\frac{100}{3}\right)}{40} = \frac{5}{6}$, $\therefore y = 39.8$
55. (a) (i) $\angle ANC = \angle BNC = 90^\circ$ (given), $CN = CN$ (common side), $AC = BC$ (given),
 $\therefore \triangle ACN \cong \triangle BCN$ (RHS), $\therefore AN = BN$ (corr. sides, $\cong \Delta$ s)
- (ii) $\therefore \triangle ACN \cong \triangle BCN$ (proved), $\therefore \angle ACN = \angle BCN$ (corr. $\angle$ s, $\cong \Delta$ s) $= \frac{2\theta}{2} = \theta$.
In $\triangle ACN$, $\frac{AN}{AC} = \sin \angle ACN$, $AN = \ell \sin \theta$ cm
 $AB = 2AN = 2\ell \sin \theta$ cm
- (b) (i) In $\triangle PQR$, $\sin 50^\circ = \frac{QR}{PR}$, $PR = \frac{20}{\sin 50^\circ}$ m ≈ 26.1 m (3 sig. fig.)
- (ii) $\angle RPS = 68^\circ - 50^\circ = 18^\circ$, $\frac{\angle RPS}{2} = \frac{18^\circ}{2} = 9^\circ$.
By (a), $RS = 2\left(\frac{20}{\sin 50^\circ}\right)(\sin 9^\circ) = 8.17$ m (3 sig. fig.)
56. (a) (i) $AC^2 = AB^2 + BC^2$ (Pyth. Theorem)
 $= (36^2 + 48^2) \text{ km}^2 = 3600 \text{ km}^2$
 $AC^2 + CE^2 = (3600 + 25^2) \text{ km}^2 = 4225 \text{ km}^2 = 65^2 \text{ km}^2 = AE^2$
 $\therefore \angle ACE$ is a right angle (Converse of Pyth. Theorem)
- (ii) $\angle DCE = 180^\circ - \angle ACE = 180^\circ - 90^\circ = 90^\circ$.
In $\triangle CDE$, $\tan 35^\circ = \frac{CD}{CE}$, $CD = 25 \tan 35^\circ \text{ km} = 17.5052 \text{ km} = 17.5 \text{ km}$ (3 sig. fig.)
- (iii) In $\triangle CDE$, $\cos 35^\circ = \frac{CE}{DE}$, $DE = \frac{25}{\cos 35^\circ} = 30.519 \text{ km} = 30.5 \text{ km}$ (3 sig. fig.)
- (b) Time for Morris' journey $= \frac{36 + 48 + 17.5052}{90} \text{ h} = 1.1278 \text{ h}$ (4 d.p.)
Time for Nick's journey $= (65 + 30.519) \div 84 \text{ h} = 1.1371 \text{ h}$ (4 d.p.)
 $\therefore 1.1278 < 1.1371$, $\therefore$ Morris arrives at city D first. *Thus, the claim is disagreed.*
57. (a) Let N be a point on PQ such that $ON \perp PQ$.
Note that $\triangle OPN \cong \triangle OQN$,
 $\therefore PN = QN$ (corr. sides, $\cong \Delta$ s), $PN = \frac{PQ}{2} = \frac{10}{2} \text{ cm} = 5 \text{ cm}$.
 $ON^2 = OP^2 - PN^2$ (Pyth. Theorem)
 $ON = \sqrt{13^2 - 5^2} = 12 \text{ cm}$ *Ans. The required vertical distance is 12 cm.*
- (b) (i) Radius of the ring $= \frac{18}{2} \text{ cm} = 9 \text{ cm}$

$$\frac{y-9}{9} = \cos(180^\circ - 140^\circ), \quad y = 9\cos 40^\circ + 9 = 15.9 \text{ (cor. to 3 sig. fig.)}$$

$$(ii) \text{ The required distance} = (y + 12 - 9) \text{ m} = (3 + 9\cos 40^\circ + 9) \text{ m} \\ = 18.9 \text{ m (3 sig. fig.)}$$

$$58. (a) \cos \angle PQR = \frac{QR}{PQ} = \frac{141}{235}, \quad \therefore \angle PQR = 53.13^\circ \text{ (4 sig. fig.)}$$

(b) (i) Let the required distance in Position (1) be h_1 cm.

$$\sin \angle PQR = \frac{h_1}{RQ}, \quad \therefore h_1 = 141 \sin 53.13^\circ = 112.8 \text{ (4 sig. fig.)}$$

Ans. The required distance in Position (1) is 112.8 cm.

Let N be a point on BC such that $RN \perp BC$.

$$\angle RNC = \angle PQC = 90^\circ$$

$$\therefore PQ \parallel RN \quad (\text{corr. } \angle\text{s equal})$$

$$\therefore \angle QRN = \angle PQR = 53.13^\circ \quad (\text{alt. } \angle\text{s, } \parallel \text{ lines})$$

$$\frac{RN}{QR} = \cos \angle QRN$$

$$\text{The required distance in Position (2)} = RN = 141 \cos 53.13^\circ \\ = 84.60 \text{ cm (4 sig. fig.)}$$

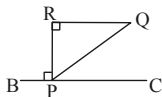
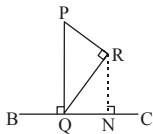
Ans. The required distance in Position (2) is 84.6 cm.

$$(ii) PR = \sqrt{PQ^2 - QR^2} \quad (\text{Pyth. Theorem})$$

$$= \sqrt{235^2 - 141^2} = 188 \text{ cm} > QR$$

Ans. The distance is greatest when the vertex P touches BC and $PR \perp BC$.

$$\text{The required distance} = PR = 188 \text{ cm.}$$



Unit 16 Areas (2): Arcs and sectors

$$1. (a) \text{ Circumferences} = 2\pi(15) = 94.2 \text{ cm, area} = \pi(15)^2 = 707.9 \text{ cm}^2$$

$$(b) \text{ Circumferences} = 2\pi\left(\frac{22}{2}\right) = 69.1 \text{ cm, area} = \pi\left(\frac{22}{2}\right)^2 = 380.1 \text{ cm}^2$$

$$2. (a) \text{ Radius} = \frac{100}{2\pi} = 15.9 \text{ cm} \quad (b) \text{ Radius} = \sqrt{\frac{84}{\pi}} = 5.17 \text{ cm}$$

$$3. (a) \text{ Perimeter} = 6 + 6 + 2 \times \frac{1}{2} \times \pi(6) = 30.8 \text{ cm. Area} = 6^2 - 2 \times \frac{1}{2} \times \pi\left(\frac{6}{2}\right)^2 = 7.73 \text{ cm}^2$$

$$(b) \text{ Perimeter} = 5 + \frac{1}{2} \times \pi(5 + 4) + \frac{1}{2} \times \pi(4) = 25.4 \text{ cm}$$

$$\text{Area} = \frac{1}{2} \times \pi\left(\frac{5+4}{2}\right)^2 - \frac{1}{2} \times \pi\left(\frac{4}{2}\right)^2 = 25.5 \text{ cm}^2$$

$$(c) \text{ Perimeter} = \pi(4 + 4 + 4) + 2 \times \frac{1}{2} \times \pi(4 + 4) + 2 \times \frac{1}{2} \times \pi(4) = 75.4 \text{ cm}$$

$$\text{Area} = \pi\left(\frac{4+4+4}{2}\right)^2 - 2 \times \frac{1}{2} \times \pi\left(\frac{4+4}{2}\right)^2 - 2 \times \frac{1}{2} \times \pi\left(\frac{4}{2}\right)^2 = 50.3 \text{ cm}^2$$

$$(d) \text{ Perimeter} = \frac{1}{2} \times \pi(6 + 10) + \frac{1}{2} \times \pi(10) + \frac{1}{2} \times \pi(6) = 50.3 \text{ cm}$$

$$\text{Area} = \frac{1}{2} \times \pi\left(\frac{6+10}{2}\right)^2 + \frac{1}{2} \times \pi\left(\frac{10}{2}\right)^2 - \frac{1}{2} \times \pi\left(\frac{6}{2}\right)^2 = 125.7 \text{ cm}^2$$

4. (a) The distance travelled = $\pi (24)(6) = 452$ m
 (b) The no. of revolutions = $\frac{1200}{24\pi} = 16$ (to the nearest integer)
5. (a) Radius = $95 \div 5 \div 2\pi = 3.02$ cm
 (b) $\frac{\text{no. of revolutions}}{5} = \frac{4.75 \text{ m}}{95 \text{ cm}}$, $\therefore$ no. of revolutions = $\frac{475}{95} \times 5 = 25$
6. Let n be the no. of revolutions of the rear wheel. $\pi (24) (36) = \pi (32) n$, $n = \frac{(24)(36)}{32} = 27$.

