Answers

Unit 1: Basic mathematics

1. (a) T (b) T (c) F (d) T (e) F

2. (a) $= 1.5 + 3 = 16.5$ (b) $= 15 - 14 = 1$

   (c) $= 4 + 12 + 3 = 16 + 3 = 19$ (d) $= 12 - \frac{8 \times 14}{28} = 12 - \frac{8}{2} = 12 - 4 = 8$

   (e) $= \frac{16 \times 117}{13} = 16 \times 9 = 144$ (f) $= 3 \times (40 - 8) = 3 \times 32 = 96$

3. (a) $= 21 - [4 + 96 \div 12] = 21 - (4 + 8) = 21 - 12 = 9$

   (b) $= \{94 - [2 \div 17]\} \div 5 = \{94 - 34\} \div 5 = 60 \div 5 = 12$

   (c) $= 6 + \{10 \div [18 - (8 + 8)] = 6 + \{10 \div [18 - 16]\}$

   $= 6 + \{10 \div 2\} = 6 + 5 = 11$

4. (a) 537 (b) 2376, 2144

5.

	6	8	9
(a) 372	✓	×	×
(b) 464	×	✓	×
(c) 689	×	×	×
(d) 774	✓	×	✓
(e) 1584	✓	✓	✓
(f) 2466	✓	×	✓

6. (a) Yes, ∵ 56 is divisible by 4.

   (b) No, ∵ 74 is not divisible by 4.

7. (a) No, ∵ 4233 is not an even number.

   (b) Yes, ∵ 5814 is even, and 5 + 8 + 1 + 4 = 18 which is a multiple of 3.

   (c) No, ∵ 7 + 0 + 1 + 2 = 10 which is not a multiple of 3.

8. (a) Yes, ∵ 8 + 5 + 9 + 5 = 27 which is a multiple of 9.

   (b) No, ∵ 2 + 1 + 9 + 7 + 9 = 28 which is not a multiple of 9.

9. (a) Yes, ∵ 472 ÷ 8 = 89 which is an integer.

   (b) Yes, ∵ 736 ÷ 8 = 92 which is an integer.

   (c) No, ∵ 364 ÷ 8 = 45.5.

10. 2 , 119 11. 51, 91

12. (a) $3 \times 3 \times 17$ (b) $2 \times 5 \times 23$ (c) $3 \times 7 \times 11$ (d) $2 \times 13 \times 19$

13. (a) $2^5 \times 3^2$ (b) $3^2 \times 5^3$ (c) $7^3 \times 11$ (d) $2^3 \times 3^3 \times 5^2$

14. (a) H.C.F. $= 2^3 \times 3 = 24$, L.C.M. $= 2^4 \times 3^2 \times 7 = 1008$

   (b) H.C.F. $= 3 \times 7 = 21$, L.C.M. $= 2^3 \times 3^3 \times 7^2 = 10{,}584$.

15. (a)
$$\begin{array}{r|rr} 2 & 1176 & 2058 \\ 3 & 588 & 1029 \\ 7 & 196 & 343 \\ 7 & 28 & 49 \\ & 4 & 7 \end{array}$$

H.C.F. $= 2 \times 3 \times 7 \times 7 = 294$

L.C.M. $= 294 \times 4 \times 7$

$= 8232$

(b) $5 \overline{)\ 165 \quad 420 \quad 1155}$
$3 \overline{)\ 33 \quad 84 \quad 231}$
$7 \overline{)\ 11 \quad 28 \quad 77}$
$11 \overline{)\ \mathbf{11} \quad 4 \quad 11}$
$1 \quad \mathbf{4} \quad 1$

H.C.F. $= 5 \times 3 = 15$
L.C.M. $= 15 \times 7 \times 11 \times 4$
$= 4620$

16. (a) $= 8.3 - 5.04 = 3.26$ (b) $= 3 + 6.6 = 9.6$

(c) $= 4.4 - 0.75 - 3.06 = 0.59$ (d) $= 8.7 - (2.87 + 5) = 8.7 - 7.87 = 0.83$

(e) $= 11.34 \div [2.7 \times 1.4] = 11.34 \div 3.78 = 3$

(f) $= 14.8 - 5.76 \div 0.45 = 14.8 - 12.8 = 2$

17. (a) $= 42 + \frac{7}{6} - 35 - \frac{7}{8} = 42 - 35 + \frac{28 - 21}{24} = 7\frac{7}{24}$

(b) $= \frac{13}{2} - \frac{5}{3} = 4\frac{5}{6}$ (c) $= 5\frac{1}{3} - \frac{40}{9} + \frac{2}{9} = 5\frac{3}{9} - 4\frac{4}{9} + \frac{2}{9} = 1\frac{1}{9}$

(d) $= \frac{35}{16} \div \frac{5 \times 25}{3 \times 12} \div 7 = \frac{35}{16} \times \frac{3 \times 12}{5 \times 25} \times \frac{1}{7} = \frac{9}{100}$

(e) $= \frac{35}{8} \div \left[\frac{7}{2} - \frac{17}{12}\right] = \frac{35}{8} \div \frac{25}{12} = \frac{35}{8} \times \frac{12}{25} = 2\frac{1}{10}$

18. The L.C.M. of 4, 6 is 12. $3 + 12 = 15$, $\therefore$ the next date is 15th March.

19. (a) $7 \overline{)\ 154 \quad 42 \quad 63}$
$22 \quad 6 \quad 9$. ***Ans.*** She should buy 7 boxes.

(b) ***Ans.*** 22 rings, 6 bracelets, 9 necklaces

20. $12 = 2 \times 2 \times 3$, $18 = 2 \times 3 \times 3$, $20 = 2 \times 2 \times 5$

L.C.M. $= 2 \times 2 \times 3 \times 3 \times 5 = 180$

Ans. The smallest possible number is 180.

21. $3 \overline{)\ 90 \quad 126 \quad 132}$
$2 \overline{)\ 30 \quad 42 \quad 44}$
$15 \quad 21 \quad 22$

Ans. The largest possible length is (3×2) cm = 6 cm.

22. (a) $= 1.82 + 1.6 = 3.42$

(b) $= \frac{4}{15} + \frac{11}{6} \times \frac{24}{10} - \frac{21}{10} = \frac{4}{15} + \frac{44}{10} - \frac{21}{10} = \frac{8 + 132 - 63}{30} = \frac{77}{30} = 2\frac{17}{30}$

(c) $= \frac{10}{11} \times \left(3 + \frac{15}{2} \times \frac{4}{1}\right) \times \left(\frac{28}{10} - \frac{4}{3}\right) = \frac{10}{11} \times 33 \times \left(\frac{28 \times 3 - 4 \times 10}{30}\right) = 30 \times \frac{44}{30} = 44$

(d) $= \left[89.7 \times \frac{2}{3} - 5(47.2 - 43.7)\right] \div 9$

$= [29.9 \times 2 - 5 \times 3.5] \div 9 = (59.8 - 17.5) \div 9 = 42.3 \div 9 = 4.7$

23. (a) $= \frac{16}{7} \div 5\frac{2}{5} = \frac{16}{7} \div \frac{32}{5} = \frac{16}{7} \times \frac{5}{32} = \frac{5}{14}$

(b) $= 1 \div \left(\frac{1}{6} \div 2\frac{1}{4}\right) = 1 \div \left(\frac{1}{6} \times \frac{4}{9}\right) = 1 \div \frac{2}{27} = \frac{27}{2} = 13\frac{1}{2}$

(c) $= \left(\frac{7}{3} - \frac{1}{6}\right) \div \left(2\frac{3}{8} - \frac{3}{4}\right) = \frac{14 - 1}{6} \div \frac{19 - 6}{8} = \frac{13}{6} \times \frac{8}{13} = \frac{4}{3} = 1\frac{1}{3}$

(d) $= 1 \div \left\{1 - \left[1 \div \left(1 + \dfrac{1}{1 - \dfrac{2}{3}}\right)\right]\right\} = 1 \div \{1 - [1 \div (1 + 3)]\} = 1 \div \left\{1 - \dfrac{1}{4}\right\} = 1 \div \dfrac{3}{4} = \dfrac{4}{3} = 1\dfrac{1}{3}$

24. (a)
$8 \overline{)\ 8424}$
$9 \overline{)\ 1053}$
$9 \overline{)\ 117}$
13

Ans. The largest prime factor is 13.

(b)
$5 \overline{)\ 6125}$
$5 \overline{)\ 1225}$
$5 \overline{)\ 245}$
$7 \overline{)\ 49}$
7

Ans. The largest prime factor is 7.

(c)
$3 \overline{)\ 1209}$
$13 \overline{)\ 403}$
31

Ans. The largest prime factor is 31.

(d)
$11 \overline{)\ 2299}$
$11 \overline{)\ 209}$
19

Ans. The largest prime factor is 19.

25.

	smallest	largest
(a)	0	8
(b)	2	8
(c)	0	6

26.

	smallest	largest
(a)	2	6
(b)	6	6
(c)	1	9

27. $1 + 3 + 4 = 8$, $\therefore$ to form a multiple of 3, ✦ may be 1, 4 or 7.
To form a multiple of 8, ✦ must be 4.
1344 is divisible by both 3 and 8, $\therefore$ yes.

28. (a) $x = 2 \times 2 \times 7 \times 7$, $y = 2 \times 2 \times 5 \times 7$
H.C.F. $= 2 \times 2 \times 7 = 28$
L.C.M. $= 2 \times 2 \times 5 \times 7 \times 7 = 980$

(b) $p = 3 \times 3 \times 5$, $q = 2 \times 3 \times 3 \times 5$, $r = 2 \times 2 \times 3 \times 5$
H.C.F. $= 3 \times 5 = 15$
L.C.M. $= 2 \times 2 \times 3 \times 3 \times 5 = 180$

29. (a) The L.C.M. of 4, 6 , 11 is $2^2 \times 3 \times 11 = 132$.
$\therefore$ the smallest value of the number $= 132 + 3 = 135$.

(b) Multiples of 132 = 264, 396, 528, ... ; $528 + 3 = 531$
531 is between 500 and 600. $\therefore$ Yes, it is possible.

30. (a)
$3 \overline{)\ 105 \quad 126 \quad 147}$
$7 \overline{)\ 35 \quad 42 \quad 49}$
$5 \quad 6 \quad 7$

Ans. The number may be 3 or 7.

(b) The number is $3 \times 7 = 21$.

31. $104 = 2^3 \times 13$, $117 = 3^2 \times 13$
Their L.C.M. $= 2^3 \times 3^2 \times 13 = 936$ which is smaller than 1000.
$\therefore$ ***Ans.*** Yes, it is possible.

32. (a) $120 = 40 \times 3$, $200 = 40 \times 5$, $280 = 40 \times 7$; their H.C.F. = 40.
$\therefore$ There are 40 people at most.

(b) $120 - 6 = 114 = 2 \times 3 \times 19$, $200 - 10 = 190 = 2 \times 5 \times 19$
$280 - 14 = 266 = 2 \times 7 \times 19$;
their H.C.F. $= 2 \times 19 = 38$, $\therefore$ there are 38 people at most.
The amount each person gets = $\$(3 \times 1 + 5 \times 2 + 7 \times 5) = \$48$

33. (a) Time for Jack to run 1 lap = $\frac{60 \times 2}{15}$ = 8 mins.

Time for Kelvin to run 1 lap = $\frac{60 \times 2}{12}$ = 10 mins.

L.C.M. of 8, 10 = 40
$\therefore$ They stopped running after 40 mins. ***Ans.*** The time was 9:10 a.m.

(b) The number of laps Kelvin ran this morning = $\frac{40}{10}$ = 4.

34. (a) Number of books Amy has now: $\left(120 \times \frac{2}{5} + 9\right) \times \left(1 - \frac{1}{3}\right)$

(b) Number of books Amy has = $(48 + 9) \times \frac{2}{3}$ = 38

$\therefore$ Number of books Betty has = 120 – 38 = 82.

35. (a) No. 2 is even, but not composite.
(b) Yes, for example, 21 is odd and composite.
(c) No. The two numbers do not have common factor other than 1, but they may have their own factors, e.g. 8 and 9.

36. (a) $303 = 3 \times 101$, $405 = 3^4 \times 5$, $505 = 5 \times 101$
(b) H.C.F. = 1 (c) The L.C.M. $= 3^4 \times 5 \times 101$

37. The sum of all digits = 4 + 3 + 5 = 12 which is a multiple of 3.
$\therefore$ 4,350,000 is divisible by 3.
The last 3 digits is 000, which is divisible by 8.
$\therefore$ 4,350,000 is divisible by 8.
$\therefore$ 4,350,000 is divisible by 24.

38. (a) $6 = 2 \times 3$, $8 = 2^3$, $9 = 3^2$; their L.C.M. $= 2^3 \times 3^2 = 72$.
the next common multiple $= 72 \times 2 = 144$
$\because$ P is a 3-digit number, and their remainders are 4,
$\therefore$ the least possible value = 144 + 3 = 147
(b) Multiples of 72: 144, 216, 288, 360, ...
If the hundreds' digit is 2, the possible values of P:
216 + 4 or 288 + 4, that is 220 or 292.

39. (a) When Q is divided by 6, the remainder is 2,
$\therefore$ Q + (6 – 2) = Q + 4 is divisible by 6.
When Q is divided by 9, the remainder is 5,
$\therefore$ Q + (9 – 5) = Q + 4 is divisible by 9.
The L.C.M. of 6 and 9 is 36.

If Q is a 2-digit number, its greatest possible value = 36 – 4 = 32.

(b) Multiples of 36: 72, 108, 144, ...

If Q is a 3-digit number, its smallest possible value = 108 – 4 = 104.

40. (a) Sum of the digits = 7 + 5 + ✦ + 4 = 16 + ✦

When ✦ = 2, the sum = 18 which is divisible by 9.

Ans. The possible values of ✦ = 0, 1, 3, 4, 5, 6, 7, 8, 9.

(b) 75✦04 – 3 = 75✦01 which is divisible by 9.

Sum of digits = 7 + 5 + ✦ + 0 + 1

= 13 + ✦ which is divisible by 9.

13 + ✦ = 18, ✦ = 5

41. (a) The L.C.M. of 3, 4, 6 is 12. ***Ans.*** The length of the side of each cube is 12 cm.

(b) No. of wooden blocks required = $\frac{12}{3}\times\frac{12}{4}\times\frac{12}{6} = 4 \times 3 \times 2 = 24$

42. (a)
$$\begin{array}{r|lll} 12 & 144 & 216 & 360 \\ \hline 6 & 12 & 18 & 30 \\ \hline & 2 & 3 & 5 \end{array}$$

Their H.C.F. = 12 × 6 = 72

Ans. The largest possible length is 72 cm.

(b) 72 = 1 × 72
= 2 × 36
= 3 × 24
= 4 ×18
= 6 × 12
= 8 × 9

Ans. The possible lengths are:
1 cm, 2 cm, 3 cm, 4 cm, 6 cm, 8 cm, 9 cm, 12 cm, 18 cm, 24 cm, 36 cm, 72 cm

43. (a) (i) $8 = 2^3$, $12 = 2^2 \times 3$, $15 = 3 \times 5$; their L.C.M. = $2^3 \times 3 \times 5 = 120$

Ans. The smallest possible number is 120.

(ii) ***Ans.*** Yes, because 10 is a factor of 120.

(iii)
$$\begin{array}{r|ll} 3 & 9 & 120 \\ \hline & 3 & 40 \end{array}$$

The L.C.M. of 9 and 120 = 3 × 3 × 40 = 360

Ans. The smallest possible number is 360.

(b) Multiples of 120: 120, 240, 360, 480, ...

After taking away 2 from the multiples: 118, 238, 358, 458, ... ;

among them, the smallest number divisible by 7 is 238.

Ans. The smallest possible number is 240.

Unit 2: Directed numbers

1. (a) < (b) > (c) > (d) >

2. (a) $-8 < -5 < -0.87 < 9.99 < 13$ (b) $-2 < -\frac{13}{7} < 0 < 1 < \frac{8}{7}$

3. (a) $\frac{25}{30} > \frac{24}{30} > \frac{15}{30} > -\frac{15}{30} > -\frac{24}{30} > -\frac{25}{30}$,

$\therefore \frac{5}{6} > \frac{4}{5} > \frac{1}{2} > -\frac{1}{2} > -\frac{4}{5} > -\frac{5}{6}$

(b) $3 > 2 > -3\frac{3}{4} > -4 > -4\frac{1}{3} > -14$, $\therefore 3 > 2 > -\frac{15}{4} > -4 > -\frac{13}{3} > -14$

4. (a) $= 7 + 2 = 9$ (b) $= -61 - 39 + 21 = -100 + 21 = -79$

   (c) $= -17 + 14 + 17 = -17 + 17 + 14 = 14$

   (d) $= +33 - 15 - (-4 + 6) = 18 - 2 = 16$

   (e) $= -11 - \frac{3}{4} + 12 + \frac{1}{3} + 5 = -11 + 12 + 5 + \frac{1}{3} - \frac{3}{4} = 6 - \frac{5}{12} = 5\frac{7}{12}$

   (f) $= 9.34 + 2.53 - 15.17 - 4.66 = 11.87 - 15.17 - 4.66 = -3.3 - 4.66 = -7.96$

5. (a) $= 9 \times 6 \div 18 = 3$ (b) $= -\frac{4 \times 3}{6 \times 10} = -\frac{1}{5}$ (c) $= \frac{2}{3} \times \frac{5}{8} \times \frac{6}{1} = \frac{5}{2} = 2\frac{1}{2}$

   (d) $= -(1.6 \times 0.25 \div 1.2 \times 0.3) = -0.1$

6. (a) $= 7 - (-19) = 26$ (b) $= -8 + (-28) = -8 - 28 = -36$

   (c) $= (-15) \times (-4) = 60$ (d) $= (-10) \div 6 = -\frac{5}{3} = -1\frac{2}{3}$

7. (a) $= -36 \div (3 \times 3) = -4$ (b) $= (-12) \div (4) \times (-6) = 18$

   (c) $= -6 - (-11 + 1) = -6 + 10 = 4$ (d) $= (-2) \times (15 + 3) = -36$

8. (a) $= +(\frac{8}{3} \times \frac{5}{2} \times \frac{1}{9}) + 1 = 1\frac{20}{27}$

   (b) $= -\frac{7}{6} + (-\frac{1}{9}) = -\frac{7}{6} - \frac{1}{9} = -\frac{21}{18} - \frac{2}{18} = -\frac{23}{18} = -1\frac{5}{18}$

   (c) $= \frac{(-5) \times (-6)}{(-10 + 8) \times 10} = \frac{30}{(-2) \times 10} = -\frac{3}{2} = -1\frac{1}{2}$

9. (a) $= (-3)^2 - 4(-5) = 9 + 20 = 29$ (b) $= (-2)(-6) + (-2)^2 = 12 + 4 = 16$

   (c) $= 3(-\frac{1}{3})^2 + 2(-\frac{1}{3}) - 4 = 3(\frac{1}{9}) - \frac{2}{3} - 4 = \frac{1}{3} - \frac{2}{3} - 4 = -4\frac{1}{3}$

   (d) $= (-8)(\frac{2}{5}) + (\frac{2}{5})(-3) + (-3)(-8) = -\frac{16}{5} - \frac{6}{5} + 24 = 24 - \frac{22}{5} = 19\frac{3}{5}$

10. Let x be the number, $x + (-26) = -53$, $x = (-53) - (-26)$, $x = -27$.

    Ans. The number is –27.

11. $27 - (-7) = 34$. ***Ans.*** The temperature drop is 34ºC.

12. Let x ºC be the temperature of Beijing, $8 - 12 = x - 5$, $x = 8 - 12 + 5$, $x = 1$.

    Ans. The temperature of Beijing is 1ºC.

13. $+250 - (-150) = 400$. ***Ans.*** The rocket is 400m above the submarine.

14. $+80 + (-150) = -70$. ***Ans.*** The shopkeeper lost $70.

15. $32(+2) + 10(-2) + 8(-1) = +64 - 20 - 8 = 36$. ***Ans.*** Andrew's test score was 36.

16. (a) $p = (-7) - (6) = -13$, $q = (-7) + (5) = -2$, $\therefore\ p - q = (-13) - (-2) = -11$

    (b) $p + q = (-13) + (-2) = -15$

17. (a) $= (-8) \times 4 + 3 = -32 + 3 = -29$ (b) $= (-8 + 6)^4 = (-2)^4 = 16$

    (c) $= \frac{4 \times [-(5 \times 5 \times 5)]}{15 \times 15} = -\frac{20}{9} = -2\frac{2}{9}$ (d) $= 36 - (-32) = 68$

    (e) $= 4 \div \frac{1}{9} = 36$

18. (a) $= (3 + \frac{1}{2} - 7 - \frac{1}{6}) \div [-4 - (\frac{4}{9})] = (-4 + \frac{1}{3}) \div (-4\frac{4}{9}) = (-3\frac{2}{3}) \div (-\frac{40}{9})$

$= \frac{11}{3} \times \frac{9}{40} = \frac{33}{40}$

(b) $= -(5 - 3) \times 36 - (-4) = -2 \times 36 + 4 = -68$ (c) $= \frac{36 - 10 - 8}{-9} = \frac{18}{-9} = -2$

19. $(3 + n)^3 = -1$, $\therefore\ 3 + n = -1$, $n = -1 - 3 = -4$

20. $= -1 - (+1) = -1 - 1 = -2$

21. (a) $= (-\frac{9}{4}) \div [\,(-\frac{1}{8}) + (-1)\,] = (-\frac{9}{4}) \div (-\frac{9}{8}) = +(\frac{9}{4} \times \frac{8}{9}) = 2$

(b) $= -\frac{1}{3} - \{\frac{-3}{4} - [-\frac{1}{3} - (\frac{1}{2})^2]\} = -\frac{1}{3} - [-\frac{3}{4} - (\frac{-7}{12})] = -\frac{1}{3} - (\frac{-1}{6}) = -\frac{1}{6}$

22. (a) $= (-81 + 1) \times (\frac{7}{2} \times \frac{5}{14} - \frac{4}{5}) = (-80) \times (\frac{5}{4} - \frac{4}{5}) = -80 \times \frac{9}{20} = -36$

(b) $= (2\frac{1}{6} - 2\frac{1}{4}) \div (-\frac{1}{3}) - [15 \times (-\frac{3}{20})] = (-\frac{1}{12}) \times (-\frac{3}{1}) - (-\frac{9}{4})$

$= \frac{1}{4} + \frac{9}{4} = 2\frac{1}{2}$

23. $= 1 - 1 \div (1 - \frac{1}{1 - \frac{1}{9}})^3 = 1 - 1 \div (1 - \frac{1}{\frac{8}{9}})^3 = 1 - 1 \div (1 - \frac{9}{8})^3 = 1 - 1 \div (-\frac{1}{8})^3$

$= 1 - 1 \times (-512) = 513$

24. (a) $= \frac{(2)(-1)}{-[0.3 + (-5)(-\frac{1}{4})]} = \frac{-2}{-(0.3 + \frac{5}{4})} = 2 \div (\frac{6}{20} + \frac{25}{20}) = 2 \div \frac{31}{20} = \frac{40}{31} = 1\frac{9}{31}$

(b) $= \frac{-[(\frac{-1}{4})(0.3) - (-2)(1)]}{-(-5)^2} = (\frac{-3}{40} + 2) \cdot \frac{1}{25} = \frac{77}{40} \cdot \frac{1}{25} = \frac{77}{1000} = 0.077$

25. (a) $\frac{x}{-25} - \frac{9}{100} = -\frac{1}{4}$, $-4x - 9 = -25$, $-4x = -16$, $\therefore\ x = 4$

(b) $(\frac{3}{4})(-\frac{2}{3}) - (\frac{4}{5})(-\frac{5}{8}x) = (\frac{1}{2})(-\frac{2}{3})$, $-\frac{1}{2} + \frac{1}{2}x = -\frac{1}{3}$,

$\frac{1}{2}x = -\frac{1}{3} + \frac{1}{2}$, $x = \frac{1}{6} \times 2$, $\therefore\ x = \frac{1}{3}$

26. (a) $= (99) \times ... \times 1 \times 0 \times (-1) \times ... \times (-99) = 0$

(b) $= (-1 + 2) + (-3 + 4) + (-5 + 6) + ... + (-49 + 50) = 1 \times 25 = 25$

27. There are 79 terms in which 40 terms are negative numbers.

$\therefore$ The product is a positive number. $= +(\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times ... \times \frac{80}{79} \times \frac{81}{80}) = \frac{81}{2} = 40\frac{1}{2}$

28. (a) $y = 1^2 + 1^3 + ... + 1^{100} = 1 + 1 + 1 + ... + 1 = 99$ ($\because$ there are 99 terms)

(b) $y = (-1)^2 + (-1)^3 + ... + (-1)^{100} = +1 - 1 + ... + 1 - 1 + 1 = 1$

29. (a) $\because$ $60 = 2 \times 2 \times 3 \times 5$, $\therefore$ the smallest value of $a + b + c = 3 + 4 + 5 = 12 > 10$,
$\therefore$ a, b, c can't be all positive.
$\because$ $abc = 60$, but a, b, c can't be all positive,
$\therefore$ two of them must be negative numbers.