Ans. The no. of revolutions of the rear wheel is 27.

7. Let the radius of the small circle be r cm.
 The difference = $2\pi (r + 5) - 2\pi r = 2\pi (5) = 10\pi$ cm
8. Area of the path = $\pi(\frac{30}{2} + 2)^2 - \pi(\frac{30}{2})^2 = 64\pi \text{ m}^2$
9. Diameter of the small circle = $16 \div 2 = 8$ cm. Area = $\pi(\frac{16}{2})^2 - \pi(\frac{8}{2})^2 = 48\pi \text{ cm}^2$
10. Radius = $\frac{40}{2\pi} = 6.366 = 6.37$ cm. Area = $\pi (6.366)^2 = 127.3 \text{ cm}^2$
11. Radius = $\sqrt{\frac{98}{\pi}} = 5.59$ cm. Circumference = $2\pi (5.59) = 35.1$ cm
12. Radius = $\frac{6.28}{2\pi}$ m, $\therefore$ area = $\pi(\frac{6.28}{2\pi})^2 = \frac{6.28^2}{34\pi} = 3.14 \text{ m}^2$
13. (a) Arc length = $2\pi(6)(\frac{40}{360}) = 4.19$ cm. Area of sector = $\pi(6)^2(\frac{40}{360}) = 12.6 \text{ cm}^2$
 (b) Angle = $\frac{70}{2\pi(14)} \times 360 = 286.5^\circ$. Area of sector = $\pi(14)^2(\frac{286.5}{360}) = 490 \text{ cm}^2$
 (c) Angle = $\frac{100\pi}{\pi 5^2} \times 360 = 1440^\circ$. Arc length = $2\pi(5)(\frac{1440}{360}) = 40\pi$ cm
 (d) Radius = $\frac{40}{2\pi} \div \frac{120}{360} = 19.1$ cm. Area of sector = $\pi(19.1)^2(\frac{120}{360}) = 382.0 \text{ cm}^2$
 (e) Radius = $\sqrt{\frac{85}{\pi} \div \frac{200}{360}} = 6.98$ cm. Arc length = $2\pi(6.98)(\frac{200}{360}) = 24.4$ cm
14. Angle = $\frac{\pi}{2\pi(5)} \times 360 = 36^\circ$ 15. Angle = $38.5 \div [\frac{22}{7} (7)^2] \times 360 = 90^\circ$
16. Distance = $2\pi(7)(\frac{35}{60}) = 25.7$ cm 17. Area = $(15)(7.5) - 2 \cdot \pi(7.5)^2(\frac{90}{360}) = 24.1 \text{ cm}^2$
18. (a) Area = $(7)(7) - \pi(7)^2(\frac{90}{360}) = 10.5 \text{ cm}^2$
 (b) Area = $(2+2)(2+2) - 4 \cdot \pi(2)^2(\frac{90}{360}) = 3.43 \text{ cm}^2$
 (c) Area = $\pi(5)^2(\frac{90}{360}) - \frac{1}{2}(5)(5) = 7.13 \text{ cm}^2$
 (d) Area = $2 \times [\pi(3)^2(\frac{90}{360}) - \frac{1}{2}(3)(3)] = 5.14$ sq. units
19. (a) Perimeter = $2(\frac{22}{7})(\frac{14}{2})(\frac{270}{360}) + 2 \cdot (4 + \frac{6}{2} + 6) = 33 + 26 = 59$ units
 Area = $[(\frac{22}{7})(\frac{14}{2})^2 - (6)(6)](\frac{270}{360}) = (154 - 36) \frac{3}{4} = 88.5$ sq. units

$$(b) \text{ Perimeter} = 2 \times \left[2 \left(\frac{22}{7} \right) \left(\frac{28}{2} \right) \left(\frac{270}{360} \right) + 2 \cdot \left(\frac{28}{2} \right) \right] = 2(66 + 28) = 188 \text{ units}$$

$$\text{Area} = 2 \times \left(\frac{22}{7} \right) \left(\frac{28}{2} \right)^2 \left(\frac{270}{360} \right) = 924 \text{ sq. units}$$

$$20. \text{ Shaded area} = \pi[(3+2)^2 - 3^2] \times \frac{60^\circ}{360^\circ} = 8.38 \text{ cm}^2$$

$$21. \text{ Perimeter} = 2\pi(4) \times \frac{300^\circ}{360^\circ} + 2\pi(10+4) \times \frac{360^\circ - 300^\circ}{360^\circ} + 10 \times 2 = 55.6 \text{ cm}$$

$$\text{Area} = \pi(4)^2 \times \frac{300^\circ}{360^\circ} + \pi(10+4)^2 \times \frac{360^\circ - 300^\circ}{360^\circ} = 145 \text{ cm}^2$$

$$22. \quad 2\pi r \times \frac{120^\circ}{360^\circ} + 2r = 36.84, \quad 3.14r \times \frac{1}{3} + r = 18.42$$

$$3.14r + 3r = 55.26, \quad 6.14r = 55.26, \quad \therefore r = 9$$

$$23. (a) \text{ Let } r \text{ cm be the radius. } 2\pi r \times \frac{1}{2} + 2r = 514, \quad 3.14r + 2r = 514, \quad 5.14r = 514,$$

$$r = 100 \quad \text{Ans. The radius of the semi-circle is } 100 \text{ cm.}$$

$$(b) \text{ Enclosed area} = \pi(100)^2 \times \frac{1}{2} = 3.14 \times 5000 = 15700 \text{ cm}^2$$

$$24. \text{ Shaded area} = \pi \left(\frac{20}{2} \right)^2 \times \frac{1}{4} - \frac{1}{2} \times \frac{20}{2} \times \frac{20}{2} = 28.5 \text{ cm}^2$$

$$25. \text{ The diagonal} = \sqrt{6^2 + 8^2} = 10 \text{ cm}, \quad \therefore \text{ radius} = \frac{10}{2} = 5 \text{ cm}$$

$$\therefore \text{ Area of shaded region} = \pi(5)^2 - 6 \times 8 = 30.5 \text{ cm}^2$$

$$26. \text{ Area of AED} = \frac{1}{2} \times 6 \times 6 - \pi(6)^2 \times \frac{45^\circ}{360^\circ} = 3.86 \text{ cm}^2. \quad \text{BD} = \sqrt{6^2 + 6^2} = \sqrt{72}$$

$$\therefore \text{ Perimeter of AED} = 2\pi(6) \times \frac{45^\circ}{360^\circ} + (\sqrt{72} - 6) + 6 = 13.2 \text{ cm}$$

$$27. (a) \text{ Perimeter} = 2\pi \left(\frac{25}{2} \right) + (150 - 25) \times 2 = 329 \text{ cm}$$

$$\text{Area} = \pi \left(\frac{25}{2} \right)^2 + (150 - 25) \times 25 = 3615.57 = 3620 \text{ m}^2$$

$$(b) \text{ Area of the path} = \pi \left(\frac{25}{2} + 1.5 \right)^2 + (150 - 25) \times (25 + 1.5 \times 2) - 3615.87$$

$$= \pi(14)^2 + 125 \times 28 - 3615.87 = 500 \text{ m}^2$$

$$28. \angle AOB = 180^\circ - 30^\circ \times 2 = 120^\circ, \quad \text{height of } \triangle OAB = 8 \times \sin 30^\circ = 4 \text{ cm}$$

$$AB = 8 \times \cos 30^\circ \times 2 = 13.8564$$

$$\therefore \text{ Area of shaded segment} = \pi(8)^2 \times \frac{120^\circ}{360^\circ} - 13.8564 \times 4 \times \frac{1}{2} = 39.3 \text{ cm}^2$$

$$29. AC = 12 \cos 40^\circ, \quad BC = 12 \sin 40^\circ$$

$$\therefore \text{ Area of shaded region} = \pi \left(\frac{12}{2} \right)^2 \times \frac{1}{2} - (12 \cos 40^\circ)(12 \sin 40^\circ) \times \frac{1}{2} = 21.1 \text{ cm}^2$$

$$30. \text{ Let } r \text{ be the original radius, then new radius} = r(1 - 40\%) = 0.6r$$

$$\therefore \text{ Percentage decrease in area} = \frac{\pi r^2 - \pi(0.6r)^2}{\pi r^2} \times 100\% = (1 - 0.36) \times 100\% = 64\%$$

$$31. \text{ Let } r_1 \text{ and } r_2 \text{ be the radii of the bigger and the smaller circles respectively.}$$

$$2\pi r_1 - 2\pi r_2 = 20, \quad 2\pi(r_1 - r_2) = 20, \quad r_1 - r_2 = \frac{10}{\pi}$$

Ans. The difference between the radii is $\frac{10}{\pi}$ m.

32. Time saved = $[2\pi(\frac{34}{2}) \times \frac{1}{2} - 29] \div 72 \times 60 = 20.3$ min

33. Total length = $2\pi(1) \times 3 - 2\pi(1) \times \frac{\angle P + \angle Q + \angle R}{360^\circ} = 6\pi - 2\pi \times \frac{180^\circ}{360^\circ} = 5\pi$ cm

34. $2\pi(4) \times \frac{\angle AOB}{360^\circ} = 2$, $\frac{\angle AOB}{360^\circ} = \frac{1}{4\pi}$

$\therefore$ Area of sector = $\pi(4)^2 \times \frac{\angle AOB}{360^\circ} = 16\pi \times \frac{1}{4\pi} = 4$ cm²

35. (a) $2\pi(18) \times \frac{\angle AOB}{360^\circ} = 2\pi(5)$, $\frac{\angle AOB}{20^\circ} = 5$, $\therefore \angle AOB = 100^\circ$

(b) $\angle AOM = 100^\circ \div 2 = 50^\circ$, $\therefore$ distance = $18 \sin 50^\circ \times 2 = 27.6$ cm

36. Let r cm be the radius. $\pi r^2 \times \frac{2}{2+5} = 77$, $\frac{22}{7} \times r^2 \times \frac{2}{7} = 77$,

$r = 77 \times \frac{49}{44} = \frac{343}{4}$, $r = \sqrt{\frac{343}{4}} = 9.26$. Ans. The radius of the circle is 9.26 cm.

37. Let r cm be the radius, then $BD = AE = 2r$ cm and $AB = ED = 2r(\frac{3}{2}) = 3r$ cm.

$3r + 3r + 2r + 2\pi r \times \frac{1}{2} = 3\pi + 24$, $8r + \pi r = 3\pi + 24$, $r(\pi + 8) = 3(\pi + 8)$, $r = 3$

$\therefore$ Area = $\pi(3)^2 \times \frac{1}{2} + (2 \times 3)(3 \times 3) = 4.5\pi + 54 = 68.1$ cm²

38. $QM = \sqrt{60^2 - (16 - 12)^2} = \sqrt{3584}$. $\cos x = \frac{4}{60}$, $x = 86.2^\circ$

$\therefore$ Reflex $\angle APC = 360^\circ - 2(86.2^\circ) = 187.6^\circ$. $y = 90^\circ - x = 3.8^\circ$

$\therefore \angle BQD = 360^\circ - 2(90^\circ) - 2(3.8^\circ) = 172.4^\circ$

$\therefore$ Length of belt = $2\pi(16) \times \frac{187.6^\circ}{360^\circ} + 2\pi(12) \times \frac{172.4^\circ}{360^\circ} + \sqrt{3584} \times 2 = 208$ cm

39. (a) Height of $\triangle ABC = 2 \sin 60^\circ$ cm

$\therefore$ Area of $\triangle ABC = \frac{1}{2} \times 2 \times 2 \sin 60^\circ = 1.732$ cm²

(b) Shaded area = $\pi(2)^2 \times \frac{60^\circ}{360^\circ} \times 3 - 1.732 \times 2 = 2.82$ cm²

40. (a) Let $AO_1 = AO_2 = O_1O_2 = r$ cm.

$r \sin 60^\circ \times 2 = \sqrt{24}$, $\therefore r \approx 2.82843 \approx 2.83$

(b) $\angle AO_1B = 60^\circ \times 2 = 120^\circ$.

Area of segment AO_2B

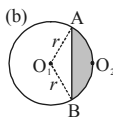
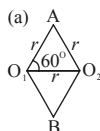
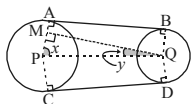
= $\pi(2.82843)^2 \times \frac{120^\circ}{360^\circ} - \frac{1}{2} \times \sqrt{24} \times \frac{2.82843}{2} = 4.9135$ (5 sig. fig.)

$\therefore$ Area of shaded region = $4.9135 \times 2 = 9.83$ cm² (3 sig. fig.)

41. (a) $\therefore EA = AB = EB$, $\therefore \triangle ABE$ is an equilateral triangle and $\angle EAB = 60^\circ$,

$\therefore \angle EAD = 90^\circ - 60^\circ = 30^\circ$

(b) $\widehat{DE} : \widehat{EB} = 2\pi S \times \frac{30^\circ}{360^\circ} : 2\pi S \times \frac{60^\circ}{360^\circ} = 1 : 2$



42. Height of $\triangle ADB = 20 \sin 45^\circ = 20\left(\frac{\sqrt{2}}{2}\right) = 10\sqrt{2}$ cm,

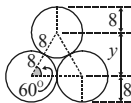
$$\therefore \text{Area of shaded region} = \left[\pi(20)^2 \times \frac{45^\circ}{360^\circ} - \frac{1}{2} \times 20 \times 10\sqrt{2} \right] \times 2 = 31.3 \text{ cm}^2$$

43. $\therefore OC = OD = 10$ cm and $\angle COE = 45^\circ$, $\therefore OE = 10 \cos 45^\circ$

$$\therefore \text{Area of OACE} = (10 \cos 45^\circ)^2 = 50 \text{ cm}^2$$

44. $y = (8+8) \sin 60^\circ = 16 \sin 60^\circ$ cm

$$\therefore \text{Height of the stack} = 8 + 16 \sin 60^\circ + 8 = 29.9$$

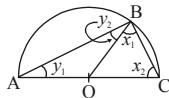


45. (a) $\therefore OA = OB = OC$ (radii),

$$\therefore x_1 = x_2 \text{ and } y_1 = y_2 \text{ (base } \angle\text{s, isos. } \triangle)$$

$$x_1 + x_2 + y_1 + y_2 = 180^\circ \text{ (}\angle \text{sum of } \triangle),$$

$$2x_1 + 2y_1 = 180^\circ, x_1 + y_1 = 90^\circ, \therefore \angle ABC = 90^\circ$$



(b) $AC = \sqrt{m^2 + n^2}$ (Pyth. Thm.), $OC = \frac{1}{2}\sqrt{m^2 + n^2}$

$$\therefore \text{Area of semi-circle} = \pi\left(\frac{1}{2}\sqrt{m^2 + n^2}\right)^2 \times \frac{1}{2} = \frac{\pi(m^2 + n^2)}{8}$$

46. $OB = 10$ cm. Let $OA = y$ cm and $AB = 2y$ cm.