(b) The positive factors of 60 include : 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60.
By inspection, $(-1) + (-4) + (+15) = 10$, and $(-1)(-4)(+15) = 60$.
$\therefore$ $a = -4$ (the smallest), $b = 15$ (the largest), $c = -1$

30. (a) $(50 - 5) \times 4 + 5 \times (-2) = 180 - 10 = 170$. ***Ans.*** He might get 170 marks.

(b) Let x be the number of correct answers.
$4x + (-3)(50 - x) = 123$, $4x - 150 + 3x = 123$, $7x = 273$, $x = 39$.
Ans. His percentage of correct answers = $\frac{39}{50} \times 100\% = 78\%$.

31. (a) False. $-(-5) = +5$ which is not a negative number.

(b) False. $-0 = +0 = 0$ (c) True (d) True (e) False. $-3 + 7 = 4$

32. (a) $(+2) - (-12) = 14$. ***Ans.*** The time in Sydney is 14h ahead that of New York.

(b) The time difference = $a - (-1) = (a + 1)$ h.

(c) Sydney – 19:00 on Thursday, London – 10:00 on Thursday
The time difference between Sydney and London = $2 - a$
$19 - 10 = 2 - a$, $9 = 2 - a$, $\therefore$ $a = -7$

(d) Vancouver – 19:00 on Monday, Hong Kong – 10:00 on Tuesday = 34:00 on Monday
$34 + b = 19$, $\therefore$ $b = 19 - 34 = -15$.
Ans. The time in Vancouver is 15 h behind that of Hong Kong.

(e) (i) The time difference between Toyko and Vancouver
$= +1 - b = 1 - (-15) = 16$h

(ii) When Toyko is 09:00 on Friday, Vancouver is $(9 - 16 + 24) = 17$:00 on Thursday.
When the person arrives at Vancouver, the time is 17:00 + 14 = 31:00 on Thursday
= 7:00 am on Friday.

33. (a) (i) < (ii) < (iii) > (b) $-2^3 > -8\frac{7}{12} > -8\frac{3}{4} > -15\frac{7}{15} > -15\frac{9}{17} > -2^4$

34. (a) T (b) F (c) T (d) F

35. (i), (ii)

36. (a) Jason (b) $(x + 3)$ cm

(c) $-5 - (-4) = -5 + 4 = -1$, $\therefore$ Winnie is shorter by 1 cm.

(d) $161 - x = 8$, $\therefore$ $x = 161 - 8 = 153$

37. Step 3, $-4 + 6 = 2$, not -10

38. (a) $= -1 - (-9 - 2) = -1 - (-11) = -1 + 11 = 10$

(b) $= -12 + (-8.5) \div \left(\frac{-3.4}{-4}\right) = -12 - \left(\frac{8.5 \times 4}{3.4}\right)$

$= -12 - \left(\frac{85 \times 2}{17}\right) = -12 - 5 \times 2 = -12 - 10 = -22$

39. (a) $= \frac{-128}{11} \times \frac{-15}{64} \times \frac{2}{1} \times \frac{-1}{11} \times \frac{121}{24} = -\frac{5}{2}$

(b) $= \frac{-8+9}{12} \times \frac{-108}{1} - 3 \times (5-2) = -9 - 9 = -18$

40. (a) $= -3 - \left[\frac{-1}{3} + \frac{2}{3} \times \frac{1}{6}\right] \times \left(\frac{-2}{3}\right) + 2 = -3 - \left(\frac{-3+1}{9}\right) \times \left(\frac{-2}{3}\right) + 2$

$= -3 - \frac{(-2)(-2)}{27} + 2 = -1 + \frac{4}{27} = -\frac{23}{27}$

(b) $= \left(\frac{4}{5} - \frac{6}{7}\right)\left(\frac{8}{9} - \frac{10}{11}\right) \div \left(-1 \div \frac{9}{2}\right) \times \frac{77}{3} = \left(\frac{28-30}{35}\right)\left(\frac{88-90}{99}\right) \div \left(-\frac{2}{9}\right) \times \frac{77}{3}$

$= \frac{-2}{35} \times \frac{-2}{99} \times \frac{-9}{2} \times \frac{77}{3} = -\frac{2}{15}$

41. (a) $= 24 - 12 \times (8-9)^4 \times 4 = 24 - 12\,(-1)^4 \times 4 = 24 - 48 = -24$

(b) $= \left[-2 + \frac{1}{3}\right]^2 \times \frac{-9}{2} \div \frac{25}{-3} = \left(\frac{-5}{3}\right)^2 \times \frac{-9}{2} \times \frac{-3}{25} = \frac{25}{9} \times \frac{9}{2} \times \frac{3}{25} = \frac{3}{2}$

42. (a) (i) $= 49 - 84 + 36 = 85 - 84 = 1$

(ii) $= -(16) - (16)\,(-2) + 36 = -16 + 32 + 36 = 52$

(b) $= \frac{1}{52}$

43. (a) The highest score = $10 \times 5 = 50$ marks.

(b) The score = $6 \times 5 + (-2)\,(10 - 6 - 3) = 30 - 2\,(1) = 28$ marks

(c) To get the second highest score, a team should answer 9 questions correctly, and did not answer 1 question. The score = $9 \times 5 + 0 = 45 < 48$.
$\therefore$ No, not possible to get 48 marks.

44. (a) At 8:30, the temperature = $38.2 + 0.3 = 38.5$ °C.

(b) From 8:00 am to 10:00 am.

(c) His total rise in temperature = $0.3 + 0.5 + 0.8 + 0.3 = 1.9$.
$38.2 + 1.9 = 40.1$ Yes, he had very high temperature.

(d) From 8:30 to noon, the final change in body temperature
$= (0.5 + 0.8 + 0.3) - (0.4 + 0.7 + 0.4 + 0.2) = 1.6 - 1.7 = -0.1 < 0$
Yes, he had a lower temperature.

Unit 3: Introduction to algebra

1. (a) $12 - 10x$ (b) $2a + 5b$ (c) $6x^2$ (d) $2a^4$
(e) $5p - 2q$ (f) $= 4n + 4n = 8n$ (g) $5xy - 3x^2$

2. (a) $\frac{1}{6}$ (b) 12 (c) $= \frac{p^2 \cdot p}{2} = \frac{p^3}{2}$ (d) $\frac{4ab}{c} - 2$

3. (a) $= 5\,(3)^2 - 3\,(4) = 45 - 12 = 33$ (b) $= (6)^2 - 3\,(6) + 8 = 36 - 18 + 8 = 26$

(c) $= 8\,(5) - (4)\,(16) = 40 - 64 = -24$ (d) $= \frac{5\,(3)^2 - 1}{2(3) + 5} = \frac{44}{11} = 4$

4. (a) $C = \frac{5}{9}(104-32) = \frac{5}{9} \times 72 = 40$ (b) $A = \frac{7}{2}(6+9) = \frac{105}{2} = 52\frac{1}{2}$

   (c) $P = 10\ (10-2)\ (10-6)\ (10-5) = 10 \times 8 \times 4 \times 5 = 1600$

   (d) $\frac{1}{f} = \frac{1}{\frac{2}{7}} - \frac{1}{1.5} = \frac{7}{2} - \frac{2}{3} = \frac{17}{6}$, $\therefore\ f = \frac{6}{17}$

5. (a) sum (b) difference (c) product (d) quotient

6. (a) $\frac{k}{3}$ (b) $x^2 - 7$ (c) $x\,(\frac{y}{2}) = \frac{xy}{2}$ (d) $(p+q) \div 2 = \frac{p+q}{2}$

7. (a) $\frac{t}{60}$ hours (b) $100n$ cents (c) $\frac{y}{1000}$ km

   (d) $(1000x + y)$ grams

8. $y - 2$ 9. $5 - x$ 10. $12 + k$ 11. $(x - 3)$ years old

12. bh cm^2 13. $\$\frac{800}{n}$ 14. $\$(2x + 3y)$ 15. $\$\frac{k}{21}$

16. $\frac{x}{y}$ h 17. $\frac{s}{x}$ km/h 18. $y \times \frac{15}{60} = \frac{y}{4}$ km

19. (a) $6x$ (b) x^6 (c) $= x^3 + x^3 = 2x^3$

    (d) $x^4 + x^2$ (e) $x^5 + x$ (f) $= x^2 + x^2 + x^2 = 3x^2$

    (g) $= x + x^4 + x = 2x + x^4$ (h) $= x^2 + 2x + x^2 = 2x^2 + 2x$

    (i) $= x + x^2 + x^2 + x = 2x + 2x^2$ (j) $= 2x + x^2 + 2x = 4x + x^2$

20. (a) $= \frac{6k \times (-18k)}{4} = -27k^2$ (b) $= -\frac{6}{7}mn - mn = -\frac{13}{7}mn$

21. (a) $30x - 6$ (b) $12 + 21a$ (c) $-9 + 54y$ (d) $-20k - 16$

22. (a) $(a+b)^2$ (b) $a^2 + b^2$ (c) $\frac{2}{5} \times n = \frac{2n}{5}$ kg

    (d) $\frac{p}{4}$ cups (e) $\frac{x}{3} \div x \times y = \frac{y}{3}$ g

23. $p \times q - \frac{k}{2} \div \frac{1}{3} = pq - \frac{3k}{2}$ 24. $(x - 7kn)$ kg

25. $10y + x$ 26. $= \frac{1000x}{96 \times 60} = \frac{25x}{144}$ minutes

27. (a) The total distance travelled = $(mx + ny)$ km

    (b) The total time taken = $(m + n)$ hours , $\therefore$ the average speed = $\frac{mx+ny}{m+n}$ km/h

28. (a) 6 years ago Paul was $(x - 6)$ years old.

    (b) 6 years ago Helen's age = $\frac{34-(x-6)}{2} = \frac{40-x}{2} = (20 - \frac{x}{2})$ years old

    (c) Helen's age now = $20 - \frac{x}{2} + 6 = (26 - \frac{x}{2})$ years old

    $\therefore$ Their total ages now = $x + (26 - \frac{x}{2}) = (\frac{x}{2} + 26)$ years old

29. (a) The value of letter G is 7.

(b) $T = 8(7) + 7(4) + 6(6) + 5(7) + 4(5) + 3(5) + 2(6)$

$= 56 + 28 + 36 + 35 + 20 + 15 + 12 = 202$

(c) $202 \div 11 = 18$ .... 4 , $\therefore$ R = 4, $x = 11 - 4 = 7$

30. (a) $= \dfrac{r-s}{r-s} = 1$ (b) $= \dfrac{a-b}{-a+b} = \dfrac{(a-b)}{-(a-b)} = -1$ (c) No.

(d) $= \dfrac{(p+q)^2}{(p+q)^2} = 1$ (e) $= \dfrac{(x-y)^2}{[-(x-y)]^2} = \dfrac{(x-y)^2}{(x-y)^2} = 1$ (f) No.

31. (a) $= 3h - 2h + h = h + h = 2h$

(b) $= y^2 + 3 + y^2 = 2y^2 + 3$

32. $\dfrac{pq}{h^2 + k^2}$

33. number of cookies left = $(n-4)\left(1-\dfrac{2}{5}\right) = \dfrac{3}{5}(n-4)$

34. Total gain or lose = $\$(-9x + 15y)$ [or: $\$(15y - 9x)$]

35. (a) $A = k^2 - \dfrac{1}{2}ab$ (b) Area = $\left[20^2 - \dfrac{1}{2}(12)(9)\right]$ cm^2

$= (400 - 54)$ cm^2 = 346 cm^2

36. (a) (i) $y - x$ (ii) $\dfrac{(x+y)^2}{y-x}$

(b) $\left[-\dfrac{1}{2} + \left(-\dfrac{1}{3}\right)\right]^2 \div \left[-\dfrac{1}{3} - \left(-\dfrac{1}{2}\right)\right] = \left(-\dfrac{1}{2} - \dfrac{1}{3}\right)^2 \div \left(-\dfrac{1}{3} + \dfrac{1}{2}\right) = \left(\dfrac{-2-3}{6}\right)^2 \div \dfrac{-2+3}{6}$

$= \left(\dfrac{-5}{6}\right)^2 \div \dfrac{1}{6} = \dfrac{25}{36} \times \dfrac{6}{1} = \dfrac{25}{6}$

37. y years ago, the son's age = $n - y$

y years ago, Mr. Lai's age = $k - (n - y) = k - n + y$

$\therefore$ The present age of Mr. Lai = $(k - n + y) + y = k - n + 2y$

Unit 4: Sequences

1. 1, 3, 6, 10, 15, 21 2. 64, 81, 100, 121, 144

3. (a) $a_n = 4 - 3n$, $\therefore$ $a_1 = 4 - 3(1) = 1$, $a_2 = 4 - 3(2) = -2$, $a_3 = 4 - 3(3) = -5$,

$a_4 = 4 - 3(4) = -8$. ***Ans.*** The first four terms are 1, –2, –5, –8.

(b) $a_n = \dfrac{n+1}{n+2}$, $\therefore a_1 = \dfrac{1+1}{1+2} = \dfrac{2}{3}$, $a_2 = \dfrac{2+1}{2+2} = \dfrac{3}{4}$, $a_3 = \dfrac{3+1}{3+2} = \dfrac{4}{5}$,

$a_4 = \dfrac{4+1}{4+2} = \dfrac{5}{6}$. ***Ans.*** The first four terms are $\dfrac{2}{3}, \dfrac{3}{4}, \dfrac{4}{5}, \dfrac{5}{6}$.

(c) $a_n = n^2 - 1$, $\therefore$ $a_1 = 1^2 - 1 = 0$, $a_2 = 2^2 - 1 = 3$, $a_3 = 3^2 - 1 = 8$,

$a_4 = 4^2 - 1 = 15$. ***Ans.*** The first four terms are 0, 3, 8, 15.

(d) $a_n = 2^n + 3$, $\therefore$ $a_1 = 2^1 + 3 = 5$, $a_2 = 2^2 + 3 = 7$, $a_3 = 2^3 + 3 = 11$,

$a_4 = 2^4 + 3 = 19$. ***Ans.*** The first four terms are *5, 7, 11, 19.*

(e) $a_n = \dfrac{n(n+1)}{2}$, $\therefore$ $a_1 = \dfrac{1(1+1)}{2} = 1$, $a_2 = \dfrac{2(2+1)}{2} = 3$, $a_3 = \dfrac{3(3+1)}{2} = 6$,

$a_4 = \frac{4(4+1)}{2} = 10$. ***Ans.*** The first four terms are 1, 3, 6, 10.

(f) $a_n = \frac{1}{2}(n+2)(3n-1)$, $\therefore\ a_1 = \frac{1}{2}(1+2)(3-1) = 3$, $a_2 = \frac{1}{2}(2+2)(6-1) = 10$,

$a_3 = \frac{1}{2}(3+2)(9-1) = 20$, $a_4 = \frac{1}{2}(4+2)(12-1) = 33$

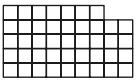
4. (a) 22, 25 (b) $\frac{1}{4}$, $\frac{1}{8}$ (c) 16, 27

5. The fifth term = 6 × 7 – 5

6. Number of dots:

Figure 1: 2 × 4 + 1 , Figure 2: 3 × 4 + 1 , Figure 3: 4 × 4 + 1

$\therefore$ Number of dots in the 7th figure = 8 × 4 + 1 = 33

7. (a)  (b) The no. of squares = 6 × 11 – 2 = 64

8. (a) The denominator of each term is greater than that of the preceding term by 1.

$\therefore$ The next two terms are $\frac{1}{6}$, $\frac{1}{7}$.

(b) The denominator of each term can be obtained by multiplying the preceding denominator by 2. The numerator of each term can be obtained by adding 4 to the preceding numerator.

$\therefore$ The next two terms are $\frac{19}{32}$, $\frac{23}{64}$

9. Both the number of rows and number of columns of each figure are one more than that of the preceding figure. $\therefore$ Number of circle in the next figure = 5 × 6 = 30

10. (a) the 1st term = $14 + (-1)^1 = 14 - 1 = 13$
the 2nd term = $14 + (-1)^2 = 14 + 1 = 15$
the 3rd term = $14 + (-1)^3 = 14 - 1 = 13$
the 4th term = $14 + (-1)^4 = 14 + 1 = 15$

(b) the 1st term = $-2^1 - 11 = -2 - 11 = -13$
the 2nd term = $-2^2 - 11 = -4 - 11 = -15$
the 3rd term = $-2^3 - 11 = -8 - 11 = -19$
the 4th term = $-2^4 - 11 = -16 - 11 = -27$

(c) the 1st term = $(-3)^1 + (-1)^2 = -3 + 1 = -2$
the 2nd term = $(-3)^2 + (-1)^3 = 9 - 1 = 8$
the 3rd term = $(-3)^3 + (-1)^4 = -27 + 1 = -26$
the 4th term = $(-3)^4 + (-1)^5 = 81 - 1 = 80$

11. (a) $k - 1 = 19$, $k = \underline{\underline{20}}$

(b) $x = \underline{\underline{6}}$. $3y + 10 = 8$, $3y = -2$, $y = \underline{\underline{-\frac{2}{3}}}$

(c) $p = \underline{\underline{21}}$. $4(q - 3) = 8$, $q - 3 = 2$, $q = \underline{\underline{25}}$

12. $10 - 4n = -58$, $10 + 58 = 4n$, $n = \frac{68}{4} = 17$. $\therefore$ The 17th term is −58.

13. If $29 - 4n = 0$, $29 = 4n$, $n = 7\frac{1}{4}$. ***Ans.*** 7 terms of this sequence are positive numbers.

14. (a) Each term can be obtained by: $2 \times$ (preceding term $- 1$) $+ 1$
$\therefore$ The next term $= (33 - 1) \times 2 + 1 = 64 + 1 = 65$.

(b) Starting from the 3rd term, each term is obtained by adding the two preceding terms.
$\therefore$ The next term $= 8 + 13 = 21$.

15. (a) Number of squares:
Figure 1: $1 + 3 = 4$, Figure 2: $1 + 3 + 5 = 9$, Figure 3: $1 + 3 + 5 + 7 = 16$

(b) (i) 9 (ii) The total number of squares $= 16 + 9 = 25$

(c) The sequence is: 4 , 9 , 16 , 25 , ...
The 1st term $= (1 + 1)^2$, the 2nd term $= (2 + 1)^2$, the 3rd term $= (3 + 1)^2$…
$\therefore$ The 10th term $= (10 + 1)^2 = 121$

16. (a) 15 (b) The term after 107 $= 107 + 5 = 112$.

(c) When a term is an odd number, the next term is formed by adding +5 to it.
When a term is an even number, the next term is formed by subtracting 1 from it.

17. (a)

Number of participants	2	3	4	5	6	7	8
Number of handshakes	1	3	6	10	15	21	28

(b) Triangular Number Pattern.

(c) The new sequence is: 2, 6, 12, 20, 30, 42, 56
$2 = 1 \times 2$, $6 = 2 \times 3$, $12 = 3 \times 4$, $20 = 4 \times 5$, $30 = 5 \times 6$, $42 = 6 \times 7$, $56 = 7 \times 8$

(d) From (c), the following pattern is observed:

No. of participants	2	3	4	5
No. of handshakes	$1 = \frac{1\times 2}{2}$	$2 = \frac{2\times 3}{2}$	$6 = \frac{3\times 4}{2}$	$10 = \frac{4\times 5}{2}$

$\therefore$ When there are n participants, the no. of handshakes $= \frac{n(n-1)}{2}$.

(e) When there are 50 participants, the no. of handshakes $= \frac{50\times 49}{2} = 1225$.

18. Number of dots:
4th figure : 26 , 5th figure : $26 + 2 \times 4 + 4 = 38$, 6th figure : $38 + 2 \times 5 + 4 = 52$
$\therefore$ Number of dots in the 7th figure $= 52 + 2 \times 6 + 4 = 68$

19. (a) The 10th term $= 52 - 9\,(10) = 52 - 90 = -38$. ***Ans.*** True.

(b) The 5th term $= 52 - 9\,(5) = 52 - 45 = 7$. The 6th term $= 52 - 9\,(6) = 52 - 54 = -2$.
$7 + (-2) = 5$ ***Ans.*** Not true.

(c) Each term is smaller than its preceding term.
The 5th term $= 7 > 0$, the 6th term $= -2 < 0$.
$\therefore$ There are only 5 positive terms. ***Ans.*** Not true.

20. (a) Number of squares in the figures:
1st : 1 , 2nd: $1 + 3 = 4$, 3rd: $4 + 3 = 7$, 4th: $7 + 3 = 10$

(b) Number of squares in the 6th figure $= (10 + 3) + 3 = 16$

(c) The number of squares added = $(4 \times 6 + 1) - 16 = 25 - 16 = 9$

21. (a) $b = 10$, $c = 13$, $d = 16$, $x = 6$, $y = 10$

(b) In each figure, the number of sticks is 3 more than that of the preceding figure.
$\therefore$ Number of sticks in the 5th figure = $16 + 3 = 19$.

(c) (i) Sequence of triangular numbers.

(ii) Number of rectangles in the 5th figure = $10 + 5 = 15$.
Number of rectangles in the 6th figure = $15 + 6 = 21$.

22. (a) Number of dots: Figure 1: $1 \times 2 = 2$, Figure 2: $2 \times 3 = 6$,
Figure 3: $3 \times 4 = 12$, Figure 4: $4 \times 5 = 20$
Following the same pattern, number of dots:
Figure 5: $5 \times 6 = \underline{30}$, Figure 6: $6 \times 7 = \underline{\underline{42}}$

(b) (i) Number of dots to be removed:
Figure 1: $2 - \frac{1}{2}(1^2 - 1) = 2 - 0 = 2$, Figure 2: $6 - \frac{1}{2}(2^2 - 2) = 6 - 1 = 5$,
Figure 3: $12 - \frac{1}{2}(3^2 - 3) = 12 - 3 = 9$, Figure 4: $20 - \frac{1}{2}(4^2 - 4) = 20 - 6 = 14$

(ii) The 10th term = $\frac{1}{2}(10^2 - 10) = \frac{1}{2} \times 90 = 45 < 50$
The 11th term = $\frac{1}{2}(11^2 - 11) = \frac{1}{2} \times 110 = 55 > 50$
$\therefore$ Yes, I agree.