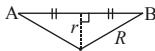
$$y^2 + (2y)^2 = 10^2, 5y^2 = 100, y^2 = 20, y = \sqrt{20}$$

$$\therefore \text{Area of the square} = (\sqrt{20} \times 2)^2 = 80 \text{ cm}^2$$

47. Let the radius of the smaller circle and the larger circle be r cm and R cm respectively.

$$\text{Area of the ring} = \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$$

But $R^2 = r^2 + \left(\frac{20}{2}\right)^2$ (Pythagoras' Thm.), $\therefore R^2 - r^2 = 100$



$$\therefore \text{Area of the ring} = \pi(100) = 100\pi \text{ cm}^2$$

48. $AB = \sqrt{3^2 + 3^2} = \sqrt{18}$.

$$\therefore \text{Area of the crescent} = \text{area of } \triangle OAB + \text{area of semi-circle} - \text{area of sector OAB}$$

$$= \frac{1}{2} \times 3 \times 3 + \frac{1}{2} \times \pi \left(\frac{3\sqrt{2}}{2}\right)^2 - \pi(3)^2 \times \frac{90^\circ}{360^\circ} = 4.5 + 2.25\pi - 2.25\pi = 4.5 \text{ cm}^2$$

49. $AB^2 + BC^2 = AC^2$ (Pyth. Thm.)

$$\therefore \text{Area of the shaded region} = \frac{1}{2} \pi \left(\frac{AB}{2}\right)^2 + \frac{1}{2} \pi \left(\frac{BC}{2}\right)^2 + 25 - \frac{1}{2} \pi \left(\frac{AC}{2}\right)^2$$

$$= \frac{1}{8} \pi (AB^2 + BC^2) + 25 - \frac{1}{8} \pi AC^2 = \frac{1}{8} \pi AC^2 + 25 - \frac{1}{8} \pi AC^2 = 25 \text{ cm}^2$$

50. Let $AB = CD = y$. Arc length on $AB = 2\pi\left(\frac{y}{2}\right) \times \frac{1}{2} = \frac{\pi y}{2}$

$$\text{Sum of arc lengths on } CD = 2\pi\left(\frac{y}{2n}\right) \times \frac{1}{2} \times n = \frac{\pi y}{2}$$

Ans. They are the same.

51. (a) Area of the sector $= \pi(15)^2 \left(\frac{72^\circ}{360^\circ}\right) = 45\pi \text{ cm}^2$

(b) Let r cm be the radius of the semi-circle. $\frac{\pi r^2}{2} = 45\pi$, $r^2 = 90$, $r = \sqrt{90}$

Length of wire A

= perimeter of the sector

$$= \left[2\pi(15) \left(\frac{72^\circ}{360^\circ} \right) + 2(15) \right] \text{ cm} = (6\pi + 30) \text{ cm} = 48.8496 \text{ cm (cor. to 4 d. p.)},$$

Length of wire B

= perimeter of the semi-circle

$$= \left(\frac{2\pi r}{2} + 2r \right) \text{ cm} = (\pi\sqrt{90} + 2\sqrt{90}) \text{ cm} = 48.7774 \text{ cm (4 d. p.)}$$

$\therefore 48.8496 \text{ cm} > 48.7774 \text{ cm}$, $\therefore$ Wire A is longer.

52. (a) (i) $\therefore OP = OQ$, $\therefore \angle OQP = \angle OPQ = 15^\circ$ (base $\angle$ s, isos. Δ),
 $\angle QON = \angle OQP + \angle OPQ = 15^\circ + 15^\circ = 30^\circ$ (ext. $\angle$ of Δ)

(ii) In ΔQON , $\frac{QN}{OQ} = \sin 30^\circ$, $QN = r \sin 30^\circ \text{ cm} = \frac{r}{2} \text{ cm}$

$$\text{Area of } \Delta POQ = \frac{1}{2}(OP)(QN) = \frac{1}{2}(r) \left(\frac{r}{2} \right) \text{ cm}^2 = \frac{r^2}{4} \text{ cm}^2$$

- (b) $\angle POQ = 180^\circ - 15^\circ - 15^\circ = 150^\circ$ ($\angle$ sum of Δ)

$$\text{reflex } \angle POQ = 360^\circ - 150^\circ = 210^\circ$$

$$\text{Area of segment PQR} = \frac{r^2}{4} + \pi r^2 \left(\frac{210^\circ}{360^\circ} \right) = \left(\frac{1}{4} + \frac{7\pi}{12} \right) r^2$$

$$\therefore \left(\frac{1}{4} + \frac{7\pi}{12} \right) r^2 = 1200, \quad r^2 = 1200 \div \left(\frac{1}{4} + \frac{7\pi}{12} \right), \quad r = 24.0 \text{ (3 sig. fig.)}$$

- (c) In ΔQON , $\frac{QN}{PQ} = \sin \angle OPQ$, $PQ = \frac{r}{2} \div \sin 15^\circ \text{ cm} = \frac{r}{2 \sin 15^\circ} \text{ cm}$

Perimeter of the segment PQR

$$= \left[2\pi r \left(\frac{210^\circ}{360^\circ} \right) + \frac{r}{2 \sin 15^\circ} \right] \text{ cm} = \left(\frac{7}{6}\pi r + \frac{r}{2 \sin 15^\circ} \right) \text{ cm} = 134 \text{ cm (3 sig. fig.)}$$

53. (a) (i) $\therefore OA = OB = OC = OD = OE$ (radii)
 and $AB = BC = CD = DE = AE$ (regular polygon),
 $\therefore \Delta AOB, \Delta BOC, \Delta COD, \Delta DOE$ and ΔAOE are congruent to each other. (SSS)
 (ii) $\angle COD = \angle AOB = \angle BOC = \angle DOE = \angle AOE$ (corr. $\angle$ s, $\cong \Delta$ s)
 $\therefore \angle COD = \frac{360^\circ}{5} = 72^\circ$

- (b) Let N be a point on CD such that $ON \perp CD$. Note that $\Delta ONC \cong \Delta OND$.

$$\therefore CN = DN \text{ (corr. sides, } \cong \Delta \text{s)} = \frac{12}{2} \text{ cm} = 6 \text{ cm},$$

$$\text{and } \angle CON = \angle DON \text{ (corr. } \angle \text{s, } \cong \Delta \text{s)} = \frac{72^\circ}{2} = 36^\circ.$$

$$\text{In } \Delta CON, \quad \frac{6}{r} = \sin 36^\circ, \quad r = \frac{6}{\sin 36^\circ} = 10.2 \text{ (3 sig. fig.)}$$

Ans. The radius of the circle is 10.2 cm.

- (c) In ΔCON , $\tan \angle CON = \frac{CN}{ON}$.

$$ON = \frac{6}{\tan 36^\circ} \text{ cm} = 8.25829 \text{ cm (5 d.p.)}$$

$$\text{Area of } \Delta COD = \frac{(CD)(ON)}{2} = \frac{(12)(8.25829)}{2} \text{ cm}^2 = 49.54974 \text{ cm}^2$$

$$\text{Area of shaded region} = \pi r^2 - 5(49.54974) = 79.6 \text{ cm}^2 \text{ (3 sig. fig.)}$$

54. (a) Area of $\Delta ABC = \frac{(AB)(BC)}{2} = \frac{(4)(4)}{2} = 8 \text{ cm}^2$

- (b) Area of semi-circle BCD = $\pi\left(\frac{BC}{2}\right)^2\left(\frac{1}{2}\right) = \pi\left(\frac{4}{2}\right)^2\left(\frac{1}{2}\right) = 2\pi \text{ cm}^2$
 $\angle BAC = \angle ACB$ (base $\angle$ s, isos. Δ) = $\frac{180^\circ - 90^\circ}{2}$ ($\angle$ sum of Δ) = 45° ,
 area of sector ABE = $\pi(4)^2\left(\frac{45^\circ}{360^\circ}\right) = 2\pi \text{ cm}^2$. $\therefore$ The claim is agreed.
- (c) Area of shaded region
 = area of semi-circle BCD – (area of ΔABC – area of sector ABE)
 = $[2\pi - (8 - 2\pi)] \text{ cm}^2 = (4\pi - 8) \text{ cm}^2$

Unit 17 Volumes (2): Cylinders

- Volume = $\pi\left(\frac{14}{2}\right)^2(0.4) = 61.6 \text{ cm}^3$
- Area = $2\pi(5)(12) + \pi(5)^2 = 145\pi \text{ cm}^2$
- Largest possible volume = $\pi\left(\frac{12}{2}\right)^2(12) = 432\pi \text{ cm}^3$
 Total surface area = $2\pi\left(\frac{12}{2}\right)(12) + 2\pi\left(\frac{12}{2}\right)^2 = 216\pi \text{ cm}^2$
- Let r cm be the base radius. $\pi r^2(10) = 471$, $r = \sqrt{\frac{471}{10\pi}} = 3.87$
Ans. The base radius is 3.87 cm.
- Let h cm be the height. $2\pi\left(\frac{14}{2}\right)h = 1056$, $h = 1056 \times \frac{1}{14} \times \frac{7}{22} = 24$
Ans. The height is 24 cm.
- Let r cm be the base radius. $2\pi r(4) = 704$, $r = 704 \times \frac{1}{8} \times \frac{7}{22} = 28$
Ans. The base radius is 28 cm.
- Diameter = 60 cm = 0.6 m, $\therefore$ the area covered = $8 \times \pi (0.6) (1.6) = 24.1 \text{ m}^2$
- (a) Volume = $\pi\left(\frac{35}{2}\right)^2(280) \times \frac{1}{2} = 134\,700 \text{ cm}^3 = 134.7 \text{ L}$ (4 sig. fig.)
Ans. The trough can hold 134.7 L of water.
 (b) Total internal surface area = $\pi(35)(28) \times \frac{1}{2} + 2 \times \frac{1}{2} \times \pi\left(\frac{35}{2}\right)^2 = 2\,501.5 \text{ cm}^2$
- Total surface area = $2\pi\left(\frac{18}{2}\right)(27) \times \frac{1}{2} + \pi\left(\frac{18}{2}\right)^2 + 6 \times 18 \times 2 + 6 \times 27 \times 2 + 18 \times 27 = 2\,040 \text{ cm}^2$.
 Volume = $\pi\left(\frac{18}{2}\right)^2(27) \times \frac{1}{2} + 6 \times 18 \times 27 = 6\,350 \text{ cm}^3$
- Total surface area = $2\pi(9) \times \frac{300}{360} \times 27 + \pi(9)^2 \times \frac{300}{360} \times 2 + 9 \times 27 \times 2 = 2\,180 \text{ cm}^2$ (3 sig. fig.)
- (a) External curved surface area = $2\pi(12)(50) = 1\,200\pi \text{ cm}^2$
 (b) Volume = $\pi(12^2 - 10^2)(50) = 2\,200\pi \text{ cm}^3$
- Let y cm be the length. $\pi\left[\left(\frac{4}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right] \times y = 30 \times 10 \times 6$, $3.75\pi \times y = 1\,800$,
 $y = \frac{480}{\pi} = 152.8$. *Ans. The length of the pipe is 152.8 cm.*

13. Internal radius = $\frac{9}{2} - 0.6 = 3.9$ cm, internal height = $10 - 0.6 = 9.4$ cm

$\therefore$ Internal surface area = $2\pi(3.9)(9.4) + \pi(3.9)^2 = 278.1$ cm²

14. Let h cm be the depth of water. $\pi(8)^2(h) = 250$, $h = \frac{125}{32\pi}$

$\therefore$ The required area = $2\pi(8)(\frac{125}{32\pi}) + \pi(8)^2 = 263.6$ cm²

15. Let h cm be the depth of water. $\pi(\frac{8}{2})^2(h) = \pi(\frac{4}{2})^2(12)$, $h = \frac{4 \times 12}{16} = 3$

Ans. The depth of water is 3 cm.

16. Let h cm be the rise in water level. $\pi(4)^2(h) = (1.5)^3 \times 8$, $h = \frac{27}{16\pi}$

$\therefore$ New water level = $6 + \frac{27}{16\pi} = 6.54$ cm

17. Let h cm be the rise in water level.

$\pi(\frac{16}{2})^2(h) = \pi(\frac{8}{2})^2(0.5) \times 50$, $64h = 400$, $h = 6.25$

Ans. The rise in water level is 6.25 cm.

18. Volume of water collected = $\pi(\frac{8}{2})^2 \times 6 \times 12 = 1152\pi$ cm³

19. (a) Total wet surface area = $2\pi(\frac{10}{2})(8) + \pi(\frac{10}{2})^2 = 330$ cm²

(b) Let h cm be the rise in water level. $\pi(\frac{10}{2})^2(h) = 2 \times 3 \times 6$, $h = \frac{36}{25\pi}$

$\therefore$ Original water level = $8 - \frac{36}{25\pi} = 7.54$ cm

20. Height of equilateral triangle = $4 \sin 60^\circ = 3.4641$ cm.

Let r cm be the base radius.

$\pi r^2(12) = \frac{4 \times 3.4641}{2} \times 18$, $\pi r^2 = 10.3923$, $r = \sqrt{\frac{10.3923}{\pi}} = 1.82$

Ans. The base radius is 1.82 cm.

21. Let r cm and h cm be the base radius and the height of original cylinder.

$\therefore 2\pi rh = 100$, $\therefore$ new curved surface area = $2\pi(2r)(2h) = 4(2\pi rh) = 4(100) = 400$ cm²

22. Original surface area = $2\pi(3r)(2r) + \pi(3r)^2 \times 2 = 30\pi r^2$

New surface area = $2\pi(3r)(2r) + 2\pi(r)(2r) + \pi[(3r)^2 - r^2] \times 2 = 32\pi r^2$

$\therefore$ Percentage change = $\frac{32\pi r^2 - 30\pi r^2}{30\pi r^2} \times 100\% = 6.67\%$

23. (a) The shortest possible length is the diagonal of the rectangle formed by flattening the semi-circular curved surface ABDC.

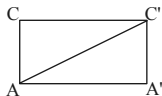
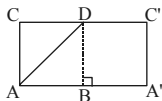
$\widehat{AB} = 2\pi(\frac{10}{2}) \times \frac{1}{2} = 5\pi$ cm

$\therefore$ Shortest possible length = $\sqrt{(5\pi)^2 + 12^2} = 19.8$ cm (Pyth. Thm.)