23. (a) $x = 6$, $y = 12$, $z = 20$

(b) (i) $a = 2$, $b = 3$, $c = 4$, $d = 5$ (ii) $a_5 = 5 \times 6 = 30$, $a_6 = 6 \times 7 = 42$

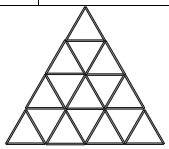
(c) (i) $q = 101$

(ii) $32 + 34 + 36 + ... + 200 = (2 + 4 + 6 + ... + 200) - (2 + 4 + 6 + ... + 30)$
$= a_{100} - a_{15} = 100 \times 101 - 15 \times 16 = 9860$

24. (a)

Figure	No. of small triangles	No of sticks
1st	1	3
2nd	4	9
3rd	9	18

(b) (i)



Number of triangles in the 4th figure = $18 + 7 = 25$

(ii) A sequence of square numbers.
(iii) Number of triangles = $20^2 = 400$

(c) Number of sticks in the figures:
1st : 3 , 2nd: $3 + 3 + 3 = 9$, 3rd: $9 + 3 + 3 = 18$
The number of sticks in the n^{th} figure can be obtained by adding $n \times 3$ to the number of sticks in the preceding figure.
$\therefore$ Number of sticks in the 5th figure = $(18 + 4 \times 3) + 5 \times 3 = 45$

25. (a) The 1st term is odd. $\because$ 4 is even, $\therefore$ $4 \times (3^n)$ must be even.
The sum of an odd number and an even number must be odd.
$\therefore$ Yes, I agree.

(b) The 5^{th} term = $161 + 4\,(3^4) = 161 + 4 \times 81 = 161 + 324 = 485$

(c) (i) $5 = 6(1) - 1$, $\therefore a = \underline{1}$. $17 = 6(3) - 1$, $\therefore b = \underline{3}$. $53 = 6(9) - 1$, $\therefore c = \underline{9}$.
$161 = 6(27) - 1$, $\therefore d = \underline{27}$. $485 = 6(81) - 1$, $\therefore e = \underline{81}$.

(ii) $b = 3a$, $c = 3b$, $d = 3c$, $e = 3d$
$\therefore$ the 6^{th} term of the given sequence = $6(3e) - 1 = 6(3\times81) - 1 = 1457$

Unit 5: Linear equations (1)

1. (a) $y = \frac{3}{5}\times 6$, $y = 3\frac{3}{5}$ (b) $3 - \frac{1}{4} = y$, $y = 2\frac{3}{4}$ (c) $m = 0$

(d) $4n = -2$, $n = -\frac{1}{2}$ (e) $-2a = -16$, $a = 8$

(f) $-9x = -18$, $x = 2$ (g) $-5x + 3 + x = 0$, $-4x = -3$, $x = \frac{3}{4}$

(h) $7x - (2x + 3) = 15 - 6x$, $11x = 18$, $x = 1\frac{7}{11}$

2. (a) $12x + 10x - 60 = 25x + 60$, $-3x = 120$, $x = -40$

(b) $2x + 1 - 15 = 30$, $2x = 44$, $x = 22$

(c) $30 + 20x = 42$, $20x = 12$, $x = \frac{3}{5}$

(d) $6 + (x + 1) = 2x$, $-x = -7$, $x = 7$

(e) $4x + 12 = 3x + 15$, $x = 3$

(f) $4y - (y - 3) = 36$, $3y = 33$, $y = 11$

3. (a) $0.2p = 2.2$, $p = 11$

(b) $0.6y + 1.2 = 1.8y + 0.6 + 1$, $-1.2y = 0.4$, $y = -\frac{1}{3}$

(c) $0.8x - 4 = 0.08$, $0.8x = 4.08$, $x = 5.1$

4. (a) $30x + 40 = 12x$, $18x = -40$, $x = -2\frac{2}{9}$

(b) $15 - 18x = -8x$, $15 = 10x$, $x = 1\frac{1}{2}$

(c) $x - 2 + 6x + 15 = 4$, $7x = -9$, $x = -1\frac{2}{7}$

(d) $2x - 16 - 3x - 6 = 4x + 7$, $-5x = 29$, $x = -5\frac{4}{5}$

(e) $3x - 2x - 6 = 4 - x$, $2x = 10$, $x = 5$

(f) $3a - 5a - 5 = 2a + 10$, $-4a = 15$, $a = -3\frac{3}{4}$

5. (a) $3k - 2(2k + 2) = 0$, $3k - 4k - 4 = 0$, $k = -4$

(b) $-6(7a - 2) + 2 - 2a = 9a - 2$, $-44a + 14 = 9a - 2$, $-53a = -16$, $a = \frac{16}{53}$

6. (a) $6+3(-x+7)=2(2x-14)$, $-3x+27=4x-28$, $-7x=-55$, $x=7\frac{6}{7}$

   (b) $15(x-3)+8(x+1)=2(7x-2)$, $23x-37=14x-4$, $9x=33$, $x=3\frac{2}{3}$

   (c) $2(x-2)-7(3x+6)=4x-1$, $-19x-46=4x-1$, $-23x=45$, $x=-1\frac{22}{23}$

   (d) $4(6x+7)-5(x-8)=120$, $19x+68=120$, $19x=52$, $x=2\frac{14}{19}$

7. (a) $0.48-5.85x=0.25(4.6x+0.8)$, $0.48-5.85x=1.15x+0.2$,
   $0.28=7x$, $x=0.04$

   (b) $\frac{15}{2}+\frac{10y}{3}+\frac{1-6y}{5}=\frac{1}{3}(-\frac{13}{20}+\frac{9y}{4}-\frac{1}{4})$, $\frac{15}{2}+\frac{10y}{3}+\frac{1-6y}{5}=-\frac{3}{10}+\frac{3y}{4}$,

   $450+200y+12(1-6y)=-18+45y$, $83y=-480$, $y=-5\frac{65}{83}$

8. (a) $7a-1=3a$, $4a=1$, $a=\frac{1}{4}$

   (b) $3m-15=5m$, $-2m=15$, $m=-7\frac{1}{2}$

   (c) $36=3x-21$, $57=3x$, $x=19$

   (d) $3\,(3)-6\,(1)+2\,(2)=14\,(6x)$, $9-6+4=84x$, $84x=7$, $x=\frac{1}{12}$

   (e) $x-(4x-1)=-(x+1)$, $-2x=-2$, $x=1$

9. $\frac{7x}{3}=5x-3x+2$, $7x=15x-9x+6$, $x=6$

10. $\frac{5y}{12}+\frac{4y-3}{60}+y=\frac{4y-3}{10}-y$, $25y+(4y-3)+60y=6(4y-3)-60y$,

    $125y=-15$, $y=-\frac{3}{25}$

11. (a) $a=3$ or $a=-3$

    (b) $3x+2=3$ or $3x+2=-3$, $3x=1$ or $3x=-5$, $x=\frac{1}{3}$ or $x=-1\frac{2}{3}$

12. When $x+3=0$, L.H.S = 0 = R.H.S. , $\therefore\ x=-3$

    When $x+3\neq 0$, $5-2x=7+x$, $-3x=2$, $\therefore\ x=-\frac{2}{3}$ ***Ans.*** $x=-3$ *or* $-\frac{2}{3}$

13. $2x+9=2$, $2x=-7$, $x=-3.5$, $\therefore\ (x+3)(x+6)=(-3.5+3)(-3.5+6)=-1.25$

14. Let $y=4x$, $\frac{y-1}{y+1}=6$, $y-1=6(y+1)$, $-5y=7$, $y=-1.4$,

    $\therefore\ (4x-1)(4x+1)=(y-1)(y+1)=(-1.4-1)(-1.4+1)=0.96$

15. $\frac{x}{4}-\left(\frac{4}{2}-\frac{9}{3}\right)=-7$, $\frac{x}{4}-(-1)=-7$, $\frac{x}{4}+1=-7$, $\frac{x}{4}=-8$, $x=-32$

16. $(-6-4)^2 - s = \frac{1}{4}(-22+10)$, $(-10)^2 - s = \frac{1}{4}(-12)$, $100 - s = -3$, $s = 103$

17. $13 - x - (110 - 10x + x) = 29 - 10x$, $13 - x - 110 + 9x = 29 - 10x$,
$-97 + 8x = 29 - 10x$, $18x = 126$, $x = 7$

18. $4(39 - 2x) - x = 94 + 2(x - 2)$, $156 - 8x - x = 94 + 2x - 4$, $156 - 9x = 90 + 2x$,
$66 = 11x$, $x = 6$

19. $6(5x + 10) + 3(x + 4) = 6(3) - 4(2x + 7)$, $30x + 60 + 3x + 12 = 18 - 8x - 28$,
$33x + 72 = -10 - 8x$, $41x = -82$, $x = -2$

20. $\frac{2x}{3} - \frac{x-1}{6} = 5 - \frac{x-2}{3}$, $2(2x) - (x - 1) = 6(5) - 2(x - 2)$, $4x - x + 1 = 30 - 2x + 4$,
$3x + 1 = 34 - 2x$, $5x = 33$, $x = \frac{33}{5}$

21. $\frac{1}{12}\left(\frac{4y-3-25y}{5}\right) + y = \frac{1}{2}\left(\frac{4y-3-10y}{5}\right)$, $\frac{-21y-3}{5} + 12y = \frac{6(-6y-3)}{5}$
$-21y - 3 + 60y = -36y - 18$, $39y - 3 = -36y - 18$, $39y + 36y = -18 + 3$, $75y = -15$,
$y = \frac{-15}{75} = -\frac{1}{5}$

Unit 6: Linear equations (2)

1. $4x + 8 = 44$, $4x = 36$, $x = 9$

2. Let x be the number, $23 + \frac{x}{7} = 44$, $\frac{x}{7} = 21$, $x = 147$. ***Ans.*** The number is 147.

3. Let x be the number, $x - 7 = \frac{2}{3}x$, $\frac{x}{3} = 7$, $x = 21$. ***Ans.*** The number is 21.

4. Let x be the number, $\frac{x-17}{7} = 5$, $x - 17 = 35$, $x = 52$. ***Ans.*** The number is 52.

5. Let x be the smallest number, then other two numbers are $(x + 1)$ and $(x + 2)$,
$x + (x + 1) + (x + 2) = 114$, $3x = 111$, $x = 37$. ***Ans.*** The numbers are 37, 38, 39.

6. Let x be the smaller number, then the larger number is $(124 - x)$,
$(124 - x) - x = 22$, $-2x = -102$, $x = 51$. ***Ans.*** The smaller number is 51.

7. Let x be the greater number, then the smaller number is $(x - 15)$,
$2x - (x - 15) = 47$, $x = 32$. ***Ans.*** The numbers are 17 and 32.

8. Let x cm be the width, then the length is $(x + 3)$ cm, $2[x + (x + 3)] = 78$,
$2x + 3 = 39$, $2x = 36$, $x = 18$. ***Ans.*** The width of the rectangle is 18 cm.

9. Let x cm be the width, then the length is $(\frac{x}{2} + 12)$ cm , $2[x + (\frac{x}{2} + 12)] = 32$,
$\frac{3x}{2} + 12 = 16$, $3x = 8$, $x = 2\frac{2}{3}$. $\therefore$ The length is $2\frac{2}{3} \times \frac{1}{2} + 12 = 13\frac{1}{3}$ cm.
Ans. The width is $2\frac{2}{3}$ cm and the area is $2\frac{2}{3} \times 13\frac{1}{3} = 35\frac{5}{9}\text{ cm}^2$.

10. Let \$$x$ be the amount Maggie has, then Kathy has \$$3x$, $x + 3x = 640$, $4x = 640$,

$x = 160$. ***Ans.*** Maggie has \$160 and Kathy has \$480.

11. Let x kg be the weight of Benson's son, then Benson's weight is $(2x + 27)$ kg ,
$2x + 27 = 73$, $2x = 46$, $x = 23$. ***Ans.*** The weight of Benson's son is 23 kg.

12. Let x be the present age of Paul, then Paul's mother is $3x$ years old now,
$3x - 4 = 35$, $3x = 39$, $x = 13$. ***Ans.*** Paul is 13 years old now.

13. Let x be the present age of the son, then the woman is $(54 - x)$ years old now,
$(54 - x) - 3 = 5(x - 3)$, $66 = 6x$, $x = 11$. ***Ans.*** The son is 11 years old now.

14. Let x be the number of years, $69 - x = 7(15 - x)$, $6x = 36$, $x = 6$.
Ans. The number of years is 6.

15. Let x be the present age of the son, then the father is $4x$ years old now,
$4x + 15 = 2(x + 15) + 5$, $2x = 20$, $x = 10$.
Ans. The present ages of the father and the son are 40 and 10 respectively.

16. Let \$$x$ be the amount A originally has, then B originally has \$$(x - 80)$,
$x - 10 = 6[(x - 80) + 10]$, $x - 10 = 6(x - 70)$, $410 = 5x$, $x = 82$.
Ans. A originally has \$82.

17. Let \$$x$ be the amount Betty originally had, $x - 43 - \frac{1}{3}(x - 43) = 48$,
$3x - 129 - (x - 43) = 144$, $2x = 230$, $x = 115$. ***Ans.*** Betty originally had \$115.

18. Let \$$x$ be the amount Adam originally had, $\frac{x}{6} \times 4 = \frac{3}{4}(x - \frac{x}{6}) + 14$,
$\frac{2x}{3} = \frac{5x}{8} + 14$, $16x = 15x + 336$, $x = 336$. ***Ans.*** Adam originally had \$336.

19. Let \$$x$ be the original price of an orange, then the new price is \$$(x + 0.4)$,
$35x + 3 = 30(x + 0.4)$, $5x = 9$, $x = 1.8$. ***Ans.*** Original price of an orange is \$1.8.

20. Let x be the number of \$2 coins, then the number of \$5 coins is $(x + 10)$,
$2x + 5(x + 10) = 134$, $7x = 84$, $x = 12$. $\therefore$ Number of \$5 coins = 12 + 10 = 22.
Ans. The total number of coins is 12 + 22 = 34.

21. Let \$$x$ be the amount received by the younger son, then the amount received
by the elder son is \$$(6000 - x)$, $\frac{1}{3}(6000 - x) = x + 500$, $6000 - x = 3x + 1500$,
$4500 = 4x$, $x = 1125$. ***Ans.*** The amount received by the younger son is \$1125.

22. Let x be the number of biscuits in the smaller portion, then the number of
biscuits in the bigger portion is $(50 - x)$, $4x = 3(50 - x) + 4$, $7x = 154$, $x = 22$.
Ans. The number of biscuits in the smaller portion is 22.

23. Let \$$x$ be the amount received by the elder brother, then the younger brother
received \$$(800 - x)$, $800 - x = \frac{3}{4}x + 30$, $770 = \frac{7}{4}x$, $3080 = 7x$, $x = 440$.
Ans. The elder and the younger brother received \$440 and \$360 respectively.

24. Let x h be the time taken he had ridden at the faster speed, then he had ridden
$(10 - x)$ h at the slower speed, $7(10 - x) + 12x = 95$, $5x = 25$, $x = 5$.
Ans. He had ridden 5 h at the faster speed.

25. Let x km be the distance between A and B, $\frac{x}{12}+\frac{x}{10}=3\frac{2}{3}$, $\frac{x}{12}+\frac{x}{10}=\frac{11}{3}$,
$5x + 6x = 220$, $11x = 220$, $x = 20$. ***Ans.*** A and B is 20 km apart.

26. Let x km/h be the power speed of the boat, $x + 6 = 2(x – 6)$, $x = 18$.
Ans. The power speed of the boat is 18 km/h.

27. Let x km/h be Car A's speed, then Car B's speed is $2x$ km/h , $4(x + 2x) = 540$,
$3x = 135$, $x = 45$. ***Ans.*** Car A's speed is 45 km/h and Car B's speed is 90 km/h.

28. Let x h be the time taken, $(45 + 75)x = 408$, $120x = 408$, $x = 3.4$.
Ans. They will meet after 3.4 h.

29. Let x m/s be the speed of Jason, $(x – 7) \times 2 \times 60 = 240$, $x – 7 = 2$, $x = 9$.
Ans. The speed of Jason is 9 m/s.

30. Let x be the smallest number, then the other two numbers are $(x + 2)$ and $(x + 4)$,
$x + (x + 2) + (x + 4) = 27$, $3x = 21$, $x = 7$. $\therefore$ The other two numbers are 9 and 11.
Ans. Their product is $7 \times 9 \times 11 = 693$.

31. Let x be the tens digit of the original number, then the units digit is $(11 – x)$,
$10x + (11 – x) = [10(11 – x) + x] + 27$, $9x + 11 = – 9x + 137$, $18x = 126$, $x = 7$.
$\therefore$ The units digit is $11 – 7 = 4$. ***Ans.*** The original two-digit number is 74.

32. Let x be the present age of the son, then the father is $5x$ years old now,
$5x + 13 = 2(x + 13) + 8$, $3x = 21$, $x = 7$.
Ans. The age of the father when his son was born was $5(7) – 7 = 28$.

33. Let \$$x$ be the cost of a scarf, then the cost of a brooch is \$$(243 – 2x)$,
$x = 2(243 – 2x) + 9$, $5x = 495$, $x = 99$,
$\therefore$ The cost of a brooch = $243 – 2(99) = 45$.
Ans. The difference in cost between a scarf and a brooch is $99 – 45 = 54$.

34. Let x be the number of \$1 coins Mandy has, then the number of 50-cent coins is
$90 – 16 – x = 74 – x$, $0.2(16) + 0.5(74 – x) + 1(x) = 53.2$, $0.5x = 13$, $x = 26$.
Ans. Mandy has 26 \$1 coins.

35. Let x be the number of 50-cent coins, then there are $(2x – 4)$ \$1 coins and the
number of 20-cent coins is $2x – 4 + 5 = 2x + 1$,
$0.2(2x + 1) + 0.5x + 1(2x – 4) = 54.2$, $2.9x = 58$, $x = 20$.
Ans. The number of \$1 coins is $2(20) – 4 = 36$.

36. Let \$$x$ be the price of 1 kg of Type A coffee, then the price of 1 kg of Type B coffee is
\$$(x + 30)$, $7x + 8(x + 30) = 33.5 \times 60$, $15x = 1770$, $x = 118$.
Ans. The price of 1 kg of Type A coffee is \$118.

37. Let x km be the distance between A and B,
$\frac{x}{8}+\frac{x}{12}=\frac{2x}{11}+\frac{21}{60}$, $\frac{x}{8}+\frac{x}{12}-\frac{2x}{11}=\frac{7}{20}$, $\frac{7x}{264}=\frac{7}{20}$, $x=\frac{7}{20}\times\frac{264}{7}$, $x = 13.2$.
Ans. The distance between A and B is 13.2 km.

38. Let x km/h be the speed for the return journey and d km be the distance of the journey,

$\frac{d}{40}+\frac{d}{x}=\frac{2d}{60}$, $\frac{1}{40}+\frac{1}{x}=\frac{1}{30}$, $\frac{1}{x}=\frac{4}{120}-\frac{3}{120}$, $\frac{1}{x}=\frac{1}{120}$, $x = 120$.

Ans. The speed for the return journey should be 120 km/h.

39. Let x h be the time taken for the later train to catch up the earlier train,
$250x = 100(x + 3)$, $150x = 300$, $x = 2$.
Ans. It will catch up the earlier train at 9pm.

40. Let x min be the time taken and y m/min be the original speed, $\frac{3}{4}y \cdot x = 2(15y)$,
$3x = 120$, $x = 40$. ***Ans.*** The required time taken is 40 min.

41. Let x h be the time Q takes to overtake P, $50x = 40(x + 0.5)$, $10x = 20$, $x = 2$,
$\therefore$ P has travelled 2.5 h before he is overtaken by Q.
Let y h be the time R starts after P , $60(2.5 – y) = 40(2.5)$, $50 = 60y$, $y = \frac{5}{6}$.
Ans. R must start $\frac{5}{6}$ h after P.

42. $\frac{x}{4}\times 4\times 16 = 4x + 60$, $16x = 4x + 60$, $12x = 60$, $x = 5$

43. Let x be the number of \$5 coins , $\frac{x}{12}=\frac{x}{15}+15$, $5x = 4x + 900$, $x = 900$.
Ans. The amount of money in the safe is 5(900) = \$4500.

44. Let x be the total number of books, $\frac{x-2}{14}=\frac{x-6}{18}+4$, $9(x – 2) = 7(x – 6) + 504$,
$2x = 480$, $x = 240$. ***Ans.*** The total number of books is 240.

45. Let x be the number of cards John has, then David has $(240 – x)$ cards,
$x – 20 = 2[(240 – x) + 20]$, $x – 20 = 2(260 – x)$, $3x = 540$, $x = 180$.
$\therefore$ John has 180 cards and David has 240 – 180 = 60 cards.
Let y be the number of cards John should give to David more,
$3[180 – (20 + y)] = 60 + (20 + y)$, $3(160 – y) = 80 + y$, $400 = 4y$, $y = 100$.
Ans. John should give David 100 more flashing cards.

46. (a) The difference = 2 × 7 = 14.
(b) Let y be the number of marbles Mary has, then Paul has $(y + 14)$ marbles,
$(y + 14) + 4 = 2(y – 4)$, $y = 26$. ***Ans.*** $x = 26 + (26 + 14) = 66$.
(c) The number of marbles Mary has = 26.

47. (a) Let x be the actual number of wrong answers, then the actual numbers of correct answers and unanswered questions are $(40 – 2x)$ and x respectively,
$4[(40 – 2x) + 2] + (–2)(x – 2) + (–1)(x) = 106$,
$4(42 – 2x) – 2x + 4 – x = 106$, $–11x = –66$, $x = 6$.
Ans. She should have got $4(40 – 2 \times 6) + (–2)(6) + (–1)(6) = 94$ marks.
(b) The actual number of correct answers = 40 – 2 × 6 = 28.

48. (a) Let x be the original number of passengers, $\frac{3}{4}(\frac{4}{5}x+4)+1=\frac{2}{3}x$,

$\frac{3x}{5}+4=\frac{2x}{3}$, $9x + 60 = 10x$, $x = 60$. ***Ans.*** Original number of passengers is 60.

(b) The total fares collected = $20(60 + 4 + 1) = \$1300$.

49. Let x be the number of type A, $\therefore$ the number of type B is $(10 – x)$

$35x + 48 (10 – x) = 389$, $35x + 480 – 48x = 389$, $480 – 13x = 389$,

$13x = 91$, $x = 7$. $\therefore$ $10 – x = 10 – 7 = 3$

Ans. She bought 7 cans of type A, 3 cans of type B.

50. Let n be the original number of students.

$21n = 12 (n + 4) + 6$, $21n = 12n + 48 + 6$, $9n = 42$, $n = 6$

$\therefore$ Number of stickers = $6 \times 21 = 126$

[OR: Let x be the total number of stickers. $\frac{x}{21}=\frac{x-6}{12}-4$, $x = 126$.]

51. (a) Let x be the present age of Vicky. $\therefore$ The present age of Doris is $(80 – x)$.

$x – 3 = 2 [(80 – x) – 3] + 11$, $x – 3 = 2 (77 – x) + 11$,

$x – 3 = 154 – 2x + 11$, $3x = 168$, $x = 56$.

Ans. Vicky is 56 years old now.

(b) The present age of Doris = $80 – 56 = 24$.

n years ago, Doris age was $(24 – n)$, Vicky age was $56 – n$.

$24 – n = \frac{1}{3}(56 – n)$, $73 – 3n = 56 – n$, $16 = 2n$, $n = 8$

$\therefore$ Doris age at that time was $24 – 8 = 16$.

52. (a) Let x be the number of Bob's marbles.

$\therefore$ The number of marbles Adam has = $x – 43$.

$x – 6 = 3 [(x – 43) + 6] – 5$, $x – 6 = 3 (x – 37) – 5$, $x – 6 = 3x – 116$,

$110 = 2x$, $x = 55$. ***Ans.*** Bob originally has 55 marbles.

(b) The number of marbles Adam has = $55 – 43 = 12$.

Let n be the number of marbles Bob gives Adam, so that they will have equal number of marbles.

$61 – n = 18 + n$, $2n = 43$, $n = 21.5$. ***Ans.*** Bob should give Adam 22 marbles.

53. (a) (i) The distance Larry travelled = $60 \times \frac{y}{60} = y$ km.

(ii) The distance Paul travelled = $45 \times \frac{y+4}{60} = \frac{3(y+4)}{4}$ km

(b) $y – \frac{3(y+4)}{4} = 1$, $4y – 3y – 12 = 4$, $y = 16$

(c) The distance Chris travelled = $16 + \frac{3(16+4)}{4} = 16 + 15 = 31$ km

$50 \times \frac{k}{60} = 31$, $k = \frac{31 \times 6}{5} = 37.2$

Unit 7: Polynomial (1)

1. (a) $\frac{5}{4}a$ (b) $-\frac{4}{15}y^2$ (c) $-\frac{5}{7}b^3$ (d) $= -\frac{4}{3}xy^2 + \frac{1}{3}xy^2 = -xy^2$

2. (a) $= 14mn - 17mn = -3mn$ (b) $-4k^3 - 2$ (c) $-7x^2 + 6x + 4$
   (d) $= xy^2 - 6x^2y + x^2y = xy^2 - 5x^2y$ (e) $= p - q - 5p - 9q = -4p - 10q$
   (f) $= 2x - 7y + 6x - y - 2x - 3y = 6x - 11y$

3. (a) 5, 4, 3, 0, –2 (b) 3, 3, 0, –5, 4 (c) 4, 3, –6, 1, 0

4. (a) 5 (b) –1 (c) 0

5. (a) $3x^4 + 3x^2 - x + 5$ (b) $-x^5 + 2x^4 - x^3 + 8x^2 + 1$

6. (a) $8 - 2y + 16y^2 - 9y^3$ (b) $-1 + 4y^2 + 2y^3 - y^4 + y^5$

7. (a) $12x$ (b) $-4a - 2b$
   (c) $-m - 3n$ (d) $5x^2 - 4$

8. (a) $-12p + 12$ (b) $-x^2 + x + 11$
   (c) $-4a^2 + 5a - 30$ (d) $13x^3 + 5x^2 - 2x + 14$

9. (a) $-7x^2 + 2x - 3$ (b) $2x^2 + 6x - 10$
   (c) $-x^2 - 10x - 4$

10. (a) $4xy^2 - 4x^2y$ (b) $= 5x^3y - 9x^3y - 6x^3y + x^3y = -9x^3y$
    (c) $3a^2b - ab^2 + 2ab + 4a - 10$

11. (a) $= 12x^2 + 3y^2 - 7x^2 - 8y^2 = -5y^2 + 5x^2$
    (b) $= -3x^2 + 2 - 11x + 10 + 8x^2 - 6x = 12 - 17x + 5x^2$
    (c) $= x^2 - 8x + 1 - 3x^2 - 5x - 2 + 4x^2 - 3x + 6 = 5 - 16x + 2x^2$
    (d) $= -7y^2 - 9xy + 4x^2 + 5xy - 6y^2 + 11x^2 - 3y^2 + 8xy + 5x^2$
    $= -7y^2 - 6y^2 - 3y^2 - 9xy + 5xy + 8xy + 4x^2 + 11x^2 + 5x^2 = -16y^2 + 4xy + 20x^2$

12. (a) $= 4p - 3q - 5p - q = -p - 4q$ (b) $= 18x^2 + 5y^2 - 3x^2 - 2y^2 = 15x^2 + 3y^2$
    (c) $= -3k^2 + 2 - 8k + 7 + 8k^2 - 8k = 5k^2 - 16k + 9$
    (d) $= 6x - y + 2x - 7y - 2x - 3y = 6x - 11y$
    (e) $= x^2 - 8x + 1 - 3x^2 - 5x - 2 + 4x^2 - 3x + 6 = 2x^2 - 16x + 5$
    (f) $= -7a^2 - 9ac + 4c^2 - 3a^2 + 8ca + 5c^2 + 5ca - 6a^2 + 11c^2 = -16a^2 + 4ac + 20c^2$

13. $= (-3x^2 + 7x - 4) - (3x^2 - 5x - 11) = -3x^2 + 7x - 4 - 3x^2 + 5x + 11 = -6x^2 + 12x + 7$

14. $= (-x^4 + x^3 - x^2 - 2x - 5) + (-5x^4 + x^2 + 4x + 8) = -x^4 + x^3 - x^2 - 2x - 5 - 5x^4 + x^2 + 4x + 8$
    $= -6x^4 + x^3 + 2x + 3$

15. (a) $= \frac{15}{12}a^2 - \frac{20}{12}a^2 = -\frac{5}{12}a^2$ (b) $= \frac{15}{40}y^3 - \frac{48}{40}y^3 + \frac{7}{40}y^3 = -\frac{26}{40}y^3 = -\frac{13}{20}y^3$

    (c) $= -\frac{11}{4}xy^2 + \frac{9}{2}xy^2 + 3\frac{2}{3}x^2y = -\frac{11}{4}xy^2 + \frac{18}{4}xy^2 + 3\frac{2}{3}x^2y = 1\frac{3}{4}xy^2 + 3\frac{2}{3}x^2y$

    (d) $= -\frac{6}{7}ab - ab = -1\frac{6}{7}ab$

Unit 8: Polynomials (2)

1. (a) $= 1 - 27 = -26$ (b) 81 (c) $= -16 + 49 = 33$

   (d) $= \dfrac{19^7 \cdot 19^7}{19^{15}} = \dfrac{19^{14}}{19^{15}} = \dfrac{1}{19}$ (e) $= \dfrac{64+27}{16-9} = \dfrac{91}{7} = 13$

2. (a) $3k^7$ (b) $3b^8$ (c) $= \dfrac{6m^6n}{18m^3n^2} = \dfrac{m^3}{3n}$ (d) $= \dfrac{25ab^2}{5a^2b} = \dfrac{5b}{a}$

   (e) $12a^6b^7$ (f) $30x^7y^2z^5$

3. (a) $-54x^6$ (b) $= 16n^7 \times (-1) = -16n^7$ (c) $= y^3 (2y^3) = 2y^6$

   (d) $= \dfrac{25a^4}{1} \times \dfrac{1}{-a^3} \times \dfrac{1}{-5a^6} = \dfrac{5a^4}{a^9} = \dfrac{5}{a^5}$

4. (a) $= \dfrac{15ab^3 \times 8ab^2}{4a^2b^4} = \dfrac{30a^2b^5}{a^2} = 30b^5$ (b) $= \dfrac{35x^6y^2}{14xy^4 \times 2x^2} = \dfrac{5x^6y^2}{4x^3y^4} = \dfrac{5x^3}{4y^2}$

   (c) $= -\dfrac{4a^2b^3 \times a^3b^2}{6a^2b} = -\dfrac{2}{3}a^3b^4$

   (d) $= \dfrac{-14x^3y^6}{1} \times \dfrac{-7}{xy^9} \times \dfrac{-2x^2y^3}{1} = -\dfrac{196x^5y^9}{xy^9} = -196x^4$

5. (a) $12y^2 - 30y + 6$ (b) $-6a^2 + 10a$

   (c) $6w^4 + 16w^3 + 36w^2$ (d) $-10x^3 - 25x^2 + 10x$

6. (a) $2x^2 + 15x + 18$ (b) $4y^2 + y - 3$

   (c) $42r^2 - 25rs + 3s^2$ (d) $2x^2 + 5x - 18$

7. (a) $= -5x^2 + 35x - x + 7 = -5x^2 + 34x + 7$

   (b) $= -8a^2 - 12ab - 12ab - 18b^2 = -8a^2 - 24ab - 18b^2$

   (c) $= -16y^4 + 2y^3 + 40y^3 - 5y^2 = -16y^4 + 42y^3 - 5y^2$

   (d) $= 4x^3 + 12x^2 - 24x - x^2 - 3x + 6 = 4x^3 + 11x^2 - 27x + 6$

   (e) $= -10x^4 + 14x^3 - 2x^2 + 15x^2 - 21x + 3 = -10x^4 + 14x^3 + 13x^2 - 21x + 3$

8. (a) $= -3\,(16x^2 + 38x - 5) = -48x^2 - 114x + 15$

   (b) $= 3x^3 + 6x^2 - 2x - 12x^2 - 24x + 8 = 3x^3 - 6x^2 - 26x + 8$

   (c) $= 9x^3 + x^2y - 4xy^2 + 18x^2y + 2xy^2 - 8y^3 = 9x^3 + 19x^2y - 2xy^2 - 8y^3$

   (d) $= -15x^2 + 30x - 35 - 6x^3 + 12x^2 - 14x = -6x^3 - 3x^2 + 16x - 35$

9. (a) $= 8x - 6 - 12x + 6 = -4x$

   (b) $= 24a^2 - 15a - 8a^2 - 12a = 16a^2 - 27a$

   (c) $= 16b - 4\,(7 + 5 - 10b) = 16b - 48 + 40b = 56b - 48$

   (d) $= 32y^2 - 24y - 8y^2 - 28y - 3y^2 + 6y = 21y^2 - 46y$

   (e) $= -15a^4 - 30a^3 + 40a^2 - 6a^4 + 6a^3 + 2a^2 - 4a$

   $= -21a^4 - 24a^3 + 42a^2 - 4a$

10. Total value of the coins $= 2\,(a^2 + 2a - 5) + 5\,(3a^2 - a + 2) = \$\,(17a^2 - a)$

11. (a) The perimeter $= 2[(3k + 4) + (15 - 2k)] = 2(k + 19) = (2k + 38)$ cm

    (b) The area $= (3k + 4)(15 - 2k) = (-6k^2 + 37k + 60)$ cm^2

12. (a) $45xy^2z^6 = 3 \cdot 15 \cdot x \cdot y^2 \cdot z^6$, $x^2yz^3 = 7 \cdot 15 \cdot x^2 \cdot y \cdot z^3$ $\therefore$ HCF $= 15xyz^3$

    (b) $6a^2b^3 = 6 \cdot a^2 \cdot b^3$, $18ab^4 = 6 \cdot 3 \cdot a \cdot b^4$; $24b^2 = 6 \cdot 4 \cdot b^2$ $\therefore$ HCF $= 6b^2$

13. (a) $33a^2bc^8 = 11 \cdot 3 \cdot a^2 \cdot b \cdot c^8$, $44ab^5c^6 = 11 \cdot 4 \cdot a \cdot b^5 \cdot c^6$

$\therefore$ LCM $= 11 \cdot 3 \cdot 4 \cdot a^2 \cdot b^5 \cdot c^8 = 132a^2b^5c^8$

(b) $15p^5q^3 = 3 \cdot 5 \cdot p^5 \cdot q^3$, $6pq^4 = 2 \cdot 3 \cdot p \cdot q^4$, $10p^2q = 2 \cdot 5 \cdot p^2 \cdot q$

$\therefore$ LCM $= 2 \cdot 3 \cdot 5 \cdot p^5 \cdot q^4 = 30p^5q^4$

14. (a) $=16x^2 - 4x\,(8 - 10x) = 16x^2 - 32x + 40x^2 = 56x^2 - 32x$

(b) $= (3y^2 - 6y + 15) - (3y^2 - 17y - 6) = 11y + 21$

(c) $= (2x^2 - 2x - 24) - (3x^2 - 13x - 10) = -x^2 + 11x - 14$

(d) $= 8k^2 + 3k - (2k - 1)\,(9k - 7) = 8k^2 + 3k - (18k^2 - 23k + 7) = -10k^2 + 26k - 7$

(e) $= (3a + 2)\,(a - 1) - 3(10 - 2a)\,(1 + a) = 3a^2 - a - 2 - 3\,(10 + 8a - 2a^2)$

$= 9a^2 - 25a - 32$

15. $(13)\,(5) - (4)\,(k) = 33$, $65 - 4k = 33$, $32 = 4k$, $k = 8$

16. The coefficient of $x^2 = 2 \times 5 + 7 \times (-4) + (-8) \times (-3) = 10 - 28 + 24 = 6$

17. Coefficient of $y^2 = (13)\,(4) + (-5)\,(-6) + (4)\,(3) - [(1)\,(15) + (9)\,(-8)] = 94 - (-57) = 151$

18. (a) $= (10x^2 + 19x + 6)\,(x - 1) = 10x^3 + 19x^2 + 6x - 10x^2 - 19x - 6$

$= 10x^3 + 9x^2 - 13x - 6$

(b) $= (-20y^2 - 17y + 10)\,(y + 3) = -20y^3 - 17y^2 + 10y - 60y^2 - 51y + 30$

$= -20y^3 - 77y^2 - 41y + 30$

(c) $= 3(-3a^2 + 5ab + 2b^2)\,(a - 5b) = 3(-3a^3 + 20a^2b - 23ab^2 - 10b^3)$

$= -9a^3 + 60a^2b - 69ab^2 - 30b^3$

19. $= \dfrac{k^k}{k \cdot k} = \dfrac{k^k}{k^2} = k^{k-2}$

20. $= 3^n \times 3^n = 3^{n+n} = 3^{2n}$

21. $\dfrac{1}{2^m} = 0.125$, $\dfrac{1}{2^m} = \dfrac{1}{8}$, $2^m = 8$, $2^m = 2^3$ $\therefore$ $m = 3$

22. $3x^3 + 3 = 15$, $3x^3 = 12$, $x^3 = 4$ $\therefore$ $\dfrac{x^6}{3} = \dfrac{(x^3)(x^3)}{3} = \dfrac{4^2}{3} = \dfrac{16}{3} = 5\dfrac{1}{3}$

23. $2^3 = x^3$ $\therefore$ $x = 2$.

$9 \times 9 \times 9 = 3^y$, $3^2 \times 3^2 \times 3^2 = 3^y$, $3^6 = 3^y$ $\therefore$ $y = 6$. $\therefore$ $y^x = 6^2 = 36$

24. (a) $(5 + 3)^2 = 8^2 = 64$, $5^2 + 3^2 = 25 + 9 = 34$. They are not equal.

(b) $= (x + y)\,(x + y) = x\,(x + y) + y\,(x + y) = x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$

(c) $= (3a - 2)\,(3a - 2) = 3a\,(3a - 2) - 2\,(3a - 2) = 9a^2 - 6a - 6a + 4$

$= 9a^2 - 12a + 4$

25. $= (-3x - 7)\,(-3x - 7) = -3x(-3x) - 3x(-7) - 7(-3x) - 7(-7) = 9x^2 + 42x + 49$

26. $= (2x - 1)\,(2x - 1)\,(2x - 1) = (2x - 1)\,(4x^2 - 4x + 1) = 8x^3 - 12x^2 + 6x - 1$

27. (a) $35 - (2x^2 + 4x + 7x + 14) = 9 - 2x^2$, $-11x = -12$, $x = 1\dfrac{1}{11}$

(b) $(y-3)(y+1) = (y-4)(y-5)$, $y^2 + y - 3y - 3 = y^2 - 5y - 4y + 20$,

$7y = 23$, $y = 3\dfrac{2}{7}$

28. The corresponding height of the triangle $= 48a^2b^3 \times 2 \div 6ab = \dfrac{96a^2b^3}{6ab} = 16ab^2$ units

29. $2\,(a + 5) + 5\,(2a^2 + 3a - 4) = 2a + 10 + 10a^2 + 15a - 20 = 10a^2 + 17a - 10$

Ans. The total value of the coins is $\$(10a^2 + 17a - 10)$.

30. $(x + 3y)(y - 2x) \div 2 = [x(y - 2x) + 3y(y - 2x)] \div 2 = [xy - 2x^2 + 3y^2 - 6xy] \div 2$
$= (-2x^2 - 5xy + 3y^2) \div 2 = -x^2 - \frac{5}{2}xy + \frac{3}{2}y^2$

Ans. The area of the triangle is $(-x^2 - \frac{5}{2}xy + \frac{3}{2}y^2)$ sq. units.

31. (a) $(12x - 2) \div 2 - 3x = 6x - 1 - 3x = 3x - 1$

Ans. The width of the rectangle is $(3x - 1)$ cm.

(b) $3x(3x - 1) - (3x - x)(3x - 1 - 1) = 9x^2 - 3x - 2x(3x - 2)$
$= 9x^2 - 3x - 6x^2 + 4x = 3x^2 + x$ ***Ans.*** The decrease in area is $(3x^3 + x)$ cm^2.

32. (a) The total number of marbles in the remaining jars
$= (x - 1) \times (y + 2) = x(y + 2) - 1(y + 2) = xy + 2x - y - 2$

(b) The number of marbles left
$= xy - (xy + 2x - y - 2) = xy - xy - 2x + y + 2 = -2x + y + 2$
[OR: $y - 2(x - 1)\ y - 2 = y - 2x + 2$]

33. Area of the trapezium $= \frac{1}{2}[(2a + b) + (4b - 3a)](5b - a) = \frac{1}{2}(5b - a)(5b - a)$
$= \frac{1}{2}(25b^2 - 10ab + a^2)$ sq. units

34. (a) Width $= \frac{1}{2}(24x - 8) - 5x = 12x - 4 - 5x = (7x - 4)$ cm

(b) Decrease in area $= 5x(7x - 4) - (5x - x)(7x - 4 - 1)$
$= 35x^2 - 20x - 4x(7x - 5) = 7x^2$ cm^2

35. (a) Length $= (20 - x) - 2x = 20 - 3x$, Width $= (5x + 2) - 2x = 3x + 2$, Height $= x$,
$\therefore$ Dimensions of the box are $(20 - 3x)$ cm $\times$ $(3x + 2)$ cm $\times$ x cm

(b) Capacity $= (20 - 3x)(3x + 2)\ x = (40 + 54x - 9x^2)\ x = (40x + 54x^2 - 9x^3)$ cm^3

36. (a) $(x + y)(x^2 - xy + y^2) = x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 = x^3 + y^3$

(b) From (a), when $x = a$, $y = 1$, $(a^3 + 1) \div (a + 1) = a^2 - a(1) + 1^2 = a^2 - a + 1$

(c) From (b), when $a = 200$, $800001 \div 201 = 200^2 - 200 + 1 = 39801$

37. $-y^3 + 6y - 4$

38. $35 - (2x^2 + 4x + 7x + 14) = 9 - 2x^2$, $21 - 2x^2 - 11x = 9 - 2x^2$, $-11x = -12$, $x = \frac{12}{11}$

39. (a) $= 3n \cdot n^2 = 3n^3$ (b) $= \frac{y}{1} \times \frac{1}{5y^3} \times \frac{1}{10y^2} = \frac{1}{50y^4}$ (c) $= \frac{6x^4}{-6x^6} = -\frac{1}{x^2}$

40. (a) Number of marbles of Dickson $= k(k + 1) = k^2 + k$

(b) Number of marbles of Kelvin
$= (k + 4)(k - 3) = k^2 - 3k + 4k - 12 = k^2 + k - 12$
Difference between Dickson and Kelvin
$= (k^2 + k) - (k^2 + k - 12) = k^2 + k - k^2 - k + 12 = 12$
Ans. Dickson has 12 more marbles than Kelvin.

(c) Number of marbles of Simon
$= (k - 3)(k + 6) = k^2 + 6k - 3k - 18 = k^2 + 3k - 18$

Difference between Simon and Kelvin

$= (k^2 + 3k - 18) - (k^2 + k - 12) = k^2 + 3k - 18 - k^2 - k + 12 = 2k - 6$

$\because\ 2k - 6 = 10\ ,\quad 2k = 16\ ,\quad \therefore\ k = 8$

41. (a) The score awarded = $120 \div 20 = 6$

(b) John's score = $12 \times 6 - (16 - 12)(x + 2) - (20 - 16)(x) = 72 - 4(x + 2) - 4x$

$= 72 - 4x - 8 - 4x = 64 - 8x$

(c) A student gets the lowest score if he answers all the questions wrong.

$120 - 20(x + 2) = 0\ ,\quad 120 - 20x - 40 = 0\ ,\quad 80 - 20x = 0\ ,\quad x = 4$

$\therefore$ John's score = $64 - 8(4) = 64 - 32 = 32$

Unit 9: Percentages (1)

1.

Fraction	$\frac{7}{8}$	$\frac{27}{2500}$	$\frac{19}{40}$	$10\frac{1}{5}$	$3\frac{1}{4}$	$\frac{33}{400}$
Decimal	0.875	0.0108	0.475	10.2	3.25	0.0825
Percentage	87.5%	1.08%	47.5%	1020%	325%	$8\frac{1}{4}\%$

2. (a) Percentage = $\frac{3}{8} \times 100\% = 37.5\%$ (b) Percentage = $\frac{1}{4} \times 100\% = 25\%$

(c) Percentage = $\frac{3}{8} \times 100\% = 37.5\%$

3. (a) $= 3 + 0.17 = 3.17$ (b) $= 3 - 0.59 = 2.41$ (c) $= 72 \times \frac{3}{4} = 54$

(d) $= 0.7 \times 0.3 = 0.21$ (e) $= 210 \div 30\% = 210 \div 0.3 = 700$

(f) $= 130 \times (1 + 15\%) = 130 \times 1.15 = 149.5$ (g) $= \frac{2}{7} \div \frac{13}{10} \times \frac{91}{100} = \frac{2}{7} \times \frac{10}{13} \times \frac{91}{100} = \frac{2}{10} = 0.2$

4. Percentage = $\frac{3}{2.5} \times 100\% = 120\%$

5. The value = $12 \times 25\% = 12 \times \frac{1}{4} = 3$

6. Let y be the number, $y \times 84\% = 1008$, $y = 1008 \times \frac{100}{84}$, $y = 1200$.

Ans. The number is 1200.

7. Percentage of female members = $\frac{60 - 15}{60} \times 100\% = 75\%$

8. Volume of water needed = $800 \times (1 - 42\%) = 800 \times 0.58 = 464$ mL

9. Let \$$y$ be his monthly income, $y \times (1 - 20\%) = 7600$, $0.8y = 7600$, $y = 9500$.

Ans. His monthly income is \$9500.

10. Number of girls = $640 \times (1 - 60\%) \times (1 - 12.5\%) = 640 \times \frac{40}{100} \times \frac{87.5}{100} = 224$

11. Let \$$y$ be the amount of money Susan's sister has, $y \times 62\% = 930$, $y = 930 \div 0.62$, $y = 1500$. ***Ans.*** The difference between the money they have is $1500 - 930 = \$570$.

12. Let y be the number of story books Peter has, $y + y \times 40\% = 84$, $1.4y = 84$, $y = 60$.

Ans. Peter has 60 story books.

13. Number of stamps Margaret will have = $80 \times (1 - 45\%) \times (1 - 25\%) = 33$

14. Percentage change = $\frac{6-18}{18} \times 100\% = \frac{-12}{18} \times 100\% = -66\frac{2}{3}\%$

15. Let y be the number, $y \times (1 + 67.5\%) = 4020$, $1.675y = 4020$, $y = 2400$.
Ans. The number is 2400.

16. Let y be the number, $y - 4 = y \times (1 - 20\%)$, $y - 4 = 0.8y$, $0.2y = 4$, $y = 20$.
Ans. The number is 20.

17. Percentage change = $\frac{50-45}{45} \times 100\% = \frac{5}{45} \times 100\% = 11\frac{1}{9}\%$

18. Let y m be the length of cloth Mrs. Ng should buy,
$y \times (1 - 5\%) = 3.8$, $0.95y = 3.8$, $y = 4$. ***Ans.*** Mrs. Ng should buy 4 m of cloth.

19. Let y be the number,
the net percentage change = $\frac{y(1+20\%)(1-20\%)-y}{y} \times 100\% = \frac{-0.04y}{y} \times 100\% = -4\%$

20. New area = $8\ (1 - 10\%) \times 12\ (1 - 10\%) = 7.2 \times 10.8 = 77.76\ m^2$

21. Population after 2 years = $50000\ (1 + 10\%)\ (1 + 10\%) = 60500$

22. Number of Chinese books = $6000 \times 70\% \times (1 - 10\%) = 3780$
Ans. The cost of the lamp is \$840.

23. Let \$$y$ be the cost of the lamp, $y \times (1 - 15\%) = 714$, $0.85y = 714$,
$y = 840$. ***Ans.*** The cost of the lamp is \$840.