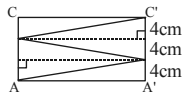
(b) The shortest possible length is the diagonal of the rectangle formed by flattening the curved surface of the cylinder.

Circumference = $2\pi(\frac{10}{2}) = 10\pi$

$\therefore$ Shortest possible length = $\sqrt{(10\pi)^2 + 12^2} = 33.6$ cm (Pyth. Thm.)



$$(c) \text{ Shortest length} = 3 \times \sqrt{(10\pi)^2 + \left(\frac{12}{3}\right)^2} = 95.0 \text{ cm (Pyth. Thm.)}$$



$$24. \text{ Time taken} = (15 \times 10 \times 5) \div (\pi \times 0.05^2 \times 10) \div 3600 = 2.65 = 2.65 \text{ hours}$$

$$25. \text{ Capacity of swimming pool} = \frac{(1.2 + 2.4) \times 50}{2} \times 18 = 1620 \text{ m}^3$$

$$\therefore \text{ Time taken} = 1620 \div \left[\pi \times \left(\frac{0.06}{2} \right)^2 \times 15 \right] \div 3600 = 10.6 \text{ hours}$$

$$26. \text{ When the base circumference is AB, volume} = \pi \left(\frac{AB}{2\pi} \right)^2 \cdot BC = \frac{AB^2 \cdot BC}{4\pi}$$

$$\text{When the base circumference is BC, volume} = \pi \left(\frac{BC}{2\pi} \right)^2 \cdot AB = \frac{BC^2 \cdot AB}{4\pi}$$

$$\text{Their ratio of volumes} = \frac{AB^2 \cdot BC}{4\pi} : \frac{BC^2 \cdot AB}{4\pi} = AB : BC = 3 : 2 \quad [\text{Or: } 2 : 3]$$

$$27. \text{ Let the original water level be } x \text{ cm. } \pi \times 9^2 \times x = \pi \times 9^2 \times 11 - \pi \times 4^2 \times 11, \\ 81x = 715, \quad x = 8.83. \quad \text{Ans. The water level was 8.83 cm.}$$

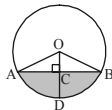
$$28. \text{ Let the rise of water level be } h \text{ cm.}$$

$$\pi(10^2)(14+h) = \pi(10^2)(14) + \pi(4^2-7^2)(14+h), \\ 1400 + 100h = 1400 + 33(14+h), \quad 100h = 462 + 33h, \quad 67h = 462, \quad h = 6.90 \\ \text{Ans. The minimum height of the cylindrical vessel} = 6.90 + 14 = 20.9 \text{ cm}$$

$$29. (a) \text{ OC} = 30 - 12 = 18 \text{ cm, OA} = \text{radius} = 30 \text{ cm,}$$

$$\therefore \text{ AC} = \sqrt{30^2 - 18^2} = 24 \text{ cm (Pyth. Thm.)}$$

$$\cos \angle AOD = \frac{OC}{OA} = \frac{18}{30}, \quad \angle AOD = 53.13^\circ$$



$$\text{Area of segment ADB} = \pi(30^2) \cdot \frac{53.13 \times 2}{360} - \frac{1}{2}(24)(18) \times 2 \approx 402.564 \text{ cm}^2$$

$$\text{Volume of water} = \frac{402.564 \times 200}{1000} \approx 80.5128 \text{ L} \approx 80.5 \text{ L}$$

$$(b) \text{ When the depth is 30 cm, the water will be in the form of a semi-cylinder.}$$

$$\therefore \text{ Water added} = \left(\frac{\pi \cdot 30^2}{2} \times 200 \right) \div 1000 - 80.5128 \text{ L} \approx 202.2 \text{ L}$$

$$30. (a) \text{ Base radius} = \frac{30}{2} \text{ cm} = 15 \text{ cm, base area} = \pi(15^2) \text{ cm}^2 = 225\pi \text{ cm}^2$$

$$(i) \text{ Volume of water} = 225\pi(8) \text{ cm}^3 = 1800\pi \text{ cm}^3$$

$$(ii) \text{ The required area} = [225\pi + 20\pi(8)] \text{ cm}^2 = 465\pi \text{ cm}^2$$

$$(b) (i) \text{ Rise in water level} = \frac{(2^3)(15)}{225\pi} = \frac{8}{15\pi} \text{ cm}$$

$$(ii) \text{ Increase in the area} = 30\pi \left(\frac{8}{15\pi} \right) \text{ cm}^2 = 16 \text{ cm}^2$$

$$31. (a) (i) \angle COD = 120^\circ - 90^\circ = 30^\circ, \quad \text{OC} = (45 - 21) \text{ cm} = 24 \text{ cm}$$

$$\text{Area of figure AODCB} = \left[\pi(45^2) \left(\frac{120^\circ}{360^\circ} \right) - \pi(24^2) \left(\frac{30^\circ}{360^\circ} \right) \right] \text{ cm}^2 = 627\pi \text{ cm}^2$$

$$(ii) \text{ OD} = \text{OC}, \quad \therefore \text{ Perimeter of figure AODCB}$$

$$= \left[2\pi(45) \left(\frac{120^\circ}{360^\circ} \right) + 45 \times 2 + 2\pi(24) \left(\frac{30^\circ}{360^\circ} \right) \right] \text{ cm} = (34\pi + 90) \text{ cm}$$

$$(b) (i) \text{ Volume of the prism} = (627\pi)(80) \text{ cm}^3 = 50160\pi \text{ cm}^3$$

$$(ii) \text{ Total surface area} = [(34\pi + 90)(80) + 627\pi(2)] \text{ cm}^2 = (3974\pi + 7200) \text{ cm}^2$$

32. (a) (i) Let h cm be the height of the cylinder.
 $\pi(4)^2(h) = 144\pi$, $h = 9$, $\therefore$ the height of the cylinder is 9 cm.
 (ii) Total surface area = $[2\pi(4)(9) + \pi(4)^2(2)] \text{ cm}^2 = 104\pi \text{ cm}^2$
- (b) (i) Length of arc PR = $2\pi(4)\frac{135^\circ}{360^\circ} = 3\pi \text{ cm}$
 (ii) Imagine that the curved surface PQR can be flattened, and when QR is shortest, PQR is a right-angled triangle in which PQ = 9 cm and PR = 3π cm.
 By Pyth. Theorem, the shortest length of the line
 $= \sqrt{(3\pi)^2 + 9^2} \text{ cm} = 13.0 \text{ cm}$ (3 sig. fig.)
33. (a) (i) $\angle BCF = 90^\circ$, $\angle FCG = \angle BCG - \angle BCF = 120^\circ - 90^\circ = 30^\circ$,
 $\angle BAC = \angle FCG = 30^\circ$ (corr. $\angle$ s, DF // AE).
 In $\triangle ABC$, $\frac{BC}{AB} = \tan \angle BAC$, $AB = \frac{\sqrt{3}}{\tan 30^\circ} = 3 \text{ cm}$
- (ii) Arc BFG = $2\pi(\sqrt{3})\left(\frac{120^\circ}{360^\circ}\right) \text{ cm} = \frac{2\sqrt{3}}{3}\pi \text{ cm}$
 $\frac{BC}{AC} = \sin \angle BAC$, $AC = \frac{\sqrt{3}}{\sin 30^\circ} = 2\sqrt{3} \text{ cm}$
 Perimeter = $\left(\frac{2\sqrt{3}}{3}\pi + 3 + 2\sqrt{3} + \sqrt{3}\right) \text{ cm} = \left(\frac{2\sqrt{3}}{3}\pi + 3 + 3\sqrt{3}\right) \text{ cm}$
- (b) Base area of the prism
 = area of $\triangle ABC$ + area of sector BFGC
 $= \frac{(3)(\sqrt{3})}{2} + \pi(\sqrt{3})^2\left(\frac{120^\circ}{360^\circ}\right) \text{ cm}^2 = \left(\frac{3\sqrt{3}}{2} + \pi\right) \text{ cm}^2$
 Volume of the prism = $\left(\frac{3\sqrt{3}}{2} + \pi\right)(2) \text{ cm}^3 = (3\sqrt{3} + 2\pi) \text{ cm}^3$
34. (a) Volume of water = $30 \times 12 \times 18 - \pi(6^2) \times 18 \text{ cm}^3 = 4444.2 \text{ cm}^3$ (1 d.p.)
 (b) (i) Let h cm be the new depth of water.
 $4444.2 + \pi(6^2)(h) = 30(18)(h)$, $h = \frac{444.2}{540 - 36\pi} = 10.4$ (1 d.p.)
Ans. The new water depth is 10.4 cm.
 (ii) Area not getting wet
 $= \pi(6^2) + 2\pi(6)(18 - h) = 399.2 \text{ cm}^2 < 400 \text{ cm}^2$
 $\therefore$ The claim is disagreed.

Unit 18 More about statistical charts

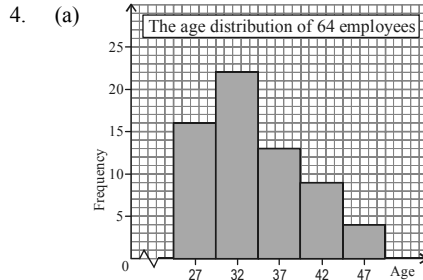
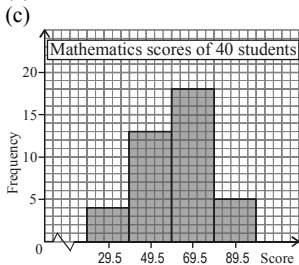
1. (a) Lower class limit = \$400, upper class limit = \$490
 (b) Lower class boundary = \$95, upper class boundary = \$195
 (c) Class mark = $\frac{200 + 290}{2} = \$245$
 (d) Class width = $\$195 - \$95 = \$100$
 (e) 100 students
 (f) The class with most students is \$300–\$390. Percentage = $\frac{30}{100} \times 100\% = 30\%$

2. (a)	Weight (kg)	40 – 44	45 – 49	50 – 54	55 – 59	60 – 64
	Tally	###	### ### ### /	### ### ###	### ///	///
	Frequency	5	16	14	9	4

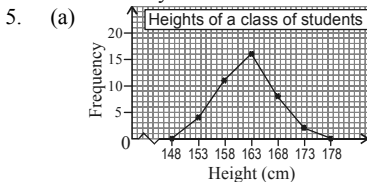
(b) Percentage of girls = $\frac{16+14}{48} \times 100\% = \frac{30}{48} \times 100\% = 62.5\%$

3. (a)	Score	Class boundaries	Class mark	Frequency
	20 – 39	19.5 – 39.5	29.5	4
	40 – 59	39.5 – 59.5	49.5	13
	60 – 79	59.5 – 79.5	69.5	18
	80 – 99	79.5 – 99.5	89.5	5

(b) Class width = $39.5 - 19.5 = 20$



- (b) None of the employees are younger than 24.5 years old or older than 49.5 years old. Besides, the ages are more concentrated in the first two classes, especially the class of 30–34 years old.



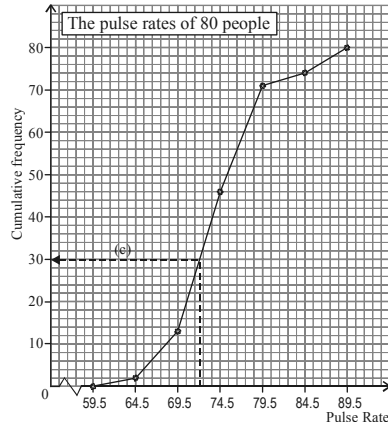
- (b) Lower boundary = $168 + 2.5 = 170.5$ cm,
upper boundary = $173 + 2.5 = 175.5$ cm
(c) Its lower class boundary = 150.5 cm,
upper class boundary = 155.5 cm
 $\therefore$ its lower class limit = 151 cm,
upper class limit = 155 cm

- (d) The class width = $158 - 153 = 5$ cm

Unit 18

6. (a)	Pulse rate less than	Cumulative frequency
	59.5	0
	64.5	2
	69.5	13
	74.5	46
	79.5	71
	84.5	76
	89.5	80

- (c) From the graph, 30 people have pulse rates less than 72.



7. (a) $60 \times \frac{1}{4} = 15$, $60 \times \frac{1}{2} = 30$, $60 \times \frac{3}{4} = 45$

From the graph,
the lower quartile = 6 500 + 1 000 = \$7 500,
the median = 10 500 + 800 = \$11 300,
the upper quartile $\approx$ 12 500 + 1 100 = \$13 600.

- (b) From the graph, 34 workers have salaries less than \$12 100.

$\therefore$ Percentage of experienced workers

$$= \frac{60 - 34}{60} \times 100\% = \frac{26}{60} \times 100\% = 43.3\%$$

8. (a) There is space between the bars of a bar chart, but there is no space between the bars of a histogram. Besides, only one axis of a bar chart is a number line and has a scale, but both axes of a histogram are number lines and have a scale.

- (b) Set A: It should be presented by a histogram. There are 100 data and have to be grouped into classes to show the pattern or characteristics of the set of data.

Set B: It should be presented by a bar chart. There are only 5 data, and they should be treated as discrete data (離散的、不連續的數據).

Set C: A bar chart is better than a histogram. Since there are only 12 data, the pattern or characteristic of the set of data can be observed easily in a bar chart. [A histogram can also be drawn, but it is neither meaningful nor necessary to group 12 data into several classes.]

9. No. of grade D students = $200 \times 30\% = 60$

No. of grades D or C students = $200 \times (30\% + 35\%) = 130$

No. of grades D, C or B students = $200 \times (1 - 10\%) = 180$

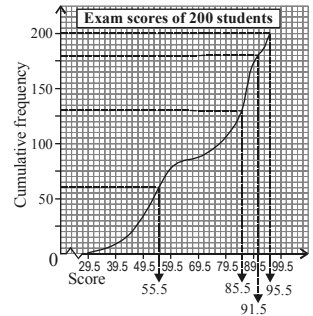
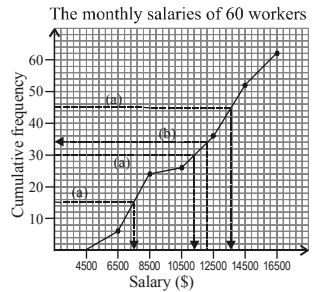
From the graph, 60 students have scores less than 55.5, 130 students have scores less than 85.5, and 180 students have scores less than 91.5. And none of the students have scores less than 29.5 or above 95.5.