24. Selling price of each skirt = $960\ (1 + 80\%) \div 12 = 960(1.8) \div 12 =$ \$144

25. Selling price of all the eggs = $6 \div 12 \times 1200 =$ \$600,
$\therefore$ Profit percentage = $\frac{600-240}{240} \times 100\% = \frac{360}{240} \times 100\% = 150\%$

26. Amount C paid = $250\ (1 + 40\%)\ (1 + 10\%) = 250\ (1.4)\ (1.1) =$ \$385

27. Selling price = $5 \times 56 + 2.5 \times 40 =$ \$380
Let \$$y$ be the total cost, $y\ (1 - 5\%) = 380$, $y = 380 \times \frac{100}{95} = 400$
Ans. The total cost is \$400.

28. Total selling price = $20000\ (1 + 20\%) + 20000\ (1 - 30\%)$
$= 24000 + 14000 =$ \$38000, total cost = $20000 \times 2 = 40000$
$\therefore$ Loss percentage = $\frac{40000-38000}{40000} \times 100\% = \frac{2000}{40000} \times 100\% = 5\%$

29. Let \$$y$ be the cost of each shirt, cost of 40 shirts = \$$40y$
selling price of all shirts = $30y \times (1 + 10\%) + 10y \times (1 - 20\%) =$ \$$41y$,
$\therefore$ Profit percentage = $\frac{41y-40y}{40y} \times 100\% = \frac{y}{40y} \times 100\% = 2.5\%$

30. Selling price = $1000\ (1 + 10\%)\ (1 - 10\%) = 1000\ (1.1)\ (0.9) =$ \$990,
$\therefore$ The storekeeper lost by $\frac{1000-990}{1000} \times 100\% = 1\%$.

31. Amount the customer can save = $900 \times 25\% =$ \$225

32. Let \$$y$ be the marked price, $y \times 20\% = 560$, $y = 560 \div 0.2$, $y = 2800$.

Ans. The marked price of the sofa is \$2800.

33. Selling price = 180 (1 – 40%) = 180 (0.6) = \$108

34. Let \$$y$ be the fee for each person,

discount percentage = $\frac{4y-3y}{4y}\times 100\% = \frac{y}{4y}\times 100\% = 25\%$

35. (a) Let \$$y$ be the marked price, $y \times 15\% = 96$, $y = 96 \div 0.15$, $y = 640$.

Ans. The marked price of the tent is \$640.

(b) Selling price = 640 – 96 = \$544

(c) Let \$$y$ be the cost of the tent, $y\,(1 + 36\%) = 544$, $1.36y = 544$,

$y = 400$. ***Ans.*** The cost of the tent is \$400.

36. (a) Let y be the total number of students, $y \times 45\% - y \times 22\% = 253$,

$0.23y = 253$, $y = 1100$. ***Ans.*** The total number of students is 1100.

(b) Number of students going home by other means = $1100 \times (1 - 45\% - 22\%)$

$= 110 \times 0.33 = 363$.

37. (a) Weight of water left = $4 \times 70\% \times (1 - 30\%) = 4 \times 0.7 \times 0.7 = 1.96$ kg.

(b) Final weight of water-melon = $4 - 4 \times 70\% \times 30\% = 4 - 4 \times 0.7 \times 0.3 = 3.16$ kg.

38. (a) Total mathematics score = $85 \times 20\% + 35 \times 80\% = 85 \times 0.2 + 35 \times 0.8 = 45$ marks.

(b) Let y marks be the examination score,

$30 \times 20\% + y \times 80\% = 70$, $6 + 0.8y = 70$, $0.8y = 64$, $y = 80$.

Ans. The examination score should be 80 marks.

39. Let y be the number of female workers, $\therefore$ no. of male workers $= y \times 60\% = 0.6y$

total number of workers $= y + 0.6y = 1.6y$,

$\therefore$ Percentage of male workers = $\frac{0.6y}{1.6y}\times 100\% = 37.5\%$

40. $\frac{x(1-60\%)}{x-10}\times 100\% = 50\%$, $\frac{0.4x}{x-10} = \frac{1}{2}$, $0.8x = x - 10$, $10 = 0.2x$, $\therefore\ x = 50$

41. Let x be the number of people in the team, $\therefore$ no. of female workers $= 0.5x$

$\frac{0.5x+4}{x}\times 100\% = 52\%$, $(0.5x + 4) \times 100 = 52x$, $50x + 400 = 52x$,

$400 = 2x$, $x = 200$. ***Ans.*** There are 200 people in the team.

42. Number of students failed in either Mathematics or English

$= 200 \times 32\% + 200 \times 27\% - 16 = 64 + 54 - 16 = 102$,

$\therefore$ Percentage of students passing both subjects = $\frac{200-102}{200}\times 100\% = 49\%$

43. Let y be the number of students in the school, $y(1 - 65\%)(1 - 60\%) = 168$,

$y \times 35\% \times 40\% = 168$, $y = 1200$, $\therefore$ There are 1200 students in the school.

Ans. Number of boys wearing glasses in the school is $1200 \times 65\% \times 30 = 234$.

44. Let a cm be the original length of the square, and y% be the percentage decrease in the width,

$a(1 + 25\%) \times a(1 - y\%) = a^2$, $125\% \times (1 - y\%) = 1$

$1 - y\% = \frac{100}{125}$, $y\% = 0.2$, $y = 20$. ***Ans.*** Percentage decrease in the width is 20%.

45. Let y be the number of eggs he originally had, $y(1 - 5\%)(1 - 80\%) = 266$,
$y(0.95)(0.2) = 266$, $y = 1400$. ***Ans.*** He originally had 1400 eggs.

46. Let \$$y$ be the cost of one egg, $\therefore$ the selling price of one egg = $\$\frac{15y}{12}$,

profit percent = $\frac{\frac{15y}{12} - y}{y} \times 100\% = \frac{0.25y}{y} \times 100\% = 25\%$

47. Let y% be the highest discount percentage, $400(1 - y\%) = 240$, $1 - y\% = 0.6$,
$y\% = 0.4$, $y = 40$. ***Ans.*** The highest discount percentage is 40%.

48. Let \$$y$ be the cost price, $y(1 + 60\%) = x(1 - 20\%)$, $1.6y = 0.8x$, $x = 2y$,
$\therefore$ Profit percent = $\frac{x - y}{y} \times 100\% = \frac{2y - y}{y} \times 100\% = 100\%$

49. The cost = $x \div (1+40\%) = \frac{100x}{140} = \frac{5x}{7}$, the selling price = $x\,(1 - 25\%) = \frac{3x}{4}$,

the profit percentage = $[(\frac{3x}{4} - \frac{5x}{7}) \div \frac{5x}{7}] \times 100\% = \frac{x}{28} \times \frac{7}{5x} \times 100\% = \frac{1}{20} \times 100\%$
$= 5\%$

50. Suppose the shopkeeper set \$$y$ as the marked price,
his cost price = $5000 \times (1 - 10\%) = 4500$, $\therefore$ $y(1 - 20\%) = 4500 \times (1 + 12\%)$,
$y = 4500 \times \frac{112}{100} \times \frac{100}{80}$, $y = 6300$. ***Ans.*** The marked price of the watch is \$6300.

51. Let \$$x$ be the cost price and y% be the discount percentage, $x(1 + 25\%)(1 - y\%) = x$,
$1.25(1 - y\%) = 1$, $1 - y\% = 0.8$, $y\% = 0.2$, $y = 20$.
Ans. The discount percentage on the marked price should be 20%.

52. (a) $x(1+15\%) = 2760$, $x = 2760 \times \frac{100}{115}$, $x = 2400$

(b) The amount Peter originally had = 12000 – 2400 = 9600,
the amount he gives Emily = 2760 – 2400 = 360
$\therefore$ Percentage decrease = $\frac{360}{9600} \times 100\% = 3.75\%$

53. Let \$$y$ be the amount she originally had,
$y(1 - 40\%)(1 - 20\%) - 342 = 1026$, $y(60\%)(80\%) = 1368$, $y = 2850$.
The total amount she had spent = 2850 – 1026 = 1824.
Ans. Percentage of money spent altogether is $\frac{1824}{2850} \times 100\% = 64\%$.

54. P = (1 + 20%) Q, $P = (\frac{120}{100})\,Q$, $Q = P \times \frac{100}{120} = P \times (\frac{100}{120} \times 100\%) = P \times (83\frac{1}{3}\%)$

Ans. $83\frac{1}{3}\%$ of P is equal to Q.

55. Percentage = $\frac{B - A}{A} \times 100\% = \frac{B - B \times 40\%}{B \times 40\%} \times 100\% = \frac{0.6B}{0.4B} \times 100\% = 150\%$
Ans. B is 150% greater than A.

56. $p = q \times 35\% = 0.36q$, $r = q(1 - 60\%) = 0.4q$, $\therefore$ p is smaller than r.

$$\frac{r-p}{r}\times 100\% = \frac{0.4q-0.36q}{0.4q}\times 100\% = \frac{0.04q}{0.4q}\times 100\% = 10\%$$

Ans. p is smaller than r by 10%.

57. (a) In 2016, Mr. Fung's age = 56 – (2022 – 2016) = 56 – 6 = 50

Adam's age = 18 – 6 = 12. ***Ans.*** The required percentage = $\frac{12}{50}\times 100\% = 24\%$

(b) Let x be the number of years required.

$(18 + n) \times 200\% = 56 + n$, $36 + 2n = 56 + n$, $n = 20$

Ans. The required year is 2022 + 20 = 2042.

58. (a) The number of black marbles = $96\times\left(1-16\frac{2}{3}\%\right)$

$= 96\times\left(1-\frac{60}{300}\%\right) = 96\times\left(1-\frac{60}{300}\%\right) = 96\times\frac{250}{300}\% = 80$

(b) Let n be the number of black marbles put into the box.

$(96 + n) \times 85\% = 80 + n$, $81.6 + 0.85n = 80 + n$, $1.6 = 0.15n$

$n = \frac{32}{3}$ which is not an integer. ***Ans.*** Not possible.

59. (a) The new length = 24 × (1 – 25%) = 18 cm,

the new width = 18 × (1 + 25%) = 22.5 cm.

(b) Original perimeter = 2 × (24 + 18) = 84 cm,

New perimeter = 2 × (18 + 22.5) = 81 cm ≠ 84 cm. ***Ans.*** No, not 0%.

(c) Original area = 24 × 18 = 432 cm^2, new area = 18 × 22.5 = 405 cm^2.

Percentage change in area

$= \frac{405-432}{432}\times 100\% = \frac{-27}{432}\times 100\% = -6\frac{1}{4}\%$

60. (a) In 2020, the number of visitors = 200,000 × (1 + 20%) = 240,000.

(b) From 2020 to 2021, the percentage decrease

$= \frac{240{,}000-200{,}000}{240{,}000}\times 100\% = 16\frac{2}{3}\%$. No, I don't agree

(c) Income in 2020 = \$60 × 240,000 = \$14,400,000.

Income in 2021 = \$60 × (1 + 15%) × 240,000 = \$13,800,000.

Ans. No, it earned less income in 2021.

61. (a) Number of female employees = 1100 × (1 + 20%) = 1320,

∴ total number of employees = 1100 + 1320 = 2420.

(b) (i) Number of local male employees = 605 × 60% = 363.

∴ Number of local female employees = 605 – 363 = 242.

(ii) Percentage of non-local employees among the females

$= \frac{242}{1320}\times 100\% = 18\frac{1}{3}\%$

62. (a) Let Ivan's original share be \$$x$.

$x \times \left(1 + 33\frac{1}{3}\%\right) = 19200$, $x \times \left(1 + \frac{1}{3}\right) = 19200$, $x = 19200 \times \frac{4}{3} = 14400$

Ans. Gary's original share is \$(43200 – 14400) = \$28800.

(b) Ivan's final amount = \$19200

Gary's final amount = \$(43200 – 19200) = \$24000.

∴ The required percentage = $\frac{19200}{24000} \times 100\% = 80\%$

(c) The decrease = \$14400 × $33\frac{1}{3}\%$ = \$4800

Percentage decrease in Gary's amount = $\frac{4800}{28800} \times 100\% = 16\frac{2}{3}\%$.

63. (a) (i) The number of female guests = 90× (1 – 40%) = 54.

(ii) Percentage of male guests = $\frac{90}{90+54} \times 100\% = \frac{90}{1444} \times 100\% = 62.5\%$

(b) The new total number of guests = 144 + k,

the new number of female guests = 54 + k

(144 + k) × 40% = 54 + k , 57.6 + 0.4k = 54 + k , 3.6 = 0.6k , $k = 6$

Ans. $k = 6$

64. (a) Let the cost price be \$$x$.

$x \times (1 + 150\%) = 45000$, $x = 45000 \div 250\% = \$18000$

Ans. The cost price is \$18000.

(b) The new sells price = \$45000 × (1 – 40%) = \$27000.

The profit percentage = $\frac{27000 - 18000}{18000} \times 100\% = \frac{9000}{18000} \times 100\% = 50\%$

(c) (i) Let \$$y$ be the new marked price.

y (1 – 20%) = 45000 , $y = 45000 \div 80\% = \$56250$

Ans. The new marked price is \$56250.

(ii) The least selling price = \$18000,

∴ the greatest discount percentage

$= \frac{56250 - 18000}{56250} \times 100\% = \frac{38250}{56250} \times 100\% = 68\%$

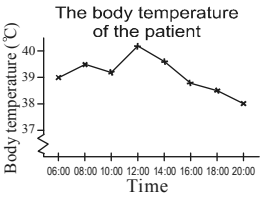
Unit 10: Introduction to statistics

1. (a) Total profit = 6 + 4 + 7 + 10 = 27 million dollars

(b) Difference in profit = 10 – 4 = 6 million dollars

(c) Percentage change = $\frac{7-4}{4} \times 100\% = \frac{3}{4} \times 100\% = 75\%$ (increase)

2. (a)



(b) Before 12:00, the body temperature bounced between 39°C and 40.2°C. After that, it decreased gradually and reached 38°C at 20:00.

3. (a) Number of students = 3 + 4 + 5 + 6 + 7 + 4 + 6 = 35

(b) Highest score = 96, lowest score = 35

(c) Percentage of students passed = $\frac{35-3-4}{35}\times 100\% = \frac{28}{35}\times 100\% = 80\%$

4. (a) Percentage = $\frac{40^\circ}{360^\circ}\times 100\% = \frac{1}{9}\times 100\% = 11\frac{1}{9}\%$

(b) Total number of students = $200 \div \frac{1}{9} = 200 \times 9 = 1800$

(c) Number of students = $1800 \times \frac{100^\circ}{360^\circ} = 1800 \times \frac{5}{18} = 500$

5. (a) Monthly income = $7500 \div \frac{150^\circ}{360^\circ} = 7500 \times \frac{12}{5} = \$18000$

(b) Amount spent on food = $18000 \times \frac{84^\circ}{360^\circ} = 18000 \times \frac{7}{30} = \$4200$

(c) $\frac{x^\circ}{360^\circ} = \frac{3000}{18000}$, $\frac{x}{360} = \frac{1}{6}$, $\therefore\ x = 60$

(d) Angle of sector of transportation = $360^\circ - 150^\circ - 84^\circ - 42^\circ - 60^\circ = 24^\circ$,

$\therefore$ Amount spent on transportation = $18000 \times \frac{24^\circ}{360^\circ} = 18000 \times \frac{1}{15} = \$1200$

6. (a) Back-to-back stem-and-leaf diagram

(b) Broken-line graph

(c) Pie chart

7. (a) The unit of the vertical axis is not shown.

(b) All bars should be equally spaced.

(c) 1. The numbers of each 'leaf' should be listed in ascending order.

2. The tens digits in the numbers of each 'leaf' should not be shown.

8. (a) Percentage = $\frac{64+9+4}{160}\times 100\% = \frac{77}{160}\times 100\% = 48\frac{1}{8}\%$

(b) Angle of sector = $360^\circ \times \frac{50}{160} = 112.5^\circ$

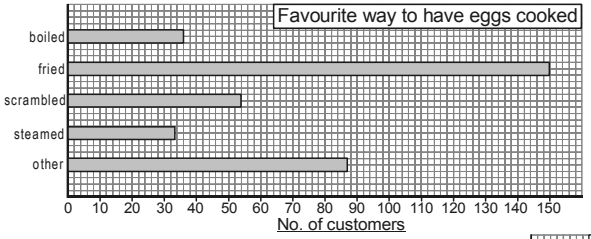
(c) Min. total number of children = 0(33) + 1(50) + 2(64) + 3(9) + 4(4) = 221

9. (a) Total number of customers = $54 \div \frac{54^\circ}{360^\circ} = 54 \times \frac{360}{54} = 360$

(b) $360° - (33° + 54° + 150° + 36°) = 360° - 273° = 87°$

$\therefore$ Number of people = $360 \times \frac{87}{360} = 87$

(c)



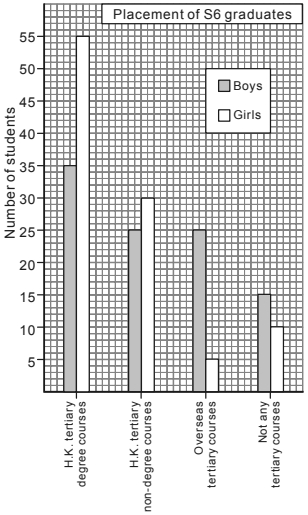
10. (a) Percentage $= \dfrac{100+100-15-10}{100+100}\times 100\%$

$= \dfrac{175}{200}\times 100\% = 87.5\%$

(b) Percentage $= \dfrac{55+30}{55+30+5}\times 100\%$

$= \dfrac{85}{90}\times 100\% = 94\dfrac{4}{9}\%$

(c)



11. (a) $a = 0$, $b = 9$

(b) Percentage of girls $= \dfrac{18}{14+18}\times 100\%$

$= 56.25\%$

(c) The exam score of the all students in the class

Stem (10)	Leaf (1)									
6	3	3	5	5	7	8	8	8	9	9
7	0	0	2	4	4	4	7	8	8	
8	2	2	3	4	4	6	9	9		
9	0	1	3	4	4					

(d) The percentage $= \dfrac{8}{32}\times 100\% = 25\%$

12. (a) 2019, \$4,000,000 (b) 2017 and 2019

(c) The profit in 2015 = \$(460 – 360) × 10000 = \$1,000,000

(d) The total profit = \$[(460 – 360) + (420 – 340) + (340 – 380) + (400 – 320) + (380 – 400) + (420 – 380)] × 10000 = \$2,400,000

13. (a) The weight of students in 1A and 1B

1A students Leaf (1 kg)	Stem (10 kg)	1A students Leaf (1 kg)
8 8 7	4	0 0 3 3 4 5 6 8
9 8 7 5 3 2	5	0 1 4 5 6 8 9
9 8 7 6 4 0 0	6	1 2 2 3
6 6 4 3	7	1

(b) The range = 76 – 40 = 36 kg

(c) Class 1A is heavier. More than half of 1A students are equal to or more than 60 kg, but more than half of 1B students are less than 60 kg.

(d) Percentage of 1A students = $\frac{9}{9+5}\times 100\% = \frac{9}{14}\times 100\% = 64\frac{2}{7}\%$

Unit 11: Introduction to geometry

1. (a) points (b) straight (c) curve
2. (a) B (b) b, $\angle B$, $\angle ABC$, $\angle CBA$
3. (a) 116° (b) 320°
4. (a) $360^\circ \times \frac{1}{5} = 72^\circ$ (b) $360^\circ \times 2\frac{2}{3} = 360^\circ \times 2 + 360^\circ \times \frac{2}{3} = 960^\circ$
5. (a) $360^\circ \times \frac{40}{60} = 240^\circ$ (b) $360^\circ \times \frac{20}{60} = 120^\circ$

   (c) $360^\circ \times \frac{2.5}{12} = 360^\circ \times \frac{25}{120} = 75^\circ$
6. (a) obtuse (b) reflex (c) acute
7. (a) equilateral (b) isosceles
8. (a) rhombus (b) rectangle (c) parallelogram

   (d) trapezium (e) square
9. (a) $x + 30^\circ + 40^\circ = 180^\circ$ ($\angle$ sum of Δ), $\therefore\ x = 110^\circ$

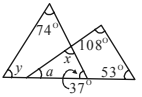
   (b) $3y = 180^\circ$ ($\angle$ sum of Δ), $\therefore\ y = 60^\circ$

   (c) $a + b + 90^\circ = 180^\circ$ ($\angle$ sum of Δ), $a + a + 90^\circ = 180^\circ$ ($a = b$), $2a = 90^\circ$,

   $\therefore\ a = \underline{\underline{45^\circ}}$, and $b = a = \underline{\underline{45^\circ}}$
10. $\angle C + 38.4° + 75.5° = 180°$ ($\angle$ sum of Δ),

    $\therefore\ \angle C = 180 - 113.9° = 66.1°$
11. $(2x+22)^\circ + (x-15)^\circ + (x+17)^\circ = 180^\circ$ ($\angle$ sum of Δ), $4x^\circ + 24^\circ = 180^\circ$,

    $4x^\circ = 156^\circ$, $\therefore\ x = 39$
12. (a) $y + 37^\circ + 74^\circ = 180^\circ$ ($\angle$ sum of Δ), $\therefore\ y = \underline{\underline{69^\circ}}$

    $a + 53^\circ + 108^\circ = 180^\circ$ ($\angle$ sum of Δ), $\therefore\ a = 19^\circ$

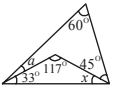
    $x + a + 37^\circ = 180^\circ$ ($\angle$ sum of Δ), $x + 19^\circ + 37^\circ = 180^\circ$, $\therefore\ x = \underline{\underline{124^\circ}}$

    (b) $3y = 180^\circ$ ($\angle$ sum of Δ), $\therefore\ y = \underline{\underline{60^\circ}}$

    $x + x + y + y = 180^\circ$ ($\angle$ sum of Δ),

    $2x + 60^\circ + 60^\circ = 180^\circ$, $2x = 60^\circ$, $\therefore\ x = \underline{\underline{30^\circ}}$
13. $x + 33^\circ + 117^\circ = 180^\circ$ ($\angle$ sum of Δ), $x = 30^\circ$

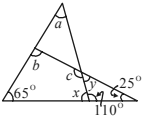
    $a + 33^\circ + x + 45^\circ + 60^\circ = 180^\circ$ ($\angle$sum of Δ),

    $a + 30^\circ + 138^\circ = 180^\circ$, $\therefore\ a = 12^\circ$
14. $b + 65^\circ + 25^\circ = 180^\circ$ ($\angle$ sum of Δ), $\therefore\ b = \underline{\underline{90^\circ}}$

    $x = 180^\circ - 110^\circ = 70^\circ$, $a + x + 65^\circ = 180^\circ$ ($\angle$ sum of Δ), $\therefore\ a = \underline{\underline{45^\circ}}$

    $y + 25^\circ + 110^\circ = 180^\circ$ ($\angle$ sum of Δ), $y = 45^\circ$

    $\therefore\ c = 180^\circ - y = 180^\circ - 45^\circ = \underline{\underline{135^\circ}}$

15. (a) The statement is correct. If there were 2 interior right angles, the remaining angle would then be 0°.

(b) Incorrect. Angles between 180° and 270° are also reflex angles.

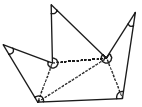
(c) Incorrect. A square has two pairs of parallel opposite sides.