$\therefore$ Grade D: scores 30–55,

Grade C: scores 56–85,

Grade B: scores 86–91,

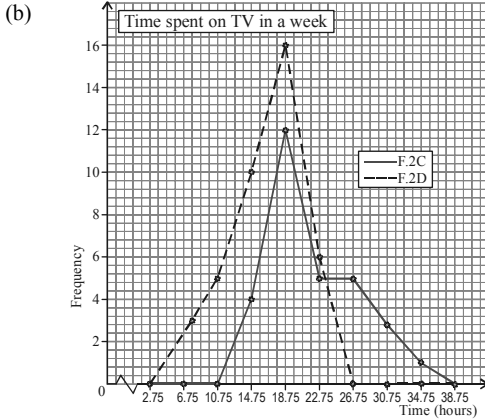
Grade A: scores 92–95



Unit 18

10. (a) No. of students of F.2C = 4 + 12 + 5 + 5 + 3 + 1 = 30
No. of students of F.2D = 3 + 5 + 10 + 16 + 6 = 40

Time spent on TV in a week (hours)	Class boundaries	Class mark	Frequency	
			F.2C	F.2D
5 – 8.5	4.75–8.75	6.75	0	3
9 – 12.5	8.75–12.75	10.75	0	5
13 – 16.5	12.75–16.75	14.75	4	10
17 – 20.5	16.75–20.75	18.75	12	16
21 – 24.5	20.75–24.75	22.75	5	6
25 – 28.5	24.75–28.75	26.75	5	0
29 – 32.5	28.75–32.75	30.75	3	0
33 – 36.5	32.75–36.75	34.75	1	0



- (c) Both frequency polygons have the greatest frequency for the class mark of 18.75 hours.
- (d) The frequency polygon of F.2D lies more to the left, while the frequency polygon of F.2C lies more to the right. This shows that on average students of F.2D spend less time on TV in a week than students of F.2C.

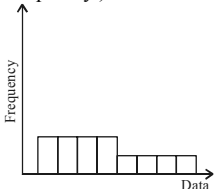
- (e) The numbers of students are different in both classes, so we have to compare the percentages.

$$\text{Percentage of F.2C students in this interval} = \frac{5}{30} \times 100\% = 16.7\%$$

$$\text{Percentage of F.2D students in this interval} = \frac{6}{40} \times 100\% = 15\%$$

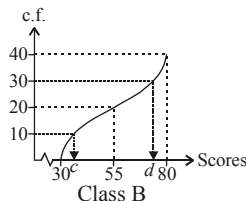
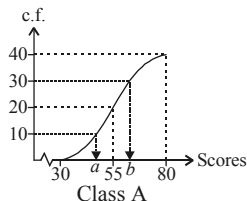
Ans. F.2C has a greater proportion of students in the interval of 21 hours to 24.5 hours.

11. Figure 11C is its corresponding cumulative frequency polygon. The horizontal line segment in the middle matches the class of zero frequency in the given histogram. (Figure 11B is wrong because a cumulative frequency polygon must not go downward. Figure 11D is wrong because it does not have any horizontal line segment matching with the class of zero frequency.)

12.  The cumulative frequency polygon is composed of 2 line segments. This shows that the first 4 classes are of equal frequencies and the last 4 classes are of equal frequencies. Besides, the total frequency of the first four classes is about twice of that of the last four classes, so the frequency of each of the first four classes should be twice of that of the last four classes.

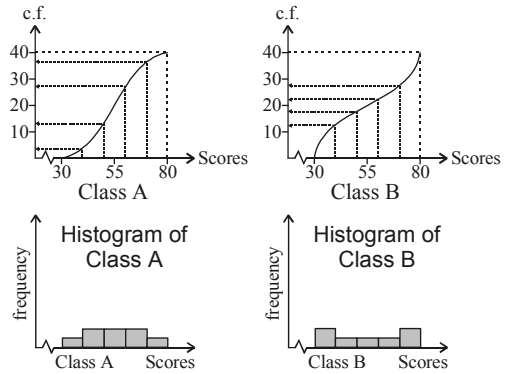
Unit 18

13. (a) In the figures below, a and b are the lower quartile and the upper quartile of Class A, while c and d are the lower quartile and the upper quartile of Class B. Since $c < a$, the average performance of the bottom 25% of Class B is poorer than that of Class A. Since $b < d$, the average performance of the top 25% of Class A is not so good as that of Class B. In sum, there are more students with low scores and high scores in Class B than in Class A.



- (b) The figures on the right show that in both Class A and Class B, the frequencies of the three class intervals in the middle are of similar frequencies.

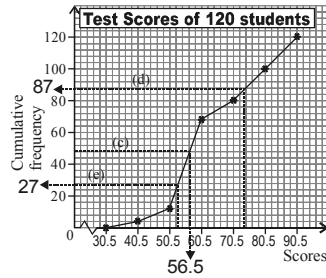
The histograms of the two classes are as follows:



14. (a)

Scores less than	Cumulative frequency
30.5	0
40.5	4
50.5	12
60.5	68
70.5	80
80.5	100
90.5	120

Score interval	Frequency
31 – 40	4
41 – 50	8
51 – 60	56
61 – 70	12
71 – 80	20
81 – 90	20



- (b) The class with most students is of scores 51–60.
 (c) No. of students failed the test = $120 \times (1 - 60\%) = 48$
 From the graph, the passing mark = 56.5.
 (d) From the graph, 87 students have scores *less than* 74.

$$\text{Percentage of students with scores greater than 74} = \frac{120 - 87}{120} \times 100\% = 27.5\%$$

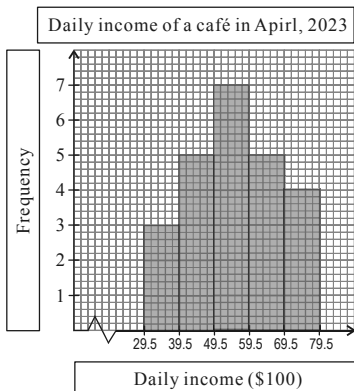
- (e) From the graph, 27 students have scores less than 52.

$$\text{Percentage of students who need to take the re-test} = \frac{27}{120} \times 100\% = 22.5\%$$

15. (a)

Daily income (\$100)	Class boundaries (\$100)	Class mark (\$100)	Frequency
30 – 39	29.5 – 39.5	34.5	3
40 – 49	39.5 – 49.5	44.5	5
50 – 59	49.5 – 59.5	54.5	7
60 – 69	59.5 – 69.5	64.5	5
70 – 79	69.5 – 79.5	74.5	4

(b)



(c) Number of days in which the daily incomes are less than \$4 950 = 3 + 5 = 8

Percentage of days in April, 2023 in which the daily incomes are less than \$4 950

$$= \frac{8}{3+5+7+5+4} \times 100\%$$

$$= \frac{8}{24} \times 100\% = 33\frac{1}{3}\% > 30\%$$

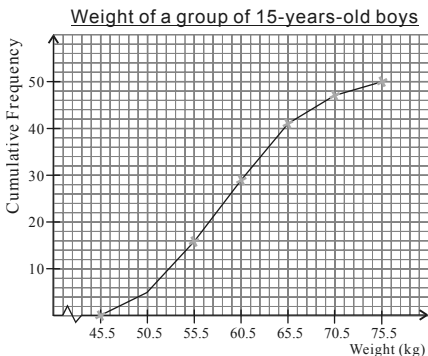
 $\therefore$ The claim is agreed.

16. (a)

Frequency distribution table		
Weight (cm)	Class Mark (cm)	Frequency
46 – 50	48	5
51 – 55	53	11
56 – 60	58	13
61 – 65	63	12
66 – 70	68	6
71 – 75	73	3

Cumulative frequency table	
Weight less than (cm)	Cumulative frequency
45.5	0
50.5	5
55.5	16
60.5	29
65.5	41
70.5	47
75.5	50

(b)



(c) (i) Number of boys less than or equal to 63 kg = 35

$$\therefore \text{The required percentage} = \frac{50-35}{50} \times 100\% = 30\%$$

(ii) The 20th percentile = 53 kg (accept 52.5 kg)

The required difference = (68 – 53) kg = 15 kg (accept 15.5 kg)

(iii) Number of overweight boys = 2

$$\therefore \text{The required percentage} = \frac{2}{50} \times 100\% = 4\%$$