16. $\angle R + 45^\circ + 90^\circ = 180^\circ$ ($\angle$ sum of Δ), $\angle R = 45^\circ$,

$\therefore$ ΔPQR is isosceles, and PQ = QR = 6 cm

$\therefore$ Area of $\Delta PQR = \frac{1}{2} \times 6 \times 6 = 18 \text{ cm}^2$

17. Divide the heptagon into 5 triangles.
Sum of interior angles of the heptagon = sum of interior angles of the 5 triangles = 180° × 5 = 900°



18. (a) $\angle A + \angle ABC + \angle C = 180^\circ$ ($\angle$ sum of Δ),
$2\angle A + 46^\circ = 180^\circ$ ($\angle A = \angle ABC$), $2\angle A = 134^\circ$, $\therefore$ $\angle A = 67^\circ$

(b) $\angle A + \angle ADB + \angle ABD = 180^\circ$ ($\angle$ sum of Δ),
$2\angle A + \angle ABD = 180^\circ$ ($\angle A = \angle ADB$),
$\angle ABD = 180^\circ - 2 \times 67^\circ = 46^\circ$; $\angle ABC = \angle A = 67^\circ$;
$\therefore$ $\angle DBC = \angle ABC - \angle ABD = 67^\circ - 46^\circ = 21^\circ$

19. $\angle ADE = 90^\circ + 60^\circ = 150^\circ$,
ΔADE is isosceles, $\therefore$ $2\angle DEA + 150^\circ = 180^\circ$
($\angle$ sum of Δ, and $\angle DEA = \angle DAE$), $\angle DEA = 15^\circ$.
Similarly, $\angle CEB = 15^\circ$. $\therefore$ $\angle AEB = \angle DEC - \angle DEA - \angle CEB = 60^\circ - 15^\circ - 15^\circ = 30^\circ$

20. (a) $28^\circ + 2\angle ABC = 180^\circ$ ($\angle$ sum of Δ, and $\angle ABC = \angle ACB$),
$\therefore$ $\angle ABC = 152^\circ \div 2 = 76^\circ$

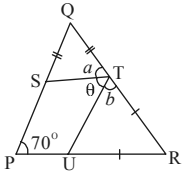
(b) $\angle BCD + \angle DBC + \angle BDC = 180^\circ$ ($\angle$ sum of Δ),
$\angle BCD + 2\angle DBC = 180^\circ$ ($\angle DBC = \angle BDC$),
$\therefore$ $\angle BCD = 180^\circ - 2 \times 76^\circ = 28^\circ$

(c) $\angle ACB = \angle ABC = 76^\circ$, $\therefore$ $\angle DCE = \angle ACB - \angle BCD = 76^\circ - 28^\circ = 48^\circ$

(d) $\angle DEC = \angle DCE = 48^\circ$, $\angle AED = 180^\circ - \angle DEC = 180^\circ - 48^\circ = 132^\circ$,
$\theta + \angle AED + \angle A = 180^\circ$ ($\angle$ sum of Δ, $\theta + 132^\circ + 28^\circ = 180^\circ$, $\therefore$ $\theta = 20^\circ$

21. $\angle DAB + \angle DBA + \angle DAC + \angle DCA = 180^\circ$ ($\angle$ sum of Δ),
$2\angle DAB + 2\angle DAC = 180^\circ$ ($\angle DAB = \angle DBA$, $\angle DAC = \angle DCA$),
$2(\angle DAB + \angle DAC) = 180^\circ$, $2\angle BAC = 180^\circ$, $\therefore$ $\angle BAC = 90^\circ$

22. Let $\angle QTS = a$, $\angle RTU = b$.
$\angle Q = 180^\circ - 2a$, $\angle R = 180^\circ - 2b$.
$70^\circ + \angle Q + \angle R = 180^\circ$,
$70^\circ + (180^\circ - 2a) + (180^\circ - 2b) = 180^\circ$,
$250^\circ - 2a - 2b = 0$, $a + b = 125^\circ$.
$a + \theta + b = 180^\circ$ (straight angle),
$\theta + 125^\circ = 180^\circ$, $\therefore$ $\theta = 55^\circ$



23. (a) At 4:15, the angle between the two hands
$= 360^\circ \div 12 \times 4\frac{15}{60} - 90^\circ = 30^\circ \times \frac{17}{4} - 90^\circ = 127.5^\circ - 90^\circ = 37.5^\circ$.

(b) At 7:50, the angle between the two hands

$= 360° \times \frac{50}{60} - 360° \div 12 \times 7\frac{50}{60} = 300° - 30° \times \frac{47}{6} = 300° - 235° = 65°.$

24. 

25. Join CD. $\because$ AC = AD = CD , $\therefore$ ACD is an equilateral Δ.
$\therefore \angle CAD = 180° \div 3 = 60°$

26. (a) $\angle PQM$, $\angle RQM$ (b) $\angle PQN$ (c) $\angle RQN$, $\angle MQN$

27. round angle > reflex angle > straight angle > obtuse angle > right angle > acute angle

28. (a) Figure R (b) Figure X

29. (a) BD (b) FE

30. (a) BCEF (or: ADHG) (b) ABCD , EFGH , CDHE , BAGF

31. $\angle C + 60° + 70° = 180°$ ($\angle$ sum of Δ) , $\angle C + 130° = 180°$, $\angle C = 50°$
$\angle ADC + \angle C + 45° = 180°$ ($\angle$ sum of Δ) , $\angle ADC + 50° + 45° = 180°$
$\angle ADC + 95° = 180°$, $\angle ADC = 180° - 95° = 85° \neq 90°$
No, he is not correct

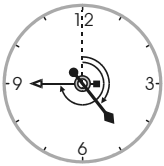
32. ΔABC is equilateral, $\therefore \angle ACD = 60°$
$\angle CED + 109° + \angle DCE = 180°$, $\angle CED + 109° + (60° - 19°) = 180°$,
$\angle CED + 150° = 180°$, $\angle CED = 30°$

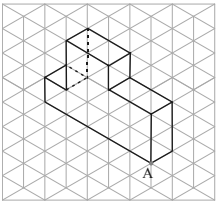
33. $35° + 4a + a = 180°$ ($\angle$ sum of Δ) , $35° + 5a = 180°$, $5a = 145°$, $a = 29°$
$b + 35° + (a + 14°) = 180°$ ($\angle$ sum of Δ) , $b + 35° + (29° + 14°) = 180°$,
$b + 78° = 180°$, $b = 102°$

34. (a) $5a° + 4a° + 90 = 180°$ ($\angle$ sum of Δ) , $9a° = 90°$, $a° = 10°$, $a = \underline{\underline{10}}$
$\angle CBD + b° + 4a° = 180°$, $(90° - 50°) + b° + 4 \times 10° = 180°$,
$b° + 80° = 180°$, $b° = 100°$, $b = 120$

(b) ΔABC is a right-angled triangle.
ΔABD is an acute-angled triangle.
ΔBCD is an obtuse-angled triangle.

35. $270° - 360° \div 12 \times 4\frac{45}{60}$
$= 360° \times \frac{3}{4} - 30° \times \frac{19}{4}$
$= 270° - 142.5° = 127.5°$



36. 

Unit 12: Areas and volumes (1)

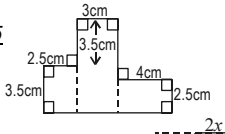
1. (a) 324.8 mm (b) 0.05 cm (c) 104000 cm
(d) 1.62 m (e) 12700 m (f) 0.096 km
(g) 5000000 cm (h) 0.0082 km

2. (a) 1800 mm^2 (b) 0.65 cm^2 (c) 420000 cm^2
(d) 0.01 m^2 (e) 9060000 m^2 (f) 0.037 km^2

3. (a) 2660 mm^3 (b) 0.24 cm^3 (c) 42000000 cm^3
   (d) 0.0001 m^3 (e) 1500 cm^3

4. Area $= \frac{6\times3}{2}+\frac{(6+4)\times5}{2}+\frac{4\times4}{2} = 9+25+8 = 42\ cm^2$

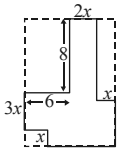
Q.5



5. Area $= 2.5\times3.5+(3.5+3.5)\times3+4\times2.5$
   $= 8.75+21+10 = 39.75\ cm^2$

6. $[(8+3x+x)+(6+2x+2)]\times2 = 56$,
   $7x+14 = 28$, $7x = 14$, $\therefore\ x = 2$

Q.6



7. (a) $\frac{15\times h}{2}=\frac{12\times9}{2}$, $15h = 108$, $\therefore\ h = 7.2$
   (b) $\frac{9\times h}{2}=\frac{3\times6}{2}$, $9h = 18$, $\therefore\ h = 2$

8. (a) Area $= 12\times8 = 96\ cm^2$ (b) $10\times h = 96$, $\therefore\ h = 9.6$

9. Area $3\times3-\frac{1\times1}{2}\times4 = 9-2 = 7\ cm^2$

10. (a) Cost of the carpet $= (12-2-2)\times(10-2-2)\times180 = 48\times180 = \$8640$
    (b) Area of the uniform path $= 12\times10-48 = 120-48 = 72\ cm^2$

11. Internal surface area $= (22\times3)\times2+(18\times3)\times2+(22\times18) = 636\ cm^2$

12. Volume of prism $= \frac{(3+8)\times5}{2}\times4 = 110\ cm^3$

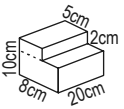
13. (a) Volume $= \frac{(30+9)\times28}{2}\times20 = 546\times20 = 10920\ cm^3$

    Total surface area $= \frac{(30+9)\times28}{2}\times2+(30+28+9+35)\times20 = 3132\ cm^2$

    (b) Volume $= 2\times7\times5+12\times6\times30 = 70+2160 = 2230\ cm^3$
    Total surface area $= (12\times6+6\times30+12\times30)\times2+(5\times2+7\times5)\times2 = 1314\ cm^2$

14. Number of rectangular blocks $= (50\times40\times45)\div(2\times4\times3) = 25\times10\times15 = 3750$

15. Volume $= [5\times2+(10-2)\times8)]\times20 = 74\times20 = 1480\ cm^3$
    Total surface area $= [5\times2+(10-2)\times8]\times2+8\times20\times2+10\times20\times2$
    $= 868\ cm^2$



or: Total surface area $= (10\times8+20\times10+8\times20)\times2-[(8-5)\times2]\times2 = 868\ cm^2$

16. Volume $= 15\times30\times20-\frac{(15-5-5)\times(30-5)}{2}\times20 = 9000-1250 = 7750\ cm^3$

17. Volume $= (7\times7-4\times4)\times8 = 33\times8 = 264\ cm^3$
    Total surface area $= 33\times2+7\times8\times4+4\times8\times4 = 66+224+128 = 418\ cm^2$

18. (a) Internal volume $= (20-1-1)\times(15-1-1)\times(10-1) = 18\times13\times9 = 2106\ cm^3$
    (b) Volume $= 20\times15\times10-2106 = 3000-2106 = 894\ cm^3$

19. Height of water level $= 6000\div(24\times25) = 6000\div600 = 10$ cm

20. The area $= 0.08\times100\times100 = 800 cm^2$
    Let x cm be the height of the triangle.
    $\frac{80x}{2} = 800$, $40x = 800$, $x = 20$.

Ans. The height of the triangle is 20 cm.

21. Let x cm be the upper base.

$\frac{(22+x)\times 5.5}{2} = 99$, $22 + x = 36$, $x = 14$.

Ans. The upper base is 14 cm.

22. Let x cm be the side of the new square, $x^2 = 500(1 - 20\%)$, $x^2 = 400$,
$\therefore$ $x = 20$, $\therefore$ Perimeter of the new square $= 20 \times 4 = 80$ cm

23. Let x cm be the side of each cube, $3x^3 = 4 \times 6 \times 8$, $x^3 = 64$, $x^3 = 4^3$, $\therefore$ $x = 4$,
$\therefore$ Total surface area of the three cubes $= (4 \times 4 \times 6) \times 3 = 288$ cm^2

24. 1m = 100cm, $\therefore$ Volume of water needed $= 80 \times 35 \times 100 = 280000$ cm^3

25. 120cm = 1.2m, 100cm = 1m,
$\therefore$ Height of the water level $= 1.5 \div (1.2 \times 1) = 1.25$ m

26. Height of water $= (4 \times 10 \times 4 \times 6) \div 50 = 19.2$ cm

27. 12mm = 1.2cm, $\therefore$ Length $= (40 \times 1.2) \div 30 = 48 \div 30 = 1.6$ cm

28. Volume of the water $= \frac{(1+2.5)\times 25}{2} \times 12 = 525$ m^3

29. 4km = 4000m, 14cm = 0.14m , $\therefore$ Time required $= (4000 \times 3 \times 0.14) \div 56 = 30$ h

30. Weight of the box $= [(60 \times 50 \times 40) - (60 - 5 - 5) \times (50 - 5 - 5) \times (40 - 5)] \times 0.8$
$= (120000 - 70000) \times 0.8 = 40000$ g $= 40$ kg

31. $\frac{4.8\times 10}{2} = \frac{8\times BC}{2}$, $8BC = 48$, $BC = 6$, $\therefore$ The area of ABCD $= 8 \times 6 = 48$ cm^2

32. Let h cm be the height of ΔABC, $\therefore$ h cm is also the height of ΔABD.

$\frac{12.5\times 4}{2} = \frac{5\times h}{2}$, $h = \frac{50}{5} = 10$ $\therefore$ Area of $\triangle ABC = \frac{(5+6)\times 10}{2} = 55$ cm^2

33. BC $= 60 - 16 - 24 = 20$ cm, $\frac{20\times AD}{2} = \frac{24\times x}{2}$, $20AD = 24x$, $\therefore$ $AD = 1.2x$ (or: $\frac{6}{5}x$)

34. Area of the parallelogram $= 4 \times 10 = 40$ sq. units

35. Shaded area is a trapezium. Area of shaded region $= \frac{(4+7)\times 4}{2} = 22$ cm^2

36. (a) Area of $\triangle BCG = \frac{5\times(8-5)}{2} = 7.5$ cm^2

(b) Shaded area = Area of ABCD + Area of ΔDGE + Area of $\triangle BCG$ – Area of $\triangle ABE$

$= 5^2 + \frac{8\times 8}{2} + 7.5 - \frac{5\times(5+8)}{2} = 25 + 32 + 7.5 - 32.5 = 32$ cm^2

37. (a) Minimum number of tiles required $= (600 \times 450) \div (20 \times 20) = 675$

(b) $\because$ $600 \div 20 = 30$ and $450 \div 20 = 22.5$,
$\therefore$ Maximum number of whole tiles $= 30 \times 22 = 660$

38. (a) Area of square ABCD $= 6 \times 6 + 16 \times 4 = 36 + 64 = 100$ cm^2

(b) The side of square ABCD $= \sqrt{100} = 10$ cm

$\therefore$ Height of each trapezium $= \dfrac{DC - SR}{2} = (10 - 6) \div 2 = 2$ cm

39. (a) No. of cubes $= 8 + 3 + 1 = 12$, $\therefore$ Volume $= 12 \times 1 = 12$ cm^3

No. of squares: bottom $= 8$, back $= 7$, left side $= 5$, right side $= 3$, others $= 19$

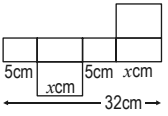
$\therefore$ Total surface area $= 8 + 7 + 5 + 3 + 19 = 42$ cm^2

(b) No. of cubes $= 11$, $\therefore$ Volume $= 11 \times 1 = 11$ cm^3

No. of squares: bottom $= 5$, left back $= 7$, right back $= 8$, others $= 20$,

$\therefore$ Total surface area $= 5 + 7 + 8 + 20 = 40$ cm^2

40. (a) Volume of the triangular prism formed $= \dfrac{6 \times 8}{2} \times 5 = 120$ cm^3



(b) Let the unknown dimension of the cuboid be x cm,

$5 + x + 5 + x = 32$, $2x = 22$, $x = 11$,

$\therefore$ Volume of the cuboid $= 3 \times 5 \times 11 = 165$ cm^3

41. Weight of the remaining solid $= \left[10 \times 12 - \dfrac{(4+8) \times 6}{2}\right] \times 32 \times 0.8$

$= (120 - 36) \times 32 \times 0.8 = 2150.4$ g

42. (a) Volume of the remaining solid $= 80 \times 80 \times 80 - 20 \times 20 \times 20 \times 6$

$= 512000 - 48000 = 464000$ cm^3

(b) Total surface area $= 80 \times 80 \times 6 + 20 \times 20 \times 4 \times 6 = 38400 + 9600 = 48000$ cm^2

43. Volume $= 24 \times 15 \times 8 - 8 \times 16 \times 2 = 2880 - 256 = 2624$ cm^3

Total surface area $= (24 \times 8 + 15 \times 8 + 24 \times 15 + 8 \times 2 + 16 \times 2) \times 2 = 1440$ cm^2

44. Volume $= 8 \times 4 \times 3 = 96$ cm^3

Total surface area $= (8 \times 4 + 3 \times 5 + 8 \times 3) \times 2 = 71 \times 2 = 142$ cm^2

45. Volume $= \left(4 \times 10 + \dfrac{5 \times 5}{2}\right) \times 2 = 52.5 \times 2 = 105$ cm^3

Total surface area $= \left(4 \times 10 + \dfrac{5 \times 5}{2} + 10 \times 2 + 1 \times 2 + 5 \times 2\right) \times 2 + 4 \times 2 = 177$ cm^2

46. Area of the water surface $= (12 \times 12 \times 12) \div 8 = 1728 \div 8 = 216$ cm^2

47. (a) Height of the triangular prism $= (5 \times 5 \times 5) \div 20 = 120 \div 20 = 6.25$ cm

(b) Original area $= 5^2 \times 6 = 150$ cm^2, new area $= 24 \times 6.25 + 20 \times 2 = 190$ cm^2,

$\therefore$ % increase in total surface area $= \dfrac{190 - 150}{150} \times 100\% = \dfrac{40}{150} \times 100\% = 26\dfrac{2}{3}\%$

48. Capacity of the swimming pool $= \dfrac{(1.2 + 2.6) \times 50}{2} \times 12 = 95 \times 12 = 1140$ m^3 $= 1140000$ L,

$\therefore$ Time required $= 1140000 \div 8000 = 142.5$ min

49. Let x be the number of marbles, $8x > 14 \times 18 \times 15 \times \left(1 - \dfrac{1}{3}\right)$, $8x > 2520$, $x > 315$.

Ans. The minimum number of marbles required is 316.

50. (a) $\dfrac{h \times 4}{2} \times 10 = \dfrac{1}{8} \times 4 \times 10 \times (12 - h)$, $20h = 5(12 - h)$, $20h = 60 - 5h$, $25h = 60$,

$\therefore$ $h = 2.4$

(b) Minimum volume of wood $= (12\times4\times10)-\left[(12-2.4)\times4+\frac{2.4\times4}{2}\right]\times10$

$= 480 - 432 = 48 \text{ cm}^3$

51. $(12 \times 15 - 4 \times 6) \times (10 + h) = 12 \times 15 \times 10$,

$156(10 + h) = 1800$, $1560 + 156h = 1800$, $156h = 240$, $\therefore\ h = 1\frac{7}{13}$

52. (a) Volume $= 126 \times 12 = 1512 \text{ cm}^3$

(b) (Perimeter of ABCD) $\times 12 + 2 \times 126 = 804$

$\therefore$ Perimeter of ABCD $= \frac{804-252}{12} = \frac{552}{12} = 46$ cm

53. (a) Area of $\Delta PTS = \frac{1}{2} \times 12 \times 9 = 54$ sq. units

(b) The height of the trapezium is equal to the perpendicular distance from T to PS.

Let the height be h. $\frac{1}{2} \times 15 \times h = 54$, $h = 7.2$

Area of the trapezium $= \frac{1}{2}(7.2)(15 + 20) = 126$ sq. units

(c) Percentage of shaded parts $= \frac{126-54}{126}\times100\% = \frac{72}{126}\times100\% = \frac{4}{7}\times100\% = 57\frac{1}{7}\%$

54. (a) Volume of one triangular prism $= \frac{1}{2}(10)(24)(15) = 1800 \text{ cm}^3$

(b) Total surface area of the cuboid $= 2 \times (24 \times 15 + 10 \times 15 + 10 \times 24)$

$= 2 \times 750 = 1500 \text{ cm}^2$

Increase in surface area $= 2 \times (15 \times 26) = 780 \text{ cm}^2$

$\therefore$ Percentage increase in total surface area $= \frac{780}{1500}\times100\% = 52\%$

55. (a) Base area $= \frac{5400}{6} = 900 \text{ cm}^2$

(b) The rise in water level $= \frac{15\times18\times4}{900} = \frac{6}{5}$ cm

$\therefore$ The new water level $= 6 + \frac{6}{5} = 7\frac{1}{5}$ cm

(c) Let the new water level be k cm.

$900k - 15 \times 4 \times k = 5400$, $840k = 5400$, $k = 6\frac{3}{7}$

Ans. The new water level is $6\frac{3}{7}$ cm.

56. (a) Total volume $= 14 \times 14 \times 4 + 12 \times (18 - 4) = 784 + 168 = 952 \text{ cm}^3$

(b) The remaining space in the container $= (14 \times 14 - 12)(6 - 4) = 184 \times 2 = 368 \text{cm}^3$

$\because 368 > 350$, $\therefore$ yes, water will overflow from the container.

57. (a) Area of $\Delta APQ = \frac{1}{2}(15)(20) = 150\ cm^2$

$\therefore$ Volume of the triangular prism $= 150 \times 20 = 3000\ cm^3$

(b) (i) $\frac{1}{2}(PQ)(12) = 150$, $PQ = 25$ cm

(ii) Area of $PQRS = PQ \times QR = 25\ cm \times 20\ cm = 500\ cm^2$

$PQ = 25$ cm, $AP = 15$ cm, $AQ = 20$ cm

$\therefore$ perimeter of $\Delta AQP = 25 + 15 + 20 = 60$ cm

Total surface area of the prism $= 60 \times 20 + 150 \times 2 = 1500\ cm^2$

(c) Volume of the remaining solid $= 25 \times 30 \times 20 - 3000 = 15000 - 3000 = \underline{12000\ cm^3}$

Total surface area of the remaining solid

= area of the cuboid − area of prism + area of $PQRS \times 2$

$= 2 \times (30 \times 20 + 25 \times 20 + 25 \times 30) - 1500 + 500 \times 2$

$= 3700 - 1500 + 1000 = \underline{\underline{3200\ cm^2}}$

58. (a) (i) Depth of water $= \frac{17500}{50 \times 25} = 14$ cm

(ii) $x = 18 - 14 = 4$

(b) (i) Volume of each stone $= \frac{1}{4}[50 \times 25 \times 4 + 30 \times 25 \times (6 - 4)]$

$= \frac{1}{4}[5000 + 1500] = 1625 cm^3$

(ii) Let the rise in water level be h cm.

$50 \times 25 \times 4 + 30 \times 25 \times (h - 4) = 12000$, $5000 + 750h - 3000 = 12000$,

$750h = 10000$, $h = 13\frac{1}{3}$. ***Ans.*** The rise in water level is $13\frac{1}{3}$ cm.

59. (a) Volume $= 6 \times 6 \times 6 = 216\ cm^3$

(b) (i) $\frac{1}{2}(8)(CM)(18) = 216$, $CM = \frac{216}{72} = 3$ cm

(ii) Increase in total surface area $= 2 \times (18 \times 3) = 108\ cm^2$

(iii) The mid-points of AE, CD and BF form a triangle parallel to ΔABC. Cutting along this triangle will form two equal triangular prisms.

The increase in total surface area $= 2 \times (\frac{1}{2} \times 8 \times 3) = 24\ cm^2$

60. (a) (i) Volume of water $= 16 \times 10 \times 6 = 960\ cm^3$

(ii) Total wet surface area $= 2 \times (16 \times 6 + 6 \times 10) + 16 \times 10$

$= 2 \times 156 + 160 = 472\ cm^2$

(b) $EH = 6$ cm, $FG = 12 + 6 = 18$ cm, $EF = 16$ cm

Area of $EFGH = \frac{1}{2}(6 + 18)(16) = 192\ cm^2$. $h = \frac{960}{192} = \underline{5}$

Total wet surface area = (primeter of EFGH) $\times h + 192$

$= (16 + 6 + 20 + 18)(5) + 192 = 300 + 192 = 492\ cm^2$

Unit 13: Angles and parallel lines

1. (a) $2x + 30^o = 4x - 50^o$ (vert. opp. $\angle$s), $80^o = 2x$, $\therefore\ x = 40^o$

   (b) $(x + 10^o) + 90^o + (x - 10^o) = 180^o$ (adj. $\angle$s on st. line), $2x = 90^o$, $\therefore\ x = 45^o$

   (c) $2x + 3x + x + 90^o = 360^o$ ($\angle$s at a pt.), $6x = 270^o$, $\therefore\ x = 45^o$

   (d) $\angle AOC = 3x - 13^o$ (vert. opp.$\angle$s), $\angle FOB = 2x + 10^o$ (vert. opp. $\angle$s),
   $\therefore\ (3x - 13^o) + 98^o + (2x + 10^o) = 180^o$ (adj. $\angle$s on st. line), $5x = 85^o$, $x = 17^o$

2. Let $\theta = \angle DBE$. $\angle DBA = 133^o - \theta$, and $\angle EBC = 101^o - \theta$,
   $\therefore\ (133^o - \theta) + \theta + (101^o - \theta) = 180^o$ (adj. $\angle$ s on st. line), $\theta = 54^o$, $\therefore\ \angle DBE = 54^o$

3. $\angle QSP = 180^o - 64^o - 49^o = 67^o$ ($\angle$ sum of Δ)
   $\angle QSR = 180^o - 24^o - 21^o = 135^o$ ($\angle$ sum of Δ)
   $\therefore\ 67° + 135° + a° = 360°$ ($\angle$s at a pt.), $\therefore\ a^o = 158^o$, $a = 158$

4. (a) $m = 32^o$ (alt.$\angle$s, AB//DE). $\angle DCE = 180^o - 70^o - m$ ($\angle$ sum of Δ),
   $\therefore\ \angle DCE = 180^o - 70^o - 32^o = 78^o$. $n = \angle DCE = 78°$ (vert. opp. $\angle$s)

   (b) $x^o = 73^o$ (alt. $\angle$s, SR//PQ), $\therefore\ x = 73$
   $\angle SQR = 180^o - 58^o - x^o$ ($\angle$ sum of Δ), $\therefore\ \angle SQR = 122^o - 73^o = 49^o$
   $y^o = \angle SQR = 49^o$ (alt. $\angle$s, PS//QR), $\therefore\ y = 49$

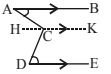
   (c) $\angle BDF = \angle ABD$ (alt. $\angle$s, DF//AB), $\angle BDE + m = 18^o + \angle CBD$;
   but $\angle BDE = \angle CBD$ (alt. $\angle$s, DE//CB), $\therefore\ m = 18^o$

   (d) $y = 55^o$ (corr. $\angle$s, AB//EC). $x + 75^o + y = 180^o$ (adj. $\angle$s on st. line),
   $\therefore x = 180^o - 75^o - 55^o = 50^o$

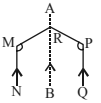
   (e) $\angle QSR + 145^o = 180^o$ (int. $\angle$s, PQ//RS), $\therefore\ \angle QSR = 180^o - 145^o = 35^o$
   $y + \angle QSR + 90^o = 360^o$ ($\angle$s at a pt.), $\therefore\ y = 360^o - 90^o - 35^o = 235^o$

   (f) $\angle ACD + 116^o = 180^o$ (int. $\angle$s, AB//CD), $\therefore\ \angle ACD = 64°$
   $(a + \angle ACD) + 47^o = 180^o$ (int. $\angle$s, EC//BD), $\therefore\ a + 64^o = 133^o$, $a = 69^o$

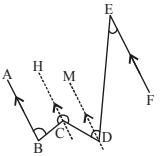
5. (a) Draw HK through C, such that HK//AB//DE.
   $\angle ACH = 56^o$ (alt. $\angle$s, AB//HK), $\angle HCD = 68^o$ (alt. $\angle$s, HK//DE)
   $x = \angle ACH + \angle HCD = 56^o + 68^o = 124^o$

   (b) Draw AB through R, such that AB//MN//PQ.
   $\angle ARM = 120^o$ (alt. $\angle$s, AB//MN),
   $\therefore\ \angle ARP = 228^o - 120^o = 108^o$
   $y = \angle ARP = 108^o$ (alt. $\angle$s, AB//PQ)

   (c) Draw HC//MD//AB//EF. $\angle MDE = 32^o$ (alt. $\angle$s, MD//EF),
   $\therefore\ \angle MDC = 68^o - 32^o = 36^o$.
   $\angle HCD + \angle MDC = 180^o$ (int. $\angle$s, HC//MN),
   $\therefore\ \angle HCD = 180° - 36 = 144°$
   $\angle HCB + 82^o = 180^o$ (int. $\angle$s, HC//AD), $\therefore\ \angle HCB = 98^o$
   $m = \angle HCB + \angle HCD = 98^o + 144^o = 242^o$

6. $a = 37^o$ (alt.$\angle$s, // lines). $d = a = 37^o$ (alt. $\angle$s, // lines). $b = 25^o$ (alt. $\angle$s, // lines)
   $(25^o + d) + c = 180^o$ (int.$\angle$s, // lines), $\therefore\ 25^o + 37^o + c = 180^o$, $c = 118^o$
   $e + (25^o + d) = 180^o$ (int.$\angle$s, // lines), $\therefore\ e + 25^o + 37^o = 180^o$, $e = 118^o$

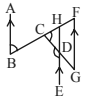
7. $a = 69^o$ (corr. $\angle$s, AB//DF), $\angle BEG = 180^o - 69^o - 45^o = 66^o$ (adj. $\angle$s on st. line)

$\therefore\ b = 180^o - 66^o = 114^o$ (int.$\angle$s, BC//EG)

8. $\angle CBE = 38^o$ (alt.$\angle$s, AC//DE). $\angle CED = 74^o$ (corr. $\angle$s, DE//FG)
$\therefore\ \angle CEB + 38^o = 74^o$, $\angle CEB = 36^o$. ***Ans.*** $\angle$ CBE is bigger.

9. $x^o + 85^o = 180^o$ (int. $\angle$s, AB//CD) , $\therefore\ x = 180 - 85 = 95$
$x^o + (x - 30)^o = y^o$ (vert. opp. $\angle$s) , $\therefore\ y = 95 + 95 - 30 = 160$

10. Produce ED to H. $\angle DHC = 59^o$ (alt. $\angle$s, AB//HE) ;
$\angle CDH = 180^o - 137^o = 43^o$ (adj. $\angle$s on st. line)
$q = 180^o - \angle DHC - \angle CDH$ ($\angle$ sum of Δ),
$\therefore\ q = 180^o - 59^o - 43^o = 78^o$



11. $\angle TSQ + 99^o = 112^o$ (alt.$\angle$s, TU//RS) , $\therefore\ \angle TSQ = 13^o$
$x^o + \angle TSQ = 101^o$ (alt.$\angle$s, PQ//SV) , $\therefore\ x = 101 - 13 = 88$

12. (a) $\angle ABE = 180^o - 83^o = 97^o$ (adj. $\angle$s on st. line)
$\because$ ABE $\neq$ DEH , $\therefore$ AC is not parallel to DF (corr. $\angle$s not equal)

(b) $\angle DBE = 180^o - 110^o - 23^o = 47^o$ (adj. $\angle$s on st. line)
$\because\ \angle DBE = 47^o = \angle BEC$, $\therefore$ DB//EC (alt. $\angle$s equal)

13. (a) $\angle RTS = 44^o$ (vert. opp. $\angle$s) , $\therefore\ 44^o + 62^o + x = 180^o$ ($\angle$ sum of Δ), $x = 74^o$

(b) $\because\ \angle RST = 74^o = \angle TPQ$, $\therefore$ PQ//RS (alt.$\angle$s equal)

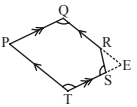
14. (a) $(2y - 11)^o + (3y + 26)^o = 180^o$ (adj. $\angle$s on st. line) , $\therefore\ 5y = 165$, $y = 33$

(b) $\angle DRS = (3 \times 33 + 26)^o = 125^o$, $\angle BQR = (158 - 33)^o = 125^o$,
$\therefore$ AB//CD (corr. $\angle$s equal)

15. If the corresponding angles are equal, then AB//CD, $\therefore\ \angle DCE = 3x + 19^o$
$\angle CDE = 68^o$ (alt. $\angle$s, BC//DE) , $\angle CDE + \angle DCE + (2x - 27^o) = 180^o$ ($\angle$ sum of Δ),
$\therefore\ 68^o + (3x + 19^o) + (2x - 27^o) = 180^o$, $5x = 120^o$, $x = 24^o$

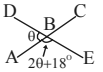
16. $\angle DAB = 47^o$ (alt.$\angle$s, AB//EF). $p + 55^o + \angle DAB = 180^o$ ($\angle$ sum of Δ)
$\therefore\ p + 55^o + 47^o = 180^o$, $p = 78^o$. $s = p = 78^o$ (corr. $\angle$s, CD//AB)
$q + s + 34^o = 180^o$ ($\angle$ sum of Δ) , $\therefore\ q + 78^o + 34^o = 180^o$, $q = 68^o$
$r + q = 180^o$ (int. $\angle$s, CD//EF) , $\therefore\ r + 68^o = 180^o$, $r = 112^o$

17. Produce QR and TS to meet at E.
$x + \angle E = 180^o$ (int. $\angle$s, PQ//TE) , $y + \angle E = 180°$ (int. $\angle$s, PT//QE)
$\therefore\ x = y$, $x : y = 1 : 1$



18. (a) Let DBA = θ.

(b) $\theta + (2\theta + 18^o) = 180^o$ (adj. $\angle$s on st. line) , $3\theta = 162^o$, $\theta = 54^o$
$\angle DBC = 2\theta + 18^o$ (vert. opp. $\angle$s), $\therefore\ \angle DBC = 2 \times 54^o + 18^o = 126^o$



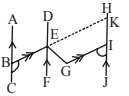
19. (a) Draw GE//CD. $\angle DEG = 180^o - 58^o = 122^o$ (int. $\angle$s, GE//CD)
$\angle BEG = 360^o - \angle DEG - 98°$ ($\angle$s at a pt.),
$\therefore\ \angle BEG = 360^o - 122^o - 98^o = 140^o$
$\because\ \angle BEG = 140^o = \angle FBA$, $\therefore$ AB//GE (corr. $\angle$s equal);
but GE//CD (construction) , $\therefore$ AB//CD

(b) $\angle CED = 360^o - 232^o - 64^o = 64^o$ ($\angle$s at a pt.)
$\angle CDE = 180^o - 73^o - \angle CED = 180^o - 73^o - 64^o = 43^o$ ($\angle$ sum of Δ)
$\therefore\ \angle CDB = 43° + 56° = 99°$. $\angle EBD = 180^o - 64^o - 56^o = 60^o$ ($\angle$ sum of Δ)

$\therefore$ $\angle ABD = 60° + 27° = 87°$. $\angle ABD + \angle CDB = 87° + 99° \neq 180°$

$\because$ The interior angles are not supplementary, $\therefore$ AB is not parallel to CD.

20. Produce BE to the point K on HJ. $\angle BKI = 180^o - x$ (int. $\angle$s, AC//HJ) ,
but $\angle BKI = y$ (corr. $\angle$s, BK//GI) , $\therefore$ $y = 180^o - x$



21. (a) $\angle PAB = b$ (alt. $\angle$s, PQ//BC) ; $\angle QAC = c$ (alt. $\angle$s, PQ//BC)
$\angle PAB + a + \angle QAC = 180°$ (adj. $\angle$s on st. line) , $\therefore$ $b + a + c = 180^o$,
i.e. the angle sum of a triangle is 180°.

(b) $\angle DCA = a$ (alt.$\angle$s, AB//DC). $\angle DCB + b = 180°$ (int. $\angle$s, AB//DC)
$\therefore$ $(\angle DCA + c) + b = 180°$, $a + c + b = 180°$,
i.e. the angle sum of a triangle is 180°.

22. $\angle BAC + \angle ACD = (p + 180^o - r) + q = (p + q) - r + 180^o = r - r + 180^o = 180^o$
$\therefore$ AB//CD (int. $\angle$s supp.)

23. (a) $(x + 10)^o = 2(x - 10)^o$ (alt. $\angle$s, AB//CD) , $\therefore$ $x + 10 = 2x - 20$, $x = 30$

(b) $\angle DCF = 360^o - 2(x - 10)^o - 255^o$ ($\angle$s at a pt.)
$\therefore$ $\angle DCF = 360^o - 2(30 - 10)^o - 255^o = 65^o$
$\angle CFE = (2x + 5)^o = (2 \times 30 + 5)^o = 65^o$
$\because$ $\angle CFE = 65^o = \angle DCF$, $\therefore$ CD//EF (alt. $\angle$s equal) ; but CD//AB , $\therefore$ AB//EF

24. $b = 2a + 24^o$ (corr. $\angle$s, AB//CD) ------ (i)
$b + (2b + 4a - 2^o) = 180^o$ (int. $\angle$s, DF//CE), $3b = 182^o - 4a$ ------ (ii)
Put (i) into (ii), $\therefore$ $3(2a + 24^o) = 182^o - 4a$, $6a + 72^o = 182^o - 4a$, $10a = 110^o$,
$\therefore$ $a = 11^o$, $b = 2(11^o) + 24^o = 46^o$

25. $x + y - 7^o + 135^o = 180^o$ ($\angle$ sum of Δ) , $\therefore$ $x + y = 52^o$ ------ (i)
$\angle DFE = 360^o - 135^o - 118^o = 107^o$ ($\angle$s at a pt.)
$\therefore$ $107^o + y + 2x - 1^o = 180^o$ ($\angle$ sum of Δ) , $2x + y = 74^o$ ------ (ii)
(ii) – (i), $\therefore$ $2x - x = 74^o - 52^o$, $x = 22^o$, $\therefore$ $y = 52^o - 22^o = 30^o$

26. $\angle EPN = 180^o - 137^o = 43^o$ (adj. $\angle$s on st. line)
$m = \angle EPN = 43^o$ (corr. $\angle$s, AB//EF). $n + 115^o = 180^o$ (int. $\angle$s, BC//DE), $\therefore$ $n = 65^o$

27. $\angle QPT = \frac{x}{2} + 60^o$ (corr. $\angle$s, PQ//WS). $\angle QPT + \angle PQS = 180°$ (int. $\angle$s, PT//QS)

$\therefore$ $(\frac{x}{2} + 60^o) + (84^o + x) = 180^o$, $\frac{3}{2}x = 36^o$, $\therefore$ $x = 24^o$

$\angle TYV = x = 24^o$ (corr.$\angle$s, PT//QS). $\angle TWU = \angle TYV = 24°$ (corr. $\angle$s, WU//YV)

$\therefore$ $15° + 24° + \frac{24}{2} + 60° + y = 180°$ (adj. $\angle$s on st. line), $y = 690°$

28. (a) (i) $\angle BQR = 35^o$ (angle of reflection). $\angle BRQ = 180^o - 108^o - 35^o$ ($\angle$ sum of Δ)
$\therefore$ $\angle BRQ = 37^o$, $\therefore$ $\angle CRS = 37^o$ (angle of reflection)

(ii) $\angle PQR = 180^o - 35^o - 35^o$ (adj. $\angle$s on st. line), $\therefore$ $\angle PQR = 110^o$
$\angle QRS = 180^o - \angle PQR$ (int. $\angle$s, QP//RS), $\therefore$ $\angle QRS = 180^o - 110^o = 70^o$
$\angle QRB + 70^o + CRS = 180^o$ (adj. $\angle$s on st. line),
but $\angle CRS = \angle QRB$ ($\angle$ of reflection), $\therefore$ $2\angle QRB + 70^o = 180^o$,

$\angle QRB = 55^o$ $\angle BQR = 35^o$ ($\angle$ of reflection),

$\therefore\ x = 180^o - 35^o - 55^o$ ($\angle$ sum of Δ), $x = 90°$

(b) $\angle BQR = a$ and $\angle BRQ = b$

$\therefore\ x + a + b = 180^o$ ($\angle$sum of Δ), $90^o + a + b = 180^o$, $a + b = 90^o$

$\angle PQR = 180^o - 2a$ and $\angle QRS = 180^o - 2b$ (adj. $\angle$s on st. line),

$\angle PQR + \angle QRS = (180^o - 2a) + (180^o - 2b) = 360^o - 2(a + b) = 360^o - 2(90^o) = 180^o$,

$\therefore$ QP//RS (int. $\angle$s supp.)

29. $(x + 27°) + (x + 80°) + (x + 73°) = 360°$ ($\angle$s at a pt.)

$3x + 180° = 360°$, $3x = 180°$, $x = 60°$. $\therefore\ \angle BOC = 60° + 73° = \underline{133°}$

30. (a) $64° \neq 74°$, L_1 is not parallel to L_3 (corr. $\angle$s not equal), $\therefore$ not true

(b) $x = 180° - 116°$ (adj. $\angle$s on st. line)

$= 64°$

$L_1 // L_4$ (corr, $\angle$s equal)

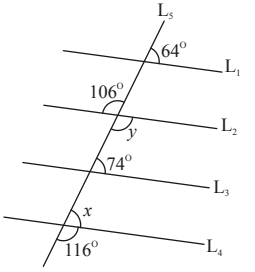
$\therefore$ true

(c) $y = 106°$ (vert. opp. $\angle$s)

$y + 74° = 106° + 74° = 180°$

$L_2 // L_3$ (int. $\angle$s supp.)

$\therefore$ true



Q. 30

31. $\angle AQR = y + 10°$ (alt. $\angle$s, EA // QR)

$\angle AQR + 4y = 180°$ (int. $\angle$s, AB // CD)

$y + 10° + 4y = 180°$, $5y = 170°$, $y = 34°$, $\therefore\ \angle EAB = 34° + 10° = \underline{\underline{44°}}$

32. Produce CD to cut AB at F.

$\angle CAB = 360° - 295°$ ($\angle$s at a pt.)

$= 65°$

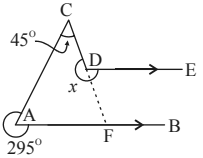
$65° + 45° + \angle CFA = 180°$ ($\angle$ sum of Δ)

$\therefore\ \angle CFA = 70°$

$\angle EDF = \angle CFA$ (alt. $\angle$s, DE // AB)

$= 70°$

$\therefore\ x = 180° + 70° = \underline{\underline{250°}}$



33. Through A, draw TW // PQ // RS.

$\angle TAP = \angle P$ (alt. $\angle$s, TW // PQ)

$\angle CAW = \angle TAP$ (vert. opp. $\angle$s)

$= \angle P$

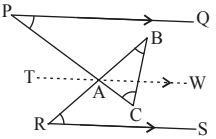
$\angle TAR = \angle R$ (alt. $\angle$s, TW // RS)

$\angle BAW = \angle TAR$ (vert. opp. $\angle$s)

$= \angle R$

$\angle CAW + \angle BAW + \angle B + \angle C = 180°$ ($\angle$ sum of Δ)

$\therefore\ \angle P + \angle R + \angle B + \angle C = \underline{\underline{180°}}$



34. (a) $\angle CBD + a = 180°$ (int. $\angle$s, AC // DE)

$\therefore\ \angle CBE + c + a = 180°$, $\angle CBE = 180° - a - c$

(b) $\angle ABD = a$ (int. $\angle$s, AC // DE)

(c) $\angle ABE = b$ (alt. $\angle$s, AC // DE)

$\therefore\ \angle ABD + c = b$, $\angle ABD = b - c$

(d) $\angle ABD = a$ and $\angle ABD = b - c$. $\therefore\ a = b - c$, $a + c = b$, yes, agree.

35. (a) $\angle BCG = 46°$ (vert. opp. $\angle$s)
$\therefore\ 37° + x + 46° = 180°$ ($\angle$ sum of Δ)
$x = 97°$
$y + x = 180°$ (int. $\angle$s, PQ // RS)
$\therefore\ y = 180° - 97° = \underline{\underline{83°}}$

(b) $\angle ADH = y = 83°$
$\angle DHI = 73° \neq \angle ADH$
$\therefore$ AE is not parallel to FI (alt. $\angle$s not equal)

36. (a) $\angle BQR + (5x + 14°) = 180°$ (int. $\angle$s, AB // RS)
$\therefore\ \angle BQR = 166° - 5x$

(b) $\angle BQF = \angle BQR + (3x + 3°)$
$= 166° - 5x + 3x + 3° = 169° - 2x$
$\angle BPD = 4x + 25°$ (vert. opps $\angle$s)
$4x + 25° = 169° - 2x$, $6x = 144°$, $x = \underline{\underline{24°}}$

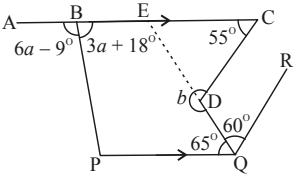
(c) $\angle TRQ + (5x + 14°) + 121° = 360°$ ($\angle$s at a pt.)
$\angle TRQ + 5 \times 24° + 14° + 121° = 360°$
$\therefore\ \angle TRQ = 360° - 255° = 105°$
$\angle RQF = 3x + 3° = 3 \times 24° + 3° = 75°$
$\angle TRQ + \angle RQF = 105° + 75° = 180°$
$\therefore$ EF // RT (int. $\angle$s supp.)

37. (a) $\angle CQB = \angle QCR$ (alt. $\angle$s, LQ // CF) ***Ans.*** $\angle CQB$.

(b) $\angle CQB = 45°$ (from (a))
$\angle BQR = 35° + \angle CQB = 35° + 45° = 80°$
$\angle BQP + \angle BQR = 180°$ (adj. $\angle$s on st. line)
$\therefore\ \angle BQP = 180° - 80° = 100°$
$\angle LBA = 100° = \angle BQP$
$\therefore$ AD is parallel to PS (corr. $\angle$s equal)

(c) $z = \underline{\underline{29°}}$ (alt. $\angle$s, AD // PS)
$x + (\angle CQB + 35°) + z = 180°$ ($\angle$ sum of Δ)
$\therefore\ x + 45° + 35° + 29° = 180°$, $x = 180° - 119° = \underline{\underline{61°}}$
$35° + y + z = 180°$ ($\angle$ sum of Δ)
$35° + y + 29° = 180°$, $y = 180° - 64° = \underline{\underline{116°}}$

38. (a) $(6a - 9°) + (3a + 18°) = 180°$ (adj. $\angle$s on st. line)
$9a = 171°$, $a = \underline{\underline{19°}}$



(b) Produce QD to meet AC at E.
$\angle CEQ = 65°$ (alt. $\angle$s, AC // PQ)
$\angle CDE + \angle CEQ + 55° = 180°$ ($\angle$ sum of Δ)
$\therefore\ \angle CDE = 180° - 55 - 65° = 60°$
$\therefore\ b = 180° + 60° = \underline{\underline{240°}}$

(c) $\angle CDQ + b = 360°$ ($\angle$s at a pt.)
$\angle CDQ = 360° - 240° = 120°$, $\angle CDQ + \angle RQD = 120° + 60° = 180°$

$\therefore$ CD // RQ (int. $\angle$s supp.)

(d) $\angle CEQ = 65°$ (from (b) , $\angle CBP = 3a + 18° = 3\,(19°) + 18° = 75° \neq \angle CEQ$

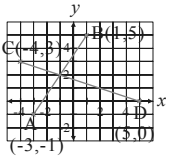
$\therefore$ BD is not parallel EQ (corr. $\angle$s not equal)

$\therefore$ BP is not parallel to DQ.

Unit 14: Introduction to coordinates

1. (a) D (b) A (c) F (d) B (e) E (f) C, G

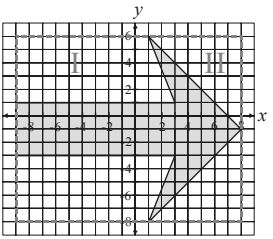
2. (a)



(b) (–1, 2)

3. (a) Distance = 6 – (– 2) = 8 units
   (b) Distance = 15 – 8 = 7 units
   (c) Distance = – 2 – (– 7) = 5 units
   (d) Distance = 9 – (– 4) = 13 units
   (e) Distance = $-3\frac{3}{4} - (-5\frac{1}{3}) = 1\frac{7}{12}$ units

Q. 5



4. Radius = $[2-(-10)] \div 2 = 6$ units

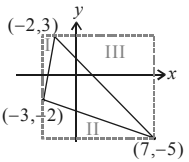
5. Area = Area of rectangle – 2 × (Area I + Area II)

$$= 14 \times 17 - 2 \times \left[\frac{(10+12)\times 5}{2} + \frac{7\times 7}{2}\right] = 238 - 159$$

= 79 sq. units

6. (a) Area = $\frac{8\times 7}{2}$ = 28 sq. units

Q. 6(b)



   (b) Area = Area of rectangle – Area I – Area II – Area III

$$= 10\times 8 - \frac{1\times 5}{2} - \frac{3\times 10}{2} - \frac{9\times 8}{2} = 80 - 2.5 - 15 - 36$$

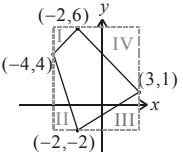
= 26.5 sq. units

   (c) Area = Area of rectangle – Area I – Area II – Area III – Area IV

$$= 7\times 8 - \frac{2\times 2}{2} - \frac{2\times 6}{2} - \frac{5\times 3}{2} - \frac{5\times 5}{2} = 56 - 2 - 6 - 7.5 - 12.5$$

= 28 sq. units

Q. 6(c)



7. (a) L (3, 0); M (7, 0); N (11, 0)

   (b) Area of ΔABC = Area of ALNC – Area of ALMB – Area of BMNC

$$= \frac{(4+9)\times 8}{2} - \frac{(4+5)\times 4}{2} - \frac{(5+9)\times 4}{2} = 52 - 18 - 28 = 6 \text{ sq. units}$$

8. (a) AB and CD are vertical.
   (b) Coordinates of H = (3, 2)
   (c) AB = 2 – (–1) = 3 , CD = 7 – (–2) = 9 , BH = 3 – (–5) = 8,

$$\therefore \text{ Area of trapezium} = \frac{(AB+CD)\times BH}{2} = \frac{(3+9)\times 8}{2} = 48 \text{ sq. units}$$

9. (a) Coordinates of B = (–4, –1); Coordinates of D = (3, 5)
   (b) AB = 3 – (–4) = 7, BC = 5 – (–1) = 6,

$\therefore$ Area of rectangle = AB × BC = 7 × 6 = 42 sq. units

10. PQ = 5 – 1 = 4, height of ΔPQR = $(2 - m)$ or $(m - 2)$.

When the height = $2 - m$, $\frac{(2-m)\times 4}{2} = 14$, $2 - m = 7$, $\therefore$ $m = -5$

When the height = $m - 2$, $\frac{(m-2)\times 4}{2} = 14$, $m - 2 = 7$, $\therefore$ $m = 9$

Ans. $m = -5$ or 9.

11. BC forms a vertical line, $\therefore$ x-coordinate of D = x-coordinate of A = 4.

BC = 2 – (–2) = 4, BC = AD = (y-coord. of A) – (y-coord. of D),

$\therefore$ y-coord. of D = y-coord. of A – BC = 3 – 4 = –1.

$\therefore$ Coordinates of D = (4, –1)

12. AB is a horizontal line, $\therefore$ AB = 1 – (–4) = 5.

Let h be height from C to AB, $\therefore$ $\frac{5\times h}{2} = 15$, $h = 6$.

In Quadrant I and Quadrant II, the y-coordinate of C = $h + 2 = 6 + 2 = 8$.

In Quadrant III and Quadrant IV, the y-coordinate of C = $2 - h = 2 - 6 = -4$.

On the other hand, the x-coordinate of C does not affect the area and can be any numbers.

Ans. In the 1st quadrant, the coordinates of C can be (1, 8).
In the 2nd quadrant, the coordinates of C can be (–4, 8).
In the 3rd quadrant, the coordinates of C can be (–7, –4).
In the 4th quadrant, the coordinates of C can be (6, –4).

13. (a) Height of ABCD = 2 – (–4) = 6, $\therefore$ BC = 48 ÷ 6 = 8 units

(b) x-coordinate of D = x-coordinate of A = –4,
y-coordinate of D = y-coordinate of A – BC = 5 – 8 = –3,
$\therefore$ coordinates of D = (–4, –3)

14. (a) The base XZ = 17 – (–8) = 25; the height = $k - 4$;

$\therefore$ $\frac{25\times(k-4)}{2} = 150$, $25(k - 4) = 300$, $k - 4 = 12$, $\therefore$ $k = 16$

(b) The base YZ = 15, $\frac{XY\times 15}{2} = 150$, $15XY = 300$, $\therefore$ XY = 20 units

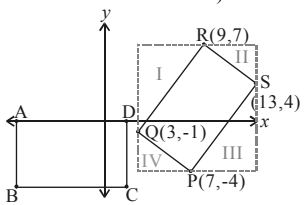
Perimeter of ΔXYZ = XY + YZ + XZ = 20 + 15 + 25 = 60 units

15. (a) Area of PQRS = Area of big rectangle – (Area I + Area II + Area III + Area IV)

$= 10\times 11 - (\frac{6\times 8}{2} + \frac{3\times 4}{2} + \frac{6\times 8}{2} + \frac{3\times 4}{2})$

= 110 – (24 + 6 + 24 + 6) = 50 sq. units

(b) DC = AB = 5, AD = BC = 50 ÷ 5 = 10,
coordinates of B = (–8, –5),
coordinates of C = (–8 + 10, –5) = (2, –5),
coordinates of D = (2, 0).



16. (a) QR is vertical and QR = 5 – (–3) = 8,

$\therefore$ Area of $\Delta OQR = \frac{8 \times 6}{2} = 24$ sq. units

Area of $\Delta PQR = \frac{1}{2}(8)[6 - (-2)] = 4(8) = 32$ sq. units.

$\therefore$ Area of $OPQ = 32 - 24 = 8$ sq. units

(b) Let $(0, y)$ be the coordinates of T, then area of $OTQR = \frac{(y+8) \times 6}{2} = 3y + 24$,

$\therefore\ 3y + 24 = 24(1 + 25\%),\ 3y = 6,\ y = 2.$

Ans. The coordinates of T are (0, 2).

17. Let B be $(0, -k)$, $\frac{1}{2}(4)(k) = 14$, $k = 7$. ***Ans.*** B(0, 7)

18. (a) D = (5, – 1) ; E = (5, 7) ; F = (–4, 7)

(b) The area of $\Delta ADB = \frac{1}{2}[5 - (-4)][4 - (-1)] = \frac{9 \times 5}{2}$ sq. units = 22.5 sq. units.

The area of $\Delta BEC = \frac{1}{2}(5 - 1)(7 - 4) = \frac{4 \times 3}{2}$ sq. units = 6 sq. units.

The area of $\Delta CFA = \frac{1}{2}[1 - (-4)][7 - (-1)] = \frac{5 \times 8}{2}$ sq. units = 20 sq. units.

(c) The area of ΔABC

$= [5 - (-4)][7 - (-1)] - (22.5 + 6 + 20) = 72 - 48.5 = 23.5$ sq. units > area of ΔADB.

$\therefore$ I do not agree.

19. (a) A(–2, 0), B(5, 0), D(–2, –4)

(b) $AB = 5 - (-2) = 7$, $CD = 3 - (-2) = 5$, $AD = 0 - (-4) = 4$

$\therefore$ area of $ABCD = \frac{1}{2}(7 + 5)(4) = 24$ sq. units

(c) EF is a horizontal line. $\therefore$ the y-coord. of F = –7

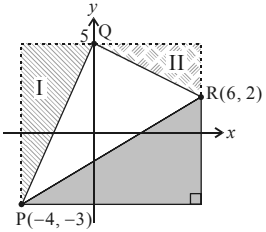
Let F be $(k, -7)$, $EF = k - (-2) = k + 2$, $DE = -4 - (-7) = 3$

$\frac{1}{2}[5 + (k + 2)](3) = 24$, $7 + k = 16$, $k = 9$. ***Ans.*** F(9, –7)

20. (a) Let S be $(6, k)$

$\frac{1}{2}[6 - (-4)](2 - k) = 25$, $5(2 - k) = 25$,

$2 - k = 5$, $k = -3$



(b) (i) Q(0, 5)

(ii) Area of $I = \frac{1}{2}(4)[5 - (-3)]$

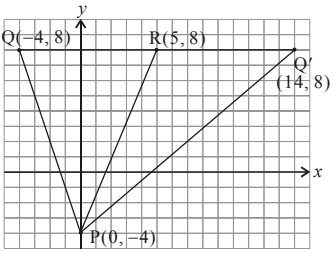
$= 2 + 8 = 16$ sq. units

Area of $II = \frac{1}{2}(6)[5 - 2] = 3 \times 3 = 9$ sq. units

$\therefore$ Area of $\Delta PQR = [5 - (-3)][6 - (-4)] - (25 + 16 + 9)$

$= 80 - 50 = 30$ sq. units.

21. (a) , (b) (ii)



(b) (i) QR = $5 - k$ or QR = $k - 5$

The height from P to QR = $8 - (-4) = 12$

When QR = $5 - k$:

$\frac{1}{2}(12)(5-k) = 54$, $5 - k = 9$, $k = \underline{\underline{-4}}$

When QR = $k - 5$:

$\frac{1}{2}(12)(k-5) = 54$, $k - 5 = 9$, $k = \underline{\underline{14}}$

(c) Area of ΔPRS = $\frac{1}{3}(54) = 18$ sq. units

Let S be $(5, a)$. The height from P to RS = 5.

When RS = $(8 - a)$:

$\frac{1}{2}(5)(8-a) = 18$, $8 - a = \frac{36}{5}$, $a = 8 - 7\frac{1}{5} = \frac{4}{5}$

When R = a − 8:

$\frac{1}{2}(5)(a-8) = 18$, $a - 8 = \frac{36}{5}$, $a = 8 + 7\frac{1}{5} = 15\frac{1}{5}$

Ans. The coordinate of S are $(5, \frac{4}{5})$ or $(5, 15\frac{1}{5})$.

22. (a) $m + 1 = 3m - 5$; $2m = 6$; $m = 3$

(b) A = (−2, 4); C = (6, 4)

The area of quadrilateral ABCD = $\left[\frac{(6+2)\times(6-4)}{2} + \frac{(6+2)\times(4+1)}{2}\right]$ sq. units

= 28 sq. units

23. (a) $6 - m = 1$; $m = 5$. ∴ B = $(-n, 5)$; C(−2, 1)

BC = $5 - 1 = 4$; CD = $(n + 3) - (-n) = 2n + 3$; $2[4 + (2n + 3)] = 22$;

$2n = 4$; $n = 2$. ***Ans.*** A(5, 5), B(−2, 5), C(−2, 1), D(5, 1)

(b) CD = $2(2) + 3 = 7$

The area of ABCD = BC × CD = 4 × 7 sq. units = 28 sq. units

(c) Let E be $(5, k)$, F be $(-2, k)$.

CF = 2BC, ∴ $1 - k = 2(4)$, $k = 1 - 8 = -7$.

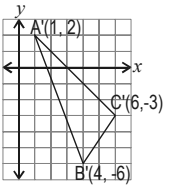
Ans. E(5, −7), F(−2, −7)

Unit 15: Transformation in the coordinate plane

1. (a) (–1, 2) (b) (14, –8)

2. (a) (2, 8) (b) (7, –9)

3. Coordinates of image = (9, 4),
   ∴ Distance = 9 – (–9) = 18 units

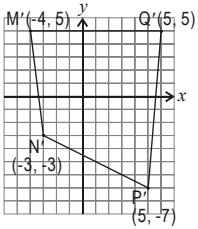
4. 

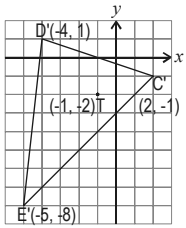
5. (a) Distance between (7, –1) and the horizontal line = 2 – (–1) = 3 units
   ∵ (7, –1) is below the line, ∴ y-coordinate of the image = –1 + 2(3) = 5,
   ∴ coordinates of the image = (7, 5)
   (b) Distance between the point and the vertical line = 4 – (–1) = 5 units
   ∵ (4, 6) is on the right of the line, ∴ x-coordinate of image = 4 – 2(5) = –6,
   ∴ coordinates of the image = (–6, 6)

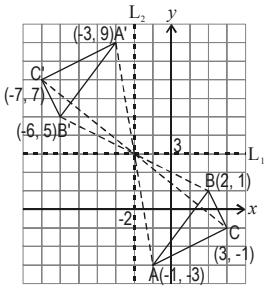
6. Rotating clockwise 270° is the same as rotating anti-clockwise 90°.
   ∴ Coordinate of A′ = (3, 1) , coordinate of B′ = (6, –6) , coordinate of C′ = (–3, –4).

7. Coordinates of P′ = (5, 3), coordinates of Q′ = (4, –1), coordinates of R′ = (–1, 2)

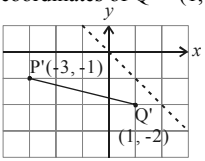
8. L_1 is a horizontal line with y-coordinate = –3.
   After reflecting along L_1:
   the x-coordinate of A's image = 1 – 2[1 – (–3)] = 1 – 2(4) = –7,
   the x-coordinate of B's image = 5 – 2[5 – (–3)] = 5 – 2(8) = –11,
   the x-coordinate of B's image = 3 – 2[3 – (–3)] = 3 – 2(6) = –9.
   ∴ After reflection along L_1, the images of A, B, C are (–5, –7), (–1, –11) and (3, –9) respectively.
   L_2 is a vertical line with x-coordinate = 4.
   After reflection along L_2 :
   coordinates of A′ = (–5 + 2×[4 – (–5)], –7) = (13, –7),
   coordinates of B′ = (–1 + 2×[4 – (–1)], –11) = (9, –11) ,
   coordinates of C′ = (3 + 2×[4 – 3], –9) = (5, –9).

9. 

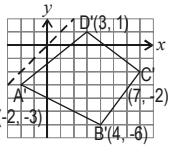
10. 

11. 

12. Coordinates of P′ = (–3, –1),
coordinates of Q′ = (1, –2)



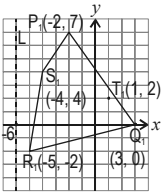
13. Coordinates of A′ = (–2, –3),
coordinates of B′ = (4, –6),
coordinates of C′ = (7, –2),
coordinates of D′ = (3, 1)



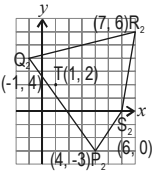
14. (a) Coordinates of $P_1 = (7, –6)$, coordinates of $P_2 = (6, 7)$,
coordinates of $P_3 = (–7, 6)$, coordinates of $P_4 = (–6, –7)$.

(b) From (a), coordinates of P_n will repeat with a period of 4,
i.e. $P_0 = P_4 = P_8 = \ldots$, $P_1 = P_5 = P_9 = \ldots$, $P_2 = P_6 = P_{10} = \ldots$, $P_3 = P_7 = P_{11} = \ldots$,
$\because$ $101 = 4(25) + 1$, $\therefore$ coordinates of P_{101} = coordinates of $P_1 = (7, –6)$

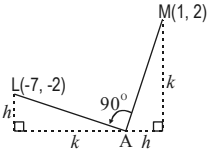
15. (a)



(b)



16. Let h= vertical distance between L and A = horizontal distance between M and A
Let k= horizontal distance between L and A = vertical distance between M and A
x-coordinate of A = $–7 + k = 1 – h$, $\therefore$ $k = 8 – h$
y-coordinate of A = $–2 – h = 2 – k$, $\therefore$ $k = 4 + h$
$8 – h = 4 + h$ ($\because$ $= k$) , $4 = 2h$, $h = 2$, $\therefore$ $k = 8 – 2 = 6$.
$\therefore$ x-coordinate of A = $–7 + 6 = –1$,
y-coordinate of A = $–2 – 2 = –4$,
Ans. Coordinates of A are (–1, –4).



17. (a) (i) B = (3, 5) , C = (3, –5) , D = (–3, –5)
ABCD is a rectangle.

(ii) The perimeter of ABCD = [(3 + 3) + (5 + 5)] × 2 units = 32 units.
The area of ABCD = (3 + 3) × (5 + 5) sq. units = 60 sq. units.

(b) The distance from A′ or D′ to L = –3 – (–5) = 2 units
The distance from B′ or C′ to L = –3 – (–5) = 8 units
$\therefore$ Coordinate of A′ = (–5 – 2, 5) = (–7, 5)
Coordinate of B′ = (–5 – 8, 5) = (–13, 5)
Coordinate of C′ = (–5 – 8, –5) = (–13, –5)
Coordinate of D′ = (–5 – 2, 5) = (–7, –5)

18. (a) Rotating A′B′C′D′ anti-clockwise 270° to ABCD, is the same as rotating ABCD clockwise 90° to A′B′C′D′.
$\therefore$ Coordinate of A′ = (3 , 2) , coordinate of B′ = (7 , 2) ,
coordinate of C′ = (7 , –6) , coordinate of D′ = (3 , –6) ,

(b) AB = 7 – 3 = 4 unit , BC = 6 – (–2) = 28 units
∴ Perimeter of ABCD = 2 × (4 + 8) = 24 units
Perimeter of A′B′C′D′ = Perimeter of ABCD = 24 units

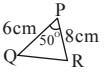
19. (a) Parallelogram
(b) Coordinate of P′ = (–3 – 3, 9 – 2) = (–6, 7)
Coordinate of Q′ = (8 + 3, 9 – 2) = (11, 7)
Coordinate of R′ = (6 + 3, –1 + 2) = (9, 1)
Coordinate of S′ = (–5 – 3, –1 + 2) = (–8, 1)
(c) The base of PQRS = 8 – (–6) = 11. The height of PQRS = 9 – (–1) = 10.
∴ Area of PQRS = 10 × 11 = 110 sq. units
The base of P′Q′R′S′ = 11 – (–3) = 17. The height of P′Q′R′S′ = 7 – 1 = 6.
∴ Area of P′Q′R′S′ = 17 × 6 = 102 sq. units
∵ 102 < 110 , ∴ yes, I agree.

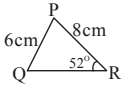
20. (a) $2m + 4 = -(-m - 5)$, $2m + 4 = m + 5$, $m = 1$, $n + 2 = 4$, $n = 2$
∴ $m = 1$ and $n = 2$
(b) P = (–6, 4) ; P' = (6, 4) ; Q = (4, 6) , Q' = (–4, 6). PP'QQ' is a trapezium.
The area of PP'QQ' = $\frac{(8+12)\times 2}{2}$ sq. units = 20 sq. units

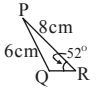
21. (a) coordinate of A_1 = (3, –5) , coordinate of B_1 = (4, –2) ,
coordinate of C_1 = (0, –4)
(b) coordinate of A_2 = (–3, –5) , coordinate of B_2 = (–4, –2) ,
coordinate of C_2 = (0, –4)
(c) $\Delta A_2B_2C_2$: (–3, –5) , (–4, –2) , (0, –4)
ΔABC : (–3, 5) , (–4, 2) , (0, 4)
All their corresponding vertices have the same *x*-coordinates, but *y*-coordinates of opposite signs. Reflecting $\Delta A_2B_2C_2$ about the *x*-axis will have ΔABC as its image.
∴ Yes, it is possible.

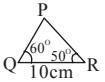
Appendix 1: Simple congruent triangles

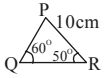
1. (a) $\Delta ABC \cong \Delta PRQ$ (b) $\Delta XYZ \cong \Delta LMN$
2. (a) $a = 55^\circ$, $b = 11$ (b) $x = 32^\circ$, $y = 5$
(c) $x = 90^\circ$, $y = 20$ (d) $a = 180^\circ - 35^\circ - 28^\circ = 117^\circ$, $b = 8.5$
3. (a) 14cm (b) 17cm (c) 8cm
4. (a) 53° (b) $\angle X = 180^\circ - 53^\circ - 66^\circ = 61^\circ$ (c) 61°
5. (a) yes, $\Delta ABC \cong \Delta ZXY$ (SAS) (b) yes, $\Delta LMN \cong \Delta ZXY$ (SSS)
(c) no (d) yes, $\Delta XYZ \cong \Delta LMN$ (AAS)
(e) yes, $\Delta ABC \cong \Delta EFD$ (RHS) (f) no
6. (a) $\Delta PQR \cong \Delta LMN$ (SAS) (b) $\Delta ABC \cong \Delta YXZ$ (RHS)
(c) $\Delta PQR \cong \Delta FED$ (AAS) (d) $\Delta MNL \cong \Delta YXZ$ (ASA)

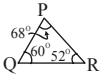
7. (a) $\Delta PQR \cong \Delta PQS$ (ASA) (b) $\Delta ABC \cong \Delta ABD$ (RHS)
   (c) $\angle EGF = 180° - 70° - 60° = 50°$, $\angle GHF = 180° - 60° - 50° = 70°$
   $\therefore\ \Delta EFG \cong \Delta HGF$ (ASA)
   (d) $\Delta OAB \cong \Delta OXY$ (or: ΔOYX) (SAS)
8. (a) $\Delta ABC \cong \Delta ZYX$ (RHS) (b) $\Delta PQR \cong \Delta XYZ$ (or: ΔYXZ) (SAS)
   (c) $\Delta PQR \cong \Delta ZYX$ (AAS)
9. (a) 

No, the two triangles will be congruent. (∵ SAS)
   (b) 

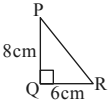
or 

   (c) 

No, the two triangles will be congruent. (∵ ASA)
   (d) 

No, the two triangles will be congruent. (∵ AAS)
   (e) 

   (f) 

No, the two triangles will be congruent. (∵ RHS)
   (g) 

No, the two triangles will be congruent. (∵ SAS)
10. $\Delta DAB \cong \Delta DEC$ (RHS)
11. $QP = QR$, $PM = RM$, $QM = QM$, $\therefore\ \Delta PQM \cong \Delta RQM$ (SSS)
12. $QP = QR$, $QM = QM$, $\angle QMP = 90° = \angle QMR$, $\therefore\ \Delta PQM \cong \Delta RQM$ (RHS)
13. $AB = CB$ ($\therefore\ AH + HB = CK + KB$), $BK = BH$, $\angle ABK = \angle CBH$,
    $\therefore\ \Delta AKB \cong \Delta CHB$ (SAS)
14. $\angle CDE = \angle FED$ ($\because\ \angle CDF + \angle FDE = \angle FEC + \angle CED$),
    $\angle CED = \angle FDE$, $DE = ED$, $\therefore\ \Delta CED \cong \Delta FDE$ (ASA)
15. $\angle PNQ = 90° = \angle PMR$, $\angle QPN = \angle RPM$, $NP = MP$, $\therefore\ \Delta RMP \cong \Delta QNP$ (ASA)
16. $\angle BAD = 60° = \angle CAE$, $BA = CA$, $AD = AE$, $\therefore\ \Delta BAD \cong \Delta CAE$ (SAS)
17. (a) $\angle CDE + \angle CDF = 90°$, $\angle CDE + \angle ADE = 90°$, $\therefore\ \angle CDF = \angle ADE$
    (b) $AD = CD$, $\angle EAD = 90° = \angle FCD$, $\angle ADE = \angle CDF$ [from (a)],
    $\therefore\ \Delta AED \cong \Delta CFD$ (ASA